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A Comparative Study of Geometric Principles in the Sulba Sutras and the Pythagorean Theorem: Historical Context and Mathematical Applications

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Abstract

This research delves into the geometric principles detailed in the ancient Sulba Sutras and their parallels with the Pythagorean theorem, traditionally attributed to the Greek mathematician Pythagoras. While the Pythagorean theorem is widely regarded as a cornerstone of Greek geometry, recent studies suggest that its formulation appeared centuries earlier in Indian mathematics, as evidenced in Baudhāyana's Sulba Sutra (circa 800 BCE). These texts offer extensive insights into the geometric constructions used in Vedic rituals, including the use of Pythagorean triples and transformations between geometric shapes. The Sulba Sutras present a rich mathematical framework, emphasizing practical applications in altar construction, land measurements, and urban planning. This study aims to bridge the research gap by comparing the geometric developments in the Sulba Sutras with Greek formulations, particularly focusing on their mathematical methods, cultural contexts, and historical significance. The findings underscore the advanced state of Indian mathematics and its profound influence on subsequent mathematical traditions, offering a nuanced understanding of the parallel evolution of geometric knowledge across cultures.

Keywords: Sulba sutras, baudhāyana, pythagorean theorem, vedic geometry, pythagorean triples, geometric transformations.

Introduction

Mathematical traditions from ancient civilizations such as India and Greece played a crucial role in shaping early knowledge systems, often emerging from practical needs related to construction, agriculture, and religious practices. One of the most significant mathematical achievements from these periods is the formulation of the Pythagorean theorem, a cornerstone of geometry. While the Greek mathematician Pythagoras, from the 6th century BC, is widely credited with this theorem, evidence from earlier Indian texts, notably the Sulbasutras, suggests that similar geometric concepts existed in India long before Pythagoras (Dutta, 1932).

The mathematics within the Vedic texts is predominantly found in the 'Vedangas', 'Samhitas', and 'Brahmanas'. Notably, the 'VedangaJyotisha', which addresses astronomical concerns, provides remarkably precise values for various astronomical parameters, including the relative sizes of planets, the distance between the Earth and the Sun, the length of a day, and the duration of a year. Additionally, the 'VedangaKalpa', which focuses on ritual practices, includes geometric handbooks known as the 'Sulba-Sutras'. These texts outline geometric principles utilized in the construction of ritual altars. Key contributors to the 'Sulba-Sutras' include Baudhayana (800 BC) and Apasthamba (600 BC), whose works are notable for their mathematical content.The Sulbasutras, part of the Vedic literature, contain detailed geometric methods used primarily for constructing ritual altars, and include formulations of what is now recognized as the Pythagorean theorem (Dutta, 2016& Joseph, 2011).

"The Pythagorean theorem is a fundamental principle in geometry (Gupta, 2021), often stated as: 'In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. Pythagoras, the ancient Greek mathematician from the 6th century BC, is commonly associated with the Pythagorean theorem. Yet, the theorem's roots extend back to the 8th century BC, long before Pythagoras. Baudhayana'sSulba-Sutra from that time includes various Pythagorean triples, such as (3, 4, 5), (5, 12, 13), (8, 15, 17), (7, 24, 25), and (12, 35, 37), as well as an early statement of the theorem: 'The area covered by a rope stretched along the diagonal of a rectangle equals the combined area of its vertical and horizontal sides.' In addition to the Pythagorean theorem, the Sulba-Sutras present important results, including geometric methods for solving linear and quadratic equations (Joseph, 2011)

Apasthamba's Sulba-Sutra, which builds on Baudhayana's work, provides a highly accurate approximation of the square root of 2.This approximation, calculated as $1+\frac{1}{3}+\frac{1}{3 x 4}+\frac{1}{3 x 4 x 34}$, yields 1.4142156, correct to five decimal places (Dutta, 2016.,O'Connor & Robertson, 2000). Ancient Greek sources, such as Cicero, Diogenes Laertius, and Proclus, mention Pythagoras in relation to the theorem, but these references emerged long after his death. Modern scholarship suggests that Euclid's presentation of the theorem in "Elements" provides a more reliable historical basis (Euclid, 300 B.C.) (Kapoor, 2008).

