New modified Inverted Weibull Distribution: Properties and Applications to COVID-19 Dataset of Nepal

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Abstract:

We have developed a four parameters new modified inverted Weibull distribution and we named it exponentiated exponential inverted Weibull distribution. Linear representation of probability density function, reliability function, hazard function, moments about the origin and its generating function, mean residual life function, order statistics, two entropies namely Renyi and q-Entropy, and mean deviation for the proposed distribution are presented. For the parameter estimation the maximum likelihood, least-square, and Cramer-Von-Mises estimation methods are used. The application of the proposed distribution is analyzed using the deaths case of the COVID-19 dataset of Nepal from 1st April to 14th May 2021.

Keywords: *Entropy, COVID-19, Inverted Weibull, Mean Deviation ,Moment.*

Introduction

Statistical models have a crucial role in modeling real-life datasets related to engineering, medicine, life sciences, etc. However classical probability models are not sufficient to fit the various varieties of the real datasets. Hence we need a flexible modified model that can address the deficiencies of the classical distributions. In this study, we have proposed a new distribution by modifying the two parameters inverted Weibull (IW) distribution. The IW distribution has been utilized to study the failure of mechanical components, parts, or a system (Murthy et al., 2004; Erto & Rapone, 1984; Calabria & Pulcini, 1994). The cumulative distribution function (CDF) and probability density function (PDF) of IW distribution with scale parameter β and shape parameter δ is

$$G\left(x;\beta,\delta\right)=\exp\left(-\beta x^{-\delta}\right);\ x>0,\beta>0,\delta>0 \tag{1.1}$$

$$g\left(x;\beta,\delta\right)=\ \beta\delta x^{-\delta}\ \exp\left(-\beta x^{-\ \delta}\right);\ x>0 \tag{1.2}$$

The IW distribution can have unimodel and decreasing hazard function this indicates that the log-normal and IW are quite similar. It has heavy-tailed distribution at the right. In the literature, we can observe that many authors have used several modified forms of IW distribution. Some of them are inverse flexible Weibull extension distribution is used by (El-Gohary et al., 2015), three parameters IW (Khan et al., 2008), generalized IW distribution (De Gusmao et al., 2011), Kumaraswamy-inverse Weibull distribution (Shahbaz et al., 2012), four parameters modified IW (Khan & King, 2012), beta IW distribution (Khan, 2010), extended IW (Okasha, 2017), transmuted IW distribution (Khan et al., 2013), Lindley inverse Weibull distribution (Joshi & Kumar, 2020), Logistic IW distribution (Chaudhary & Kumar, 2020). Using two parameters IW distribution we have introduced a new probability model and we named it exponentiated exponential inverted Weibull (EEIW) distribution. Using the similar approach (Sapkota, 2020) has defined exponentiated exponential logistic distribution. This proposed model was generated using the beta exponentiated-X family of distribution introduced by (Alzaatreh et al., 2013) with PDF and CDF as

$$f\left(x;\lambda,\theta,\omega\right)=\frac{\lambda}{\mathrm{B}(\theta,\omega)}g\left(x\right)\left[1-G\left(x\right)\right]^{\lambda\omega-1}\left[1-\left[1-G\left(x\right)\right]^{\lambda}\right]^{\theta-1};x>0 \tag{1.3}$$

and $$F\left(x;\lambda,\theta,\omega\right)=1-I_{\left[1-G\left(x\right)\right]^{\lambda}}\left[\lambda\left(\omega-1\right)+1,\theta\right] \tag{1.4}$$

The prime objective of this work is to suggest a more flexible model for analyzing COVID-19 pandemic data. We have illustrated some prime properties of the EEIW distribution and illustrate its applicability. The different sections of the proposed study are arranged as follows. The new EEIW distribution is introduced and various distributional properties are discussed in Section 2. Some prime properties of the proposed model are presented in section 3. To estimate the model parameters, we have employed three well-known estimation methods namely maximum likelihood estimation (MLE), least-square estimation (LSE), and Cramer-Von-Mises (CVM) methods in Section 4. In Section 5 we have considered the deaths case of COVID-19 real dataset to analyze and explore the applications of the proposed distribution. Finally, Section 6 ends up with some general concluding remarks.

New distribution

Inserting equations (1.1) and (1.2) in equations (1.3) and (1.4) and setting $\omega=1$ we get the CDF and PDF of exponentiated exponential inverted Weibull distribution with parameter space $\Theta=\left(\beta,\delta,\lambda,\theta\right)$ as

$$F\left(x;\Theta\right)=\left[1-\left\{1-\exp\left(-\beta x^{-\delta}\right)\right\}^{\lambda}\right]^{\theta};x>0,\Theta>0 \tag{2.1}$$

$$f\left(x;\Theta\right)=\beta\delta\lambda\theta x^{-(\delta+1)}\exp\left(-\beta x^{-\delta}\right)\left\{1-\exp\left(-\beta x^{-\delta}\right)\right\}^{\lambda-1}\left[1-\left\{1-\exp\left(-\beta x^{-\delta}\right)\right\}^{\lambda}\right]^{\theta-1} \tag{2.2}$$

Reliability function $X\sim EEIW\left(\Theta\right)$ is

$$R(x)=1-F(x)$$

$$=1-\left[1-\left\{1-\exp\left(-\beta x^{-\delta}\right)\right\}^{\lambda}\right]^{\theta};x>0,\Theta>0. \tag{2.3}$$

Hazard rate function (HRF)

The HRF $EEIW\left(\Theta\right)$ can be obtained as

$$h(x)=\frac{f(x)}{R(x)}$$

$$=\frac{\beta\delta\lambda\theta x^{-(\delta+1)}Z(x)\left\{1-Z(x)\right\}^{\lambda-1}\left[1-\left\{1-Z(x)\right\}^{\lambda}\right]^{\theta-1}}{1-\left[1-\left\{1-Z(x)\right\}^{\lambda}\right]^{\theta}}. \tag{2.4}$$

where $Z(x) = \exp\left(-\beta x^{-\delta}\right)$.

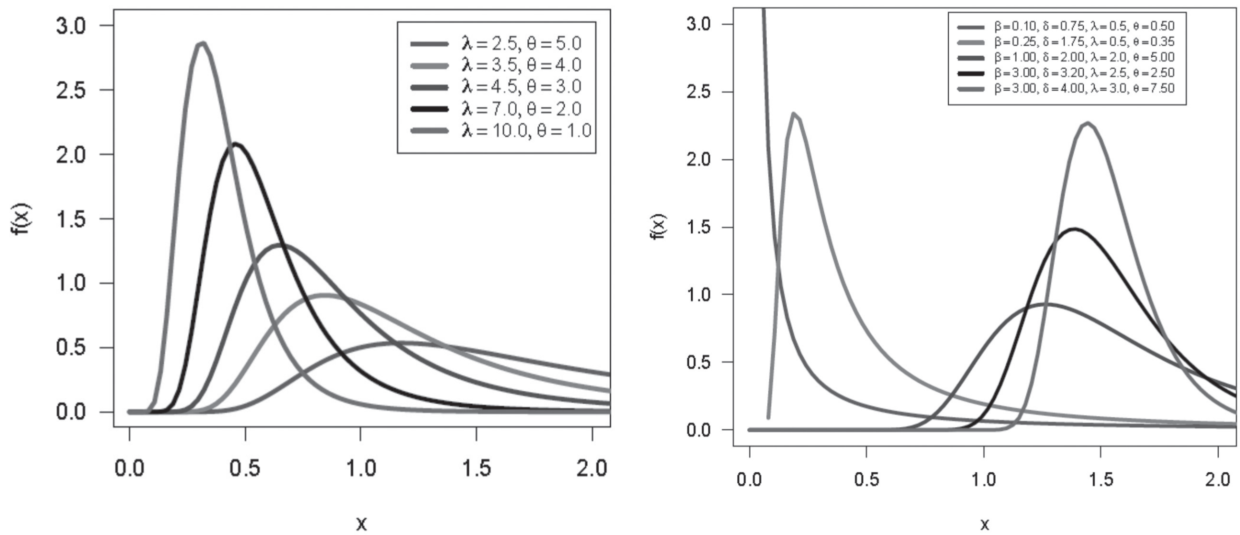

Figure 1 PDF plots for λ *and* θ keeping constant β *and* δ parameters (left) and for various values of all parameters (right).