The Sulbasutras, authored by mathematicians such as Baudhayana and Apasthamba, not only present early examples of Pythagorean triples but also reveal advanced geometric principles used in Vedic rituals. These texts demonstrate a profound understanding of geometry, including methods for constructing right-angled triangles and calculating areas, which were critical for altar construction. Baudhayana'sSulbasutra, for example, predates the formal Greek Pythagorean theorem by centuries and offers a practical application of the principle in the form of rope geometry: "The area covered by a rope stretched along the diagonal of a rectangle equals the combined area of its vertical and horizontal sides" (Kapoor, 2008&Pearce, 2002).

Among the Sulbasutras, works by Bodhayana, Apastamba, and Katyayana describe geometric principles akin to the Pythagorean theorem. Notably, Bodhayana'sSulbasutra, one of the earliest known Indian mathematical texts, presents a formulation resembling the Pythagorean theorem. Bodhayana's statement, "the diagonal of a rectangle produces the same area as the squares on the length and breadth," reflects an early expression of this theorem (Dutta, 2016). This principle was later reaffirmed by Apastamba and Katyayana, emphasizing its practical utility in altar construction (Seidenberg, 1978).

Vedic society utilized geometry for practical applications beyond religious rituals. Agricultural land measurements, tax assessments, and other secular activities required accurate geometric calculations. For instance, local administrators converted irregularly shaped plots of land, such as rectangles or triangles, into squares of equivalent area to facilitate taxation and land management (Kapoor, 2014).

Parallel to Greek developments, ancient Indian mathematics, as detailed in the Sulbasutras, also exhibits advanced geometric principles. The Sulbasutras, integral to the Kalpa Sutras, offer extensive guidance on constructing sacrificial altars used in Vedic rituals. The term "Sulbasutra" translates to "rules of the cord," highlighting the use of measuring cords in these constructions (Filliozat, 2004). These texts contain detailed instructions on geometric constructions, including the creation of squares, rectangles, and circles, as well as the use of right-angled triples (Seidenberg, 1978&Dutta, 2016).

This study seeks to explore the parallel development of geometric knowledge in ancient India and Greece, particularly focusing on the comparison between the Pythagorean theorem and its counterparts in the Sulbasutras. By analyzing their mathematical formulations, historical contexts, and applications, the research aims to shed light on the cross-cultural contributions to early geometry. Furthermore, this analysis will emphasize the role of Indian mathematics in the global history of mathematical discovery, exploring how these ancient principles influenced subsequent developments in both Indian and Greek traditions (Rajaram, 2014)

Through a comparative lens, this research aims to fill the gap in literature that typically isolates these mathematical traditions, instead providing a unified understanding of their significance and interconnectedness.

Statement of Problem

This research addresses the need for a comparative analysis of the geometric principles found in the Sulbasutras and the Pythagorean theorem, examining their distinct historical contexts, mathematical approaches, and practical applications. It seeks to identify the similarities and differences between these ancient mathematical traditions and their influence on subsequent developments in mathematics.

Research objective

This research aims to compare the geometric principles of the Sulbasutras and the Pythagorean theorem, exploring their historical contexts, practical applications, and mathematical methods, while assessing their influence on later mathematical traditions.

Research Methodology

In this research, a comprehensive literature review was conducted, focusing on primary texts and secondary analyses related to the Sulbasutras and the Pythagorean theorem. The study employed a comparative analysis method to systematically evaluate the geometric principles detailed in both the Sulbasutras and Greek mathematical traditions. Historical and cultural contexts were examined through archaeological findings and historical records to understand their influence on mathematical development. Mathematical techniques from both traditions were analyzed, applying theoretical frameworks to assess accuracy and application. involved detailed content analysis of ancient texts, practical applications in rituals and construction, and an evaluation of their impact on later mathematical practices. Findings were synthesized to highlight the similarities, differences, and influences between these geometric principles, culminating in a comprehensive report documenting the research outcomes.