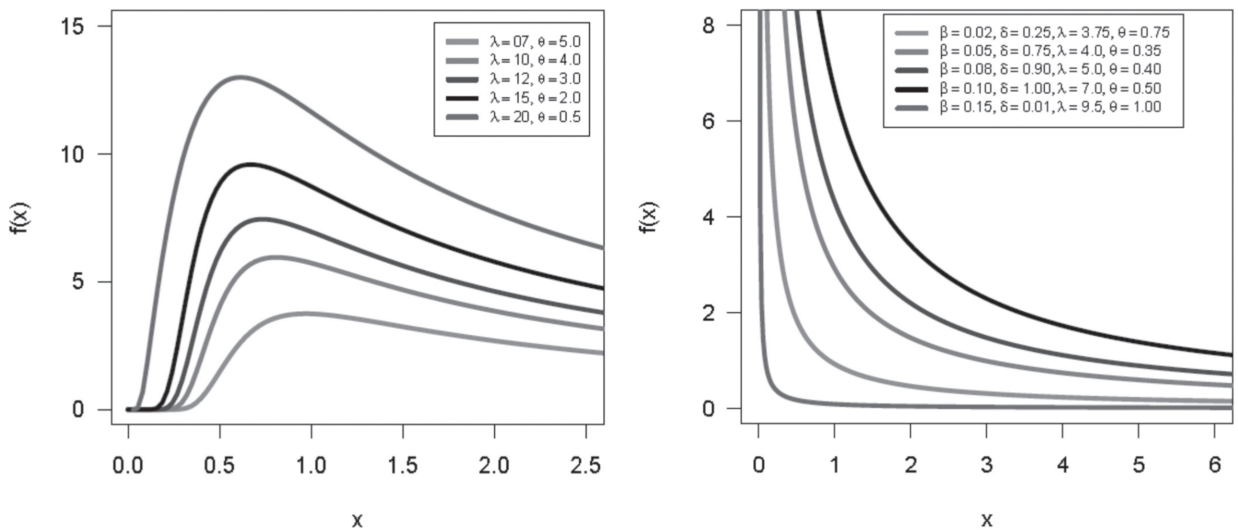

Figure 2 plots of hazard function for λ *and* θ keeping constant β *and* δ parameters (left) and for various values of all parameters (right).

Properties of EEIW distribution

Quantile function (QF)

The QF can be obtained by taking the inverse function of (2.1) as

$$Q(y) = F^{-1}(y)$$

Hence QF is obtained as,

$$Q(y) = \left[-\frac{1}{\beta}\log\left\{1-\left(1-y^{\frac{1}{\theta}}\right)^{1/\lambda}\right\}\right]^{-1/\delta} \quad ;0 < y < 1 \tag{3.1}$$

where y is the uniform random variable of U (0,1).

The random numbers can be generated for EEIW distribution using (3.1) as

$$x=\left[-\frac{1}{\beta}\log\left\{1-\left(1-u^{\frac{1}{\theta}}\right)^{1/\lambda}\right\}\right]^{-1/\delta} \quad ;0<u<1.$$

Linear Representation

Using the binomial expansion $(1-w)^n=\sum_{i=0}^{\infty}(-1)^i\binom{n}{i}w^i;\quad |w|<1$ we can express the PDF of EEIW as the mixture of two parameters IW distribution as

$$f(x)=\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}W_{ij}\,x^{-(\delta+1)}\exp\left(-\beta(j+1)\,x^{-\delta}\right) \tag{3.2}$$

$$W_{ij}=\beta\delta\lambda\theta(-1)^{i+j}\binom{\theta-1}{i}\binom{\lambda i+\lambda-1}{j}$$

Now (3.2) can be expressed as

$f(x)=\sum_{j=0}^{\infty}W_j\; f^*(x;\beta(j+1),\delta)$ where $f^*(x;\beta(j+1),\delta)$ is the PDF of two-parameter IW distribution with scale parameter $\beta(j+1)$ and shape parameter δ and

$$W_j=\frac{\lambda\theta}{(j+1)!}\sum_{i=0}^{\infty}(-1)^{i+j}\binom{\theta-1}{i}\frac{(\lambda i+\lambda-1)!}{(\lambda i+\lambda-1-j)!}$$

Moments

The r[th] moment about the origin of EEIW distribution using (3.2) can be obtained as

$$\mu_r^{'}=\int_0^{\infty}x^r f(x)\,dx$$

$$=\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}W_{ij}\int_0^{\infty}x^{k-(\delta+1)}\exp\left(-\beta(j+1)x^{-\delta}\right)dx$$

$$=\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}W_{ij}\left(\frac{-1}{\delta}\right)\frac{\Gamma\left((\delta-k)/\delta\right)}{\left[\beta(j+1)\right]^{1-\frac{k}{\delta}}} \tag{3.3}$$

where $\int_0^{\infty}x^n e^{-ax}dx=\frac{\Gamma(n+1)}{a^{n+1}}$ is a standard gamma integral. Hence mean and variance of EEIW are

$$\text{Mean}=\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}W_{ij}\left(\frac{-1}{\delta}\right)\frac{\Gamma\left((\delta-1)/\delta\right)}{\left[\beta(j+1)\right]^{1-\frac{1}{\delta}}}$$

$$Variance = \mu_2^{'} - (Mean)^2$$

$$= \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} W_{ij} \left(\frac{-1}{\delta} \right) \frac{\Gamma\left((\delta-2)/\delta \right)}{\left[\beta(j+1) \right]^{2-\frac{1}{\delta}}} - \left[\sum_{i=0}^{\infty} \sum_{j=0}^{\infty} W_{ij} \left(\frac{-1}{\delta} \right) \frac{\Gamma\left((\delta-1)/\delta \right)}{\left[\beta(j+1) \right]^{1-\frac{1}{\delta}}} \right]^2$$