Literature Review

Mathematics in ancient civilizations, notably India and Greece, developed out of practical necessities and religious or philosophical motivations, shaping the foundation of many modern mathematical concepts. Both traditions, though distinct in their approaches and priorities, contributed significantly to early mathematical thought, particularly in geometry. This review examines these contributions in a historical and comparative context, focusing on the Vedic period in India, where the Sulbasutras played a key role which gave rise to formalized geometry, exemplified by the Pythagorean theorem.

The Indus Valley Civilization (3000–1500 BC) is one of the earliest known examples of sophisticated mathematical application. Archaeological excavations from the cities of Harappa and Mohenjo-Daro reveal the use of advanced urban planning techniques, suggesting an understanding of geometry, measurement, and standardization. The weights and measures discovered in these sites, including uniform measuring scales, indicate an early grasp of mathematical concepts necessary for large-scale construction and city planning (Sykorova, 2006). The decimal system, found in evidence from this period, further supports the notion that the Indus Valley peoples employed advanced mathematics in daily life, long before its formalization in later Indian texts (O'Connor & Robertson, 2000).

Following the decline of the Indus Valley Civilization, the **Vedic period** (1500–500 BC) witnessed the development of more structured mathematical thought, particularly through the composition of the **Sulbasutras**. These ancient texts were primarily concerned with the construction of ritual altars, which were essential to Vedic religious ceremonies. In order to construct these altars with precise dimensions and shapes, the Sulbasutras detailed methods of transforming geometric shapes, calculating areas, and using what is now known as the Pythagorean theorem (Rajaram, 2014&Seidenberg, 1978). These texts mark one of the earliest known applications of geometric principles in history.

The Sulbasutras, including those by **Baudhayana**, **Apastamba**, **Manava**, and **Katyayana**, provide detailed procedures for constructing right-angled triangles, a key geometric form in the context of religious altars. For instance, **Baudhayana'sSulbasutra**, which predates Pythagoras by centuries, includes formulations of the Pythagorean theorem and Pythagorean triples, demonstrating a clear understanding of the relationship between the sides of a right-angled triangle (Kapoor, 2014). This suggests that Indian mathematicians were aware of the theorem long before it was formally recorded in Greek mathematics (Dutta, 2016& Joseph, 2011). Baudhayana's theorem, as it is sometimes called, provides both a geometric and numerical explanation of the principle, emphasizing its practical applications in ritual altar construction rather than formal proofs.

While Indian mathematicians in the **Vedic period** focused on practical applications, their Greek counterparts, notably **Pythagoras** and **Euclid**, were more concerned with the theoretical underpinnings of mathematical principles. Greek mathematicians developed a systematic approach to mathematics, emphasizing logical rigor and formal proofs. **Euclidean geometry**, as outlined in **Euclid's Elements**, represents one of the earliest comprehensive systems of geometric theory. Euclid's rigorous approach to the Pythagorean theorem, which included formal proofs, contrasts with the practical, application-driven methods found in the Sulbasutras (Kapoor, 2008&Pearce,2002). This divergence reflects the different cultural and intellectual priorities in ancient India and Greece—Indian texts were primarily concerned with religious rituals, while Greek mathematics pursued theoretical abstraction.

However, the Sulbasutras are remarkable for their **intuitive grasp of geometric principles**. Though they lack formal proofs, these texts reveal a sophisticated understanding of area, proportion, and transformation of shapes. This practical knowledge was directly tied to the performance of Vedic rituals, where precision in altar construction was seen as essential for proper religious observance (Osborn, 2014). The Sulbasutras also illustrate early techniques for transforming geometric shapes, such as converting squares into rectangles or circles, a skill necessary for creating altars of specific dimensions while maintaining proportional relationships (Kapoor, 2008).