Moment Generating Function (MGF)

Let $X \sim EEIW(\Theta)$, then the MGF of X can be defined using (3.3) as

$$M_x(t) = E\left(e^{tx}\right) = \sum_{l=0}^{\infty} \frac{t^l}{l!} \mu_r^{'}$$

$$= \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{m=0}^{\infty} W_{ij} \left(\frac{-1}{\delta} \right) \frac{t^m}{m!} \frac{\Gamma\left((\delta-k)/\delta \right)}{\left[\beta(j+1) \right]^{1-\frac{k}{\delta}}}$$

Mean Residual Life Function

The mean residual life function of EEIW can be defined as

$$\Lambda(x) = \frac{\int_t^{\infty} x f(x) dx}{1-F(x)} - x$$

$$= \frac{\sum_{i=0}^{\infty} \sum_{j=0}^{\infty} W_{ij} \left(\frac{-1}{\delta} \right) \Gamma\left((\delta-1)/\delta, \beta(j+1)x \right)}{\{1-F(x)\} \left[\beta(j+1) \right]^{1-\frac{1}{\delta}}} - x$$

here $\int_x^{\infty} t^a e^{-bt} dt = \frac{\Gamma(a+1, bt)}{b^{a+1}}$ is the upper incomplete gamma function.

Entropies

Rényi entropy

The Rényi entropy for EEIW distribution can be defined as

$$D_\gamma(X) = \frac{1}{1-\gamma} \log \int_{-\infty}^{\infty} \left[f(x) \right]^\gamma dx; \quad \gamma > 0 \text{ and } \gamma \neq 1. \tag{3.4}$$

After expanding $\left[f(x) \right]^\gamma$ (3.4) can be expressed as

$$D_\gamma(X) = \frac{1}{1-\gamma} \log \left[\sum_{i=0}^{\infty} \sum_{j=0}^{\infty} W_{ij}^* \left(\frac{-1}{\delta} \right) \frac{\Gamma\left(1-\lambda(\delta+1) \right)}{\left[\beta(j+\gamma) \right]^{1-\lambda(\delta+1)}} \right]. \tag{3.5}$$

where, $W_{ij}^* = (\beta\delta\lambda\theta)^\gamma (-1)^{i+j} \binom{\gamma(\theta-1)}{i} \binom{\lambda i + \gamma(\lambda-1)}{j}$

The q-entropy is defined by

$$P_\gamma(X)=\frac{1}{1-\gamma}\log\left(1-\int_{-\infty}^{\infty}\left[f(x)\right]^\gamma dx\right),\quad \gamma>0 \text{ and } \gamma\neq 1.$$

And using (3.5) q-entropy takes the form

$$P_\gamma(X)=\frac{1}{1-\gamma}\log\left(1-\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}W_{ij}^*\left(\frac{-1}{\delta}\right)\frac{\Gamma\left(1-\lambda(\delta+1)\right)}{\left[\beta(j+\gamma)\right]^{1-\lambda(\delta+1)}}\right).$$

Mean Deviation

The mean deviation taken from the mean can be obtained as

$$Z(\mu)=E|X-\mu|$$

$$=2\mu F(\mu)-2\mu+\int_{\mu}^{\infty}xf(x)dx$$

$$=2\mu F(\mu)-2\mu+\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}W_{ij}\left(\frac{-1}{\delta}\right)\frac{\Gamma\left((\delta-1)/\delta,\beta(j+1)\mu\right)}{\left[\beta(j+1)\right]^{1-\frac{1}{\delta}}}$$

Similarly mean deviation from the median is

$$Z(M_d)=E|X-M_d|$$

$$=M_dF(M_d)-M_d-\mu+2\int_{M_d}^{\infty}x\,f(x)dx$$

$$=M_dF(M_d)-M_d-\mu+2\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}W_{ij}\left(\frac{-1}{\delta}\right)\frac{\Gamma\left((\delta-1)/\delta,\beta(j+1)M_d\right)}{\left[\beta(j+1)\right]^{1-\frac{1}{\delta}}}$$

here μ and M_d are the values of the mean and median.