Beyond the Vedic period, the mathematical landscape in India continued to evolve, particularly within **Jain** and **Buddhist traditions**. By the 3rd century BC, **Jain mathematicians** were making significant contributions to **combinatorics** and **logarithms**. Texts such as the **Bhagvati Sutra** and the **Sathananga Sutra** delve into permutations, combinations, and early concepts related to the binomial expansion, showing the depth of Jain mathematical thought (Kapoor, 2008). These developments highlight the growing sophistication of Indian mathematics, extending beyond geometry into more abstract areas of mathematics.

In the **Buddhist tradition**, mathematicians engaged with concepts such as **indeterminate numbers** and **infinite sets**, which reflected broader philosophical inquiries into the nature of infinity and the universe (Joseph, 2011). The mathematical explorations of **Nagarjuna** (c. 150–250 AD), a renowned Buddhist philosopher, exemplify this blend of philosophy and mathematics. His work on magic squares, specifically a **4x4 magic square**, demonstrates properties where the sums of the numbers in each row, column, and diagonal are equal, showcasing the interplay between mathematical curiosity and metaphysical inquiry (Aczel, 2015).

Pingala, a mathematician from the **3rd century BC**, made significant strides in combinatorial mathematics, specifically through his **Chandah Sutra**, which laid the groundwork for what would later be formalized as the **Pascal triangle**. Pingala's work on prosody, while ostensibly concerned with poetic meters, also contained mathematical insights into binomial coefficients and combinatorics (Shah, 1991&Dutta, 2002). His contributions were further developed by later scholars such as **Halayudha**, whose commentary on Pingala's work elaborated on its combinatorial applications.

Baudhayana'sSulbasutra provides specific Pythagorean triples and describes methods for constructing right-angled triangles, indicating a sophisticated understanding of these principles predating Greek contributions (Sharma, 1985). This suggests that Indian mathematicians were aware of the theorem's principles well before Pythagoras(Dutta, 2016).

While the Sulbasutras demonstrate practical applications of the Pythagorean theorem, they do not always include formal proofs. In contrast, Greek mathematics, especially through Euclidean geometry, is noted for its rigorous proofs and systematic approach to geometric results (Kapoor, 2008&Pearce, 2002). Indian texts, focusing more on practical applications for rituals and constructions, reflect a different set of mathematical priorities and practices.

The Sulbasutras were primarily utilized for constructing ritual altars, integrating mathematical principles into religious practices (Osborn, 2014). Although Greek mathematics also had practical applications, it often pursued theoretical exploration and proofs (Dutta, 2016).

The Sulbasutras, dating from the Vedic period (1500-500 BC), offer an early glimpse into mathematical thought with a focus on constructing ritual altars. Despite their functional orientation, these texts are significant for their early use of geometric principles, which laid the groundwork for future mathematical developments.

The Sulbasutras include some of the earliest known formulations of what is now the Pythagorean theorem. Baudhayana'sSulbasutra provides methods for constructing right-angled triangles and demonstrates an understanding of Pythagorean triples (Kapoor, 2014&Joseph, 1990). This early application of geometric principles underscores the sophistication of Indian mathematics prior to Greek formalization.Unlike Greek mathematics, which emphasized rigorous proofs, the Sulbasutras concentrate on practical applications. The geometric results are presented through methods and rules rather than formal proofs, showcasing an intuitive grasp of area and proportion (Joseph, 1990).

The Sulbasutras illustrate the early development of mathematical principles in ancient India, emphasizing practical applications in ritual construction and early geometric principles. These texts, along with later contributions from Jainism and Buddhism, highlight the rich mathematical tradition of ancient India and its influence on subsequent mathematical developments (Kak, 1980).

Influence on Global Mathematics

The mathematical principles described in the **Sulba Sutras** had a profound influence on other ancient civilizations. American mathematician **Abraham Seidenberg** concluded that the elements of ancient geometry found in Egypt and Babylon likely originated from the ritual systems described in the **Sulba Sutras** (Seidenberg, 1978). This suggests that Vedic mathematical knowledge may have contributed to the development of geometry in other parts of the ancient world, indicating the global significance of Indian mathematical advancements (Kak, 1980).