Order statistics

Let $x_1,\ldots,x_n$ be independently and identically distributed random variables with their corresponding CDF F(x). Let $x_{(1)}<\ldots<x_{(n)}$ be the corresponding order of a random sample from a population of size n then the PDF of t^{th} order statistic is

$$f_{X_{(t)}}(x_t)=\frac{f(x_{(t)})}{\mathrm{B}(t,n-t+1)}\sum_{k=0}^{n-t}(-1)^k\binom{n-t}{l}\left[F(x_{(r)})\right]^{t+l-1}$$

where B (.) is the beta function. Now the PDF of order statistics is

$$f_{X_{(t)}}(x_t)=\frac{f(x_{(t)})}{\mathrm{B}(t,n-t+1)}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{n-t}W_{ijk}^{'}e^{-\beta jx^{-\delta}}$$

here $W_{ijk}^{'}=(-1)^{i+j+k}\binom{\lambda i}{j}\binom{n-t}{l}\binom{\theta(t+k-1)}{i}.$

Estimation Methods

Maximum Likelihood Estimation (MLE) Method

To estimate the parameters of the EEIW distribution, we have used the MLE method (Casella & Berger, 1990). Let, $x_1,...,x_n$ be a random sample from $EEIW\left(\Theta\right)$, and the likelihood function, $L\left(\beta,\delta,\lambda,\theta\right)$ is given by,

$$L\left(\Theta;x_1,...,x_n\right)=f\left(x_1,...,x_n/\Theta\right)=\prod_{i=1}^{n}f(x_i/\Theta)$$

$$L\left(x;\Theta\right)=\left(\beta\delta\lambda\theta\right)^n\prod_{i=1}^{n}x_i^{-(\delta+1)}\exp\left(-\beta x_i^{-\delta}\right)\left\{1-\exp\left(-\beta x_i^{-\delta}\right)\right\}^{\lambda-1}\left[1-\left\{1-\exp\left(-\beta x_i^{-\delta}\right)\right\}^{\lambda}\right]^{\theta-1} \quad (4.1)$$

Now log-likelihood density of (4.1) is

$$\ell=n\log\left(\beta\delta\lambda\theta\right)-\left(\delta+1\right)\sum_{i=1}^{n}\log(x_i)-\beta\sum_{i=1}^{n}x_i^{-\delta}+\left(\lambda-1\right)\sum_{i=1}^{n}\log\left\{1-\exp\left(-\beta x_i^{-\delta}\right)\right\}$$
$$+\left(\theta-1\right)\sum_{i=1}^{n}\log\left[1-\left\{1-\exp\left(-\beta x_i^{-\delta}\right)\right\}^{\lambda}\right] \quad (4.2)$$

To get the MLEs of the proposed distribution analytically we have to maximize (4.2) with respect to model parameters.

Method of Least-Square Estimation (LSE)

The ordinary least square estimates for the EEIW distribution can be calculated by minimizing

$$K\left(X;\Theta\right)=\sum_{i=1}^{n}\left[F(X_i;\Theta)-\frac{i}{n+1}\right]^2 \quad (4.3)$$

with respect to unknown parameters $\Theta=\left(\beta,\delta,\lambda,\theta\right)$.

Consider $F(X_i)$ denotes the distribution function of the ordered random variables $X_{(1)}<\ \ldots<X_{(n)}$ where $\{X_1,\ldots,X_n\}$ is a random sample of size n from a distribution function F(.). The least-square estimators of $\Theta=\left(\beta,\delta,\lambda,\theta\right)$ can be obtained by minimizing the following equation

$$K\left(X;\Theta\right)=\sum_{i=1}^{n}\left[\left[1-\left\{1-\exp\left(-\beta x_i^{-\delta}\right)\right\}^{\lambda}\right]^{\theta}-\frac{i}{n+1}\right]^2. \quad (4.4)$$

Method of Cramer-Von-Mises estimation (CVME)

The Cramer-Von-Mises estimates $\Theta=\left(\beta,\delta,\lambda,\theta\right)$ are obtained by minimizing the function

$$W\left(X;\Theta\right)=\frac{1}{12n}+\sum_{i=1}^{n}\left[F\left(x_{i:n}\mid\Theta\right)-\frac{2i-1}{2n}\right]^2$$
$$=\frac{1}{12n}+\sum_{i=1}^{n}\left[\left[1-\left\{1-\exp\left(-\beta x_i^{-\delta}\right)\right\}^{\lambda}\right]^{\theta}-\frac{2i-1}{2n}\right]^2 \quad (4.5)$$

Illustration with the death case of COVID-19 data

In this section, we have considered the real data set related to the number of death that occurred during the period of 1st April to 14th May 2021 daily in Nepal and only the days taken at which the number of deaths was greater than or equal to 1 (Tharu et al., 2021). The dataset is

1, 1, 4, 2, 1, 1, 13, 5, 3, 5, 4, 5, 8, 8, 11, 10, 5, 5, 14, 28, 12, 18, 17,
35, 33, 19, 27, 37, 55, 58, 54, 50, 53, 88, 139, 225, 168, 214, 203

Exploratory study of Dataset

From the exploratory study, we observed that the minimum number of death is 1, the maximum is 225 and the average death per day is 43 during the period (Table 1). The clear picture of the dataset is depicted by plotting the histogram and TTT (Total time on test) plot to view the actual shape of the hazard function. The TTT plot is convex which indicates the increasing HRF of the data.