The Scientific Approach in the Sulba Sutras

The **Sulba Sutras** not only provide mathematical instructions but also imply an early form of the scientific method, as evidenced by the logical structure of the texts. The **Nyaya-Sutras** formalized the scientific method during the Vedic period, suggesting that proofs of mathematical theorems, such as the **Pythagorean theorem**, were likely known but transmitted orally through the **Gurukul** system. Although the actual proofs may have been lost, the **Sulba Sutras** make it clear that the authors understood why these mathematical principles were true, as demonstrated by **Baudhayana's** method of combining squares to form a third square (Kapoor, 2008&Dutta, 2016).

Legacy and Implications for Modern Mathematics

The legacy of Vedic mathematics is evident in modern applications of geometry and algebra. The **Vedanga Jyotisya** and other Vedic texts emphasize the importance of mathematics, stating that it holds the highest position among all branches of knowledge. The **Sulba Sutras** laid the foundation for many mathematical concepts that continue to be relevant today. The geometrical insights and methods described in these ancient texts not only contributed to the religious and cultural life of Vedic society but also influenced mathematical developments across the world.

Panini's work on Sanskrit grammar, which introduced a scientific notational model, also reflects the Vedic emphasis on formal logic and abstract reasoning. Linguists such as **Frits Staal** have highlighted the advanced understanding of context-sensitive rules in Panini's grammar, which parallels the logical structures found in modern computer science and linguistics (Joseph, 2011&Kapoor, 2008).

Resultsand Discussion

The **Sulba Sutras**, composed by ancient Vedic mathematicians such as Baudhāyana, Apastamba, Manava, and Katyayana, contain some of the earliest known geometric principles. These texts document advanced geometry long before modern mathematical formulations, particularly for constructing altars used in Vedic rituals. Among the key contributions is **Baudhāyana's theorem**, an early expression of what would later become known as the **Pythagorean theorem**.

The Sulbasutras also describe techniques for transforming geometric figures essential for constructing ritual altars. These methods, involving calculations of areas and perimeters, reflect an advanced understanding of spatial relationships and measurement (Kapoor, 2008).

Some of the expression in Sulba Sutra are as follows.

दीर्घचतुरस्रस्यायारज्जु : पार्श्वमानीतिर्यङ्मानौ च यत् पृथकग्भुते कुरुतस्तदुभयं करोति ॥

वोद्धायन सुल्वसूत्र १४८

(The long four-strand rope, measuring laterally and measuring horizontally, does both what it does in the separately)

VoddhayanaSulvasutra

दीर्घस्वाऽक्ष्णयारज्जु:पार्श्वमानौ च यत् पृथग्भुते कुरुतस्तदुभयं करोति ॥

आवस्तम्व सुल्वसूत्र १४८ ॥

(The long rope side measuring laterally and measuring horizontally measurement do both what they do in the separate world.)

According to the Pythagorean theorem:$c^2 = a^2 + b^2$

Steps to solve:

Define the lateral and horizontal measurements:

Suppose the lateral measurement a is the distance measured along one axis (say vertically or along one dimension).

The horizontal measurement b is the distance along another axis perpendicular to the lateral measurement.

Use the Pythagorean theorem: The long rope, representing the hypotenuse c, satisfies the relationship:$c^2 = a^2 + b^2$ or $c=\sqrt{a^2 + b^2}$

Example calculation:

Suppose the lateral measurement a=3units, and the horizontal measurement b=4units. Using the Pythagorean theorem:

$c=\sqrt{3^2 + 4^2}$

$=\sqrt{9 + 16} = \sqrt{25}=5$

So, the length of the long rope side c is 5 units.

AvastamvaSulva Sutra 148 ||

दीर्घचतुरस्रस्याक्ष्णया रज्जुरः तिर्यङ्मानौ पार्श्वमानौ च यत् पृथग्भुते कुरुतस्तदुभयं करोति ॥

कात्यायन सुल्वसूत्र २ ॥

(The rope with its long square does both the horizontal and measuring horizontally measurement do both what they do in the separate world.)

Example Calculation:

Suppose the lateral (vertical) measurement a=6units and the horizontal measurement b=8units. Then, the length of the rope $c$is:

$c^2 = 6^2 + 8^{\ 2}$=36+64=100

c=$\sqrt{100}$=10units

Katyayana Sulva Sutra 2 ||

Baudhāyana's Contribution to Mathematics

Baudhāyana's work, dating back to around 800 BCE, is recorded in the **Sulba Sutras**. His theorem predates **Pythagoras** and is expressed as: $c^2 = a^2 + b^2$

where c is the hypotenuse, and a and b are the other two sides of a right-angled triangle. This theorem was used in Vedic altar construction, ensuring precise geometric proportions.

Pythagorean Triples in Baudhāyana's Work

Baudhāyana's **Sulba Sutra** also includes a list of **Pythagorean triples**, integers that satisfy the Pythagorean theorem:$c^2 = a^2 + b^2$

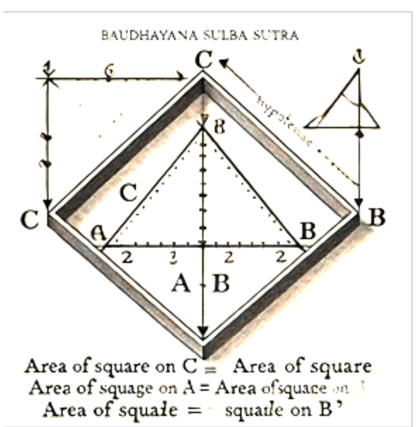
Examples include:

- (3, 4, 5)
- (5, 12, 13)
- (7, 24, 25)
- (8, 15, 17)
- (12, 35, 37)

These triples were used practically for creating right-angled corners in altars.

Geometrical Demonstrations

Baudhāyana provided both **numerical examples** and **geometrical demonstrations** of his theorem. He demonstrated that the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the other two sides:



Area of the square on c=Area of the square ona+Area of the square on b.

Figure 1: geometrical demonstrations of Pythagorean Triples in Baudhāyana's Work
This approach was critical for ensuring that altars adhered to the required dimensions.

Comparison with Pythagoras

While **Pythagoras** (circa 570–495 BCE) is credited with the formal proof of the Pythagorean theorem, Baudhāyana's formulation predates him by several centuries. Baudhāyana's method was practical, using geometric constructions, whereas Pythagoras provided a deductive proof.

Broader Mathematical Contributions

The geometric constructions described in the Sulba Sutras begin with establishing a *prãcï*, a line oriented east-west, which is subsequently used as a center line or line of symmetry in various geometric constructions.

Baudhāyana explored methods to convert one geometric shape into another while preserving the area:

The Area Sutra is a principle from Vedic mathematics attributed to Baudhāyana. It provides a method for calculating the area of geometric shapes, particularly rectangles, triangles, and parallelograms, using simple sutras or formulas.

One of the key sutras related to the area is:
Rectangle: The area of a rectangle is obtained by multiplying its length and width.
Area=length×width
Triangle: The area of a triangle can be calculated as:
Area=$\frac{1}{2}$×base×height
Parallelogram: The area of a parallelogram is:
Area=base×height
Baudhāyana's methods often involve simplifying complex geometric problems into basic calculations using these fundamental principles.
Some geometrical theorems form Baudhayana Sulba Sutra are illustrated below

1. Construction of a Square with a Given Side Length

As outlined in Verses I, 22-28 of the BaudhayanaŚulba Sutra (BSS), the procedure starts with a *prãcï* and a center point (line EP in Figure 2). By constructing circles with specified centers and radii, a square ABCD is formed where E and W are the midpoints of AB and CD, respectively. The steps are depicted in Figure 1(Price, 2001)

This is explained diagrammatically in fig 2.