Table 1

Some descriptive statistics of the dataset

Summary statistic	Min.	1st Quartile	Median	Mean	3rd Quartile	Max.	Skewness	Kurtosis
No. of Deaths	1.00	5.00	14.00	42.03	51.50	225.00	1.8733	2.3077

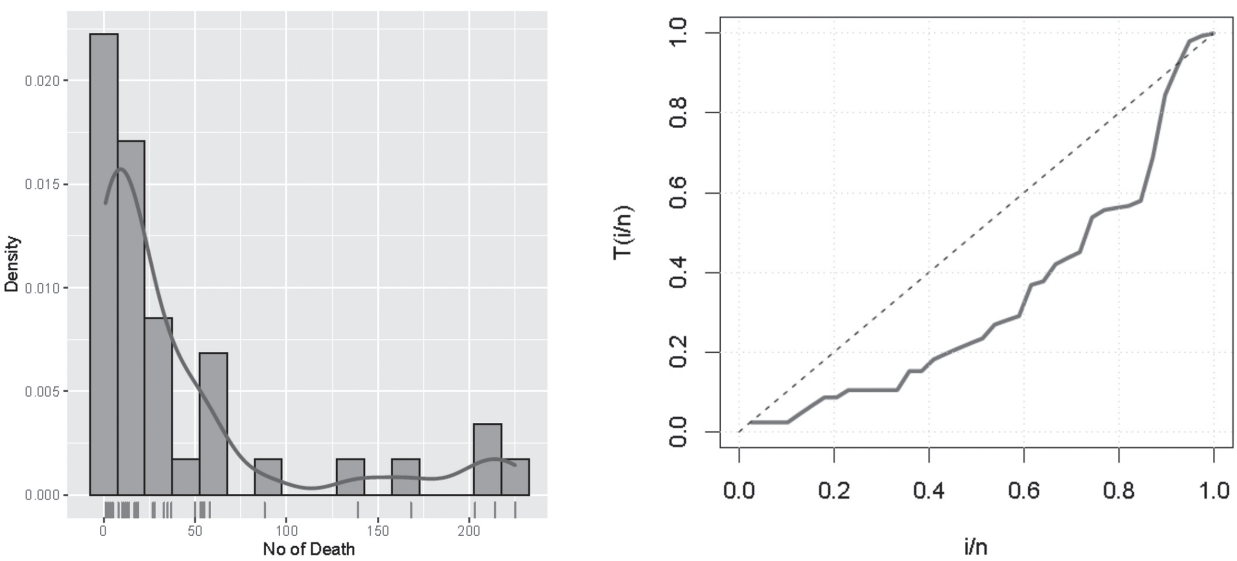

Figure 3 Histogram and TTT plot of deaths of the COVID-19 dataset

Estimation of the Unknown Parameters

In this study, three estimation methods are utilized to estimate the parameters of the proposed model and we have presented MLEs along with their standard deviation and CI also (Table 2). Using maxLik() function (Henningsen & Toomet, 2011) in R software (R Core Team, 2022) and (Mailund, 2017) by maximizing the likelihood function (4.2). In Table 3 MLEs, LSEs and CVMEs for EEIW distribution are displayed.

Table 2

MLEs with 95% asymptotic confidence interval (CI)

Parameters	MLEs	Standard Error	95 % CI
beta	22.0684	0.12578	21.8219 - 22.3149
delta	0.4409	0.03434	0.3736 - 0.5082
lambda	11.9232	0.12574	11.6768 - 12.1697
theta	0.1667	0.02968	0.1085 - 0.2249

Table 3

Estimated values via MLE, LSE and CVME Methods

Estimation Method	beta	delta	lambda	theta
MLE	22.0684	0.4409	11.9232	0.1667
LSE	0.0761	5.8439	0.1032	3.9424
CVME	0.0405	6.3911	0.0986	4.5425

Validity Test of the EEIW distribution

To test the validity of the proposed model we have presented the Kolmogorov-Smirnov (KS) test and found the value of test statistic 0.0857 and p-value 0.9367 also we have plotted the KS plot and Q-Q plot (Figure 4). These results support that the EEIW fits data very well.

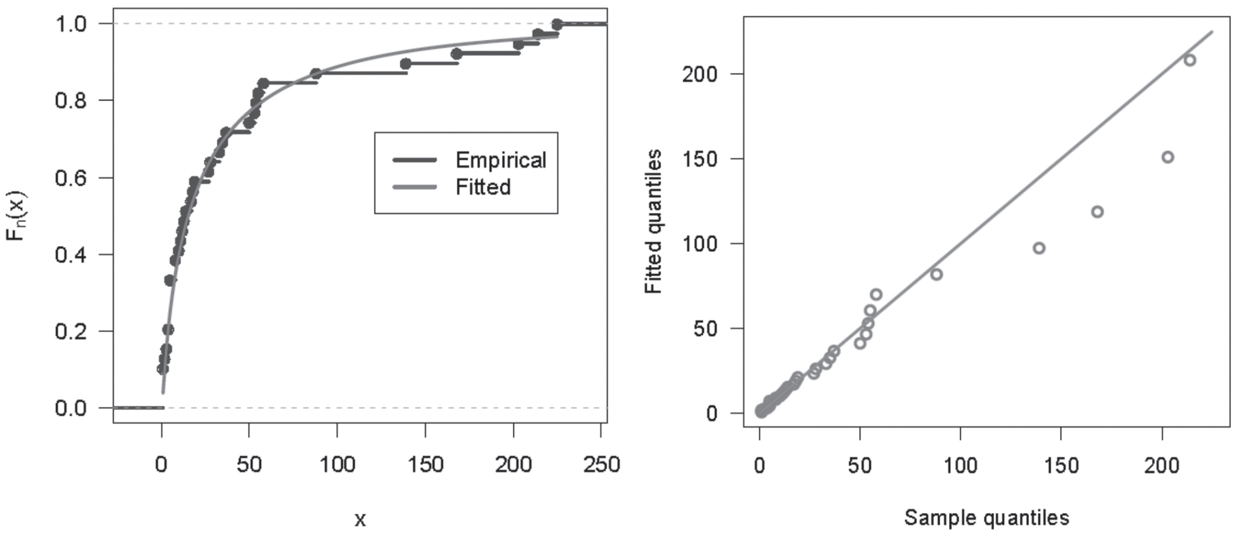

Figure 4 KS plot(left) and Q-Q plot (right)

Model Selection

To compare the performance of EEIW distribution with some modified forms of IW distributions such as generalized inverse Weibull (GIW) (De Gusmao et al. 2011), inverse Weibull (IW) (Afify et al., 2021), Lindley inverse Weibull (LIW) (Joshi & Kumar, 2020), The Kumaraswamy-inverse Weibull (KSM-IW) (Shahbaz et al., 2012), logistic inverse Weibull (LGT-IW) (Chaudhary & Kumar, 2020), and modified inverse Weibull (MIW) (Khan & King, 2012). The Akaike information criterion (AIC), Bayesian information criterion (BIC), Corrected Akaike information criterion (CAIC) and Hannan-Quinn information criterion (HQIC) for each model are presented in Table 5. Further, the Kolmogorov-Smirnov (KS), the Anderson-Darling (W) and the Cramer-Von Mises (A^2) test statistics are presented in Table 6. It is observed that the EEIW distribution gets the minimum value of the test statistic and a higher p-value hence the proposed model is the best among six competing models for COVID-19 dataset. Further, this is also verified by a graphical view (Figure 5).

Table 4

MLEs for the distributions under study

Distribution	Estimated parameters						
	$\hat{a}$	$\hat{b}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\delta}$	$\hat{\lambda}$	$\hat{\theta}$
EEIW	-	-	-	22.0684	0.4409	11.9232	0.1667

Distribution	Estimated parameters						
	$\hat{a}$	$\hat{b}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\delta}$	$\hat{\lambda}$	$\hat{\theta}$
GIW	-	-	1.2822	0.6878	-	3.1952	-
IW	-	-	2.265×10^{-07}	0.6901	-	0.2649	-
LIW	-	-	5.9082	0.2343	-	-	16.2468
KSM-IW	2.47667	17.35540	2.42955	0.22647	-	-	-
LGT-IW	-	-	4.7450	0.1656	-	1.0802	-
MIW	-	-	2.795	0.5756	-	1.483	7.128×10^{-07}