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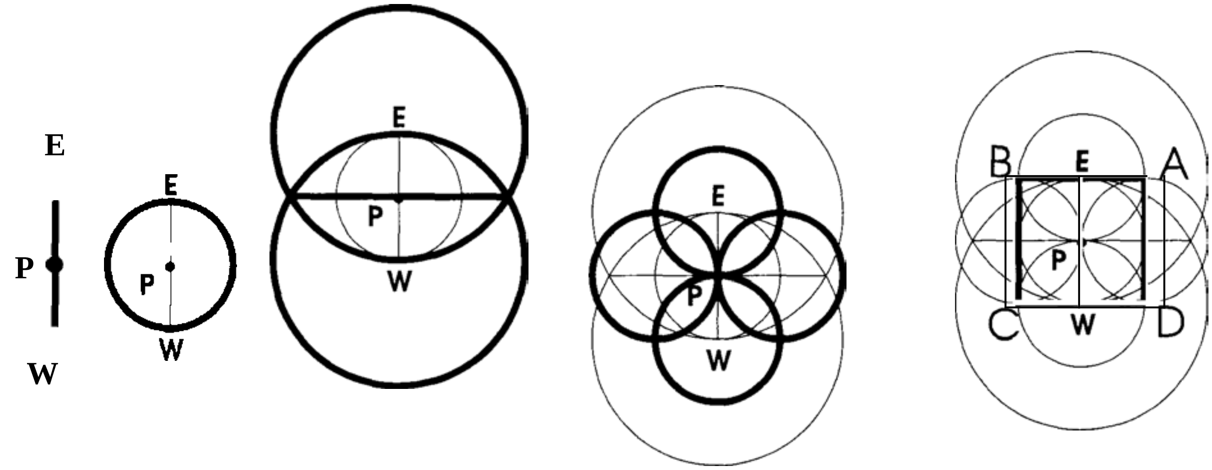


Figure 2: Construction of a Square with a Given Side Length (Price, 2001).

2. Construction of a Square Equal to the Sum and Difference of Two Unequal Squares

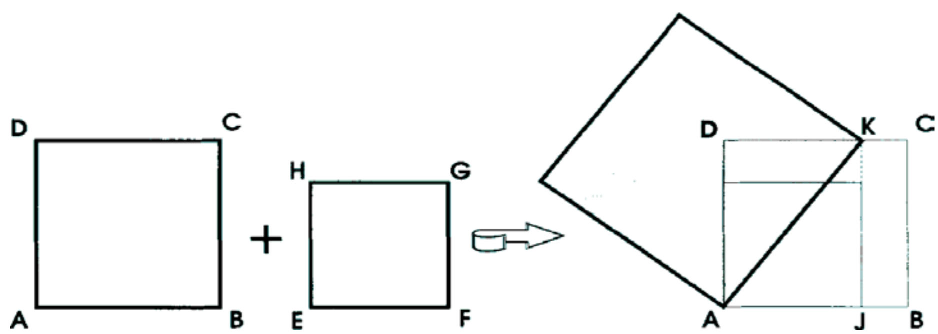
2. a. Square Equal to the Sum of Two Unequal Squares

Verse I, 50 of the BaudhayanaŚulba Sutra (BSS) describes the method for constructing a square whose area equals the sum of the areas of two unequal squares. Given squares ABCD and EFGH where AB>EF:

1. Point out points J and K on square ABCD where AJ=DK=EF
2. The line segment AK is the side of a new square whose area is equal to the sum of the areas of the two original squares. This is shown in fig.3.

Figure 3: Square Equal to the Sum of Two Unequal Squares(Price, 2001).

2. b. Square Equal to the Difference of Two Unequal Squares



Verse I, 51 of Sulbashutra provides a method for constructing a square with an area equal to the difference between the areas of two unequal squares. For a rectangle with sides AB=a and AD=b:

1. Draw an arc DL with center A (refer to Figure 4. Price, 2001).
2. The side JL of the resulting square satisfies the equation:

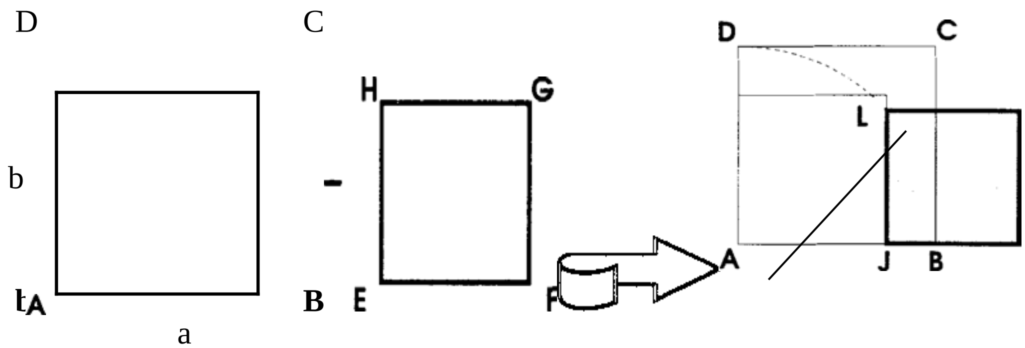
This result follows from the equation:

$(AJ)^2 + (JL)^2 = (AL)^2$

where AL=AB and AJ=EF. The derived side JL of the new square confirms that:

$(JL)^2 = (AB)^2 - (EF)^2$

Figure 4: Square Equal to the Sum of Two Unequal Squares(Price, 2001).



3. Theorem on the Square of the Diagonal

Verse I.48 of the BaudhayanaŚulba Sutra (BSS) states:

"The diagonal of a rectangle produces both (areas) which its length and breadth produce separately.

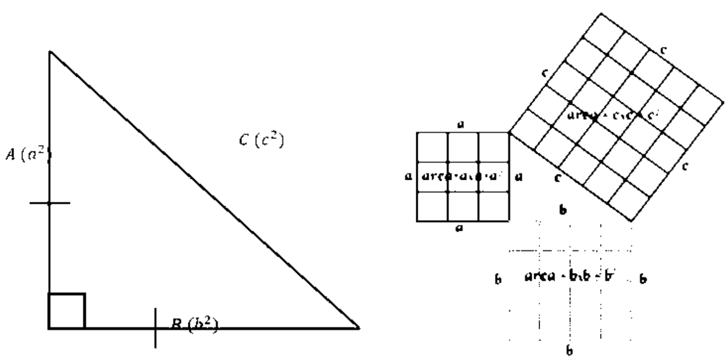


Figure 5: theoremon the Square of the Diagonal- (Das 2024)

Although this verse does not directly mention areas, it is essentially a formulation of the Pythagorean theorem, which states that for a right triangle with sides a,b, and hypotenuse c, the relationship:

$$c^2 = a^2 + b^2$$

This principle can be observed in rectangles with various dimensions such as 3 and 4, 12 and 5, 15 and 8, 7 and 24, 12 and 35, and 15 and 36. which is illustrated in **fig. 3**(Price, 2001).

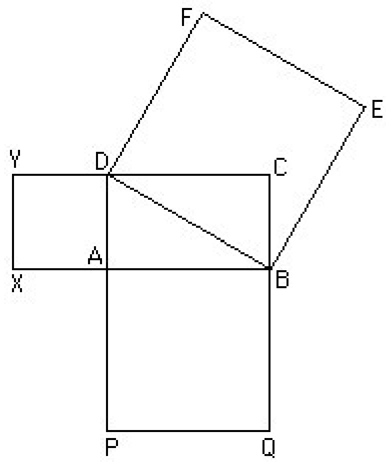


Figure 6: Pythagoras version of BaudhayanaŚulbaSutra

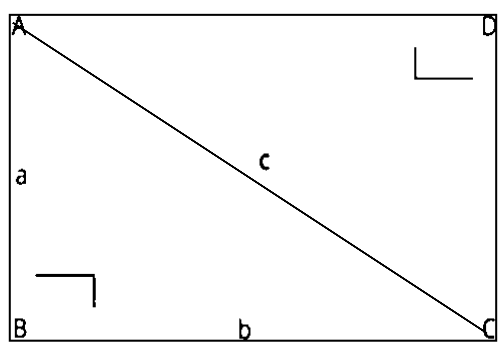


Figure 7: Relating