Table 5

Log-likelihood (LL), AIC, BIC, CAIC and HQIC

Distribution	ll	AIC	BIC	CAIC	HQIC
EEIW	-177.2989	362.5978	369.252	363.7743	364.9853
LIW	-177.9936	362.9873	366.978	362.673	363.7779
KSM-IW	-177.9974	363.9947	370.649	365.1712	366.3822
LGT-IW	-179.1483	364.2966	369.2873	364.9823	366.0872
GIW	-180.006	366.012	371.0026	366.6977	367.8026
IW	-180.0074	366.0147	371.0054	366.7004	367.8053
MIW	-181.5528	371.1056	377.7598	372.282	373.4931

Table 6

The goodness-of-fit statistics and their corresponding p-value

Distribution	KS(p-value)	AD(p-value)	CVM(p-value)
EEIW	0.0857 (0.9367)	0.2947 (0.9418)	0.0297 (0.9786)
GIW	0.0873(0.9273)	0.6485(0.6023)	0.0760(0.7181)
IW	0.0885(0.9202)	0.6654(0.5874)	0.0814(0.6863)
LIW	0.0978(0.8495)	0.3159(0.9253)	0.0354(0.9569)
KSM-IW	0.0984(0.8443)	0.3167(0.9247)	0.0357(0.9558)
LGT-IW	0.1017(0.8145)	0.3481(0.8976)	0.0407(0.932)
MIW	0.1321(0.5043)	1.3277(0.2234)	0.2049(0.2588)

Prediction of Mortality Rates

The mortality rate of a death cases in Nepal due to COVID-19 during the period 1st April- 14th May, 2021has been predicted based on MLEs using the EEIW distribution (Table 7).

Table 7

Prediction of Deaths

No. of Death	0-40	40-80	80-120	120-160	160-200	200-240	240 and above
Probability	0.7247	0.1308	0.0560	0.0297	0.0176	0.0112	0.0299

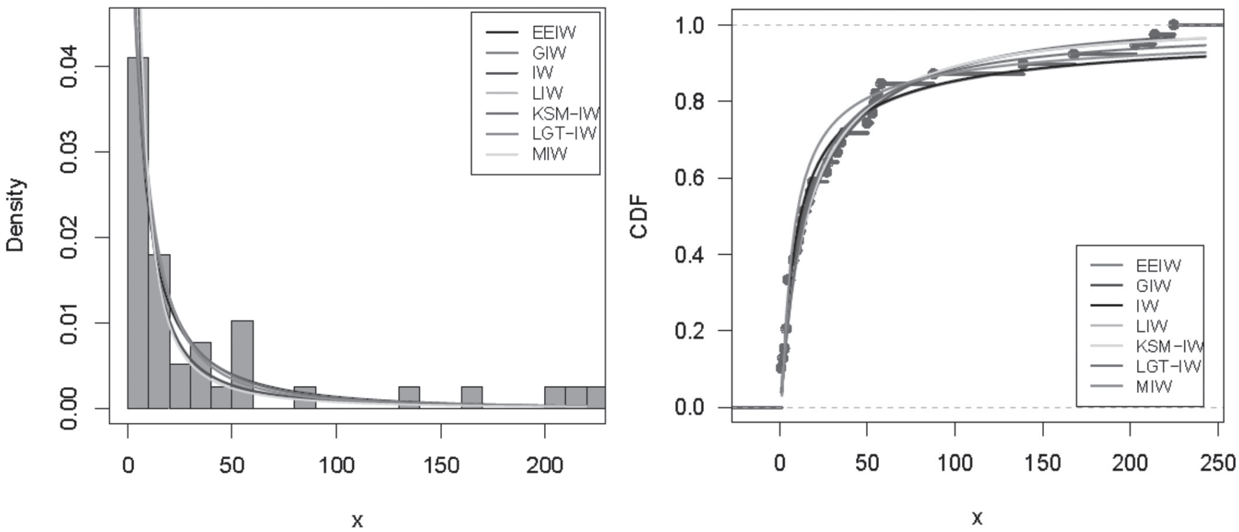

Figure 5: The Histogram and density function of fitted distributions (left) and fitted CDF (right)

Conclusions

In this study, we suggest a univariate continuous distribution having four parameters named exponentiated exponential inverse Weibull distribution. Some important statistical properties of the EEIW distribution are discussed such as a linear mixture of IW densities, moment and moment generating function, mean deviation, entropy and order statistics. The parameters of the proposed distribution are estimated by using maximum likelihood estimation (MLE), least-square estimation (LSE), and Cramer-Von-Mises estimation (CVME) methods and we found that the MLEs are quite better than LSEs and CVMs for the dataset under study. The death case of COVID-19 data set was analyzed to explore the applicability and suitability of the proposed model and found that the proposed model is quite better than six other existing models taken into consideration. We hope this model may be an alternative in the field of survival analysis, probability theory and applied statistics.

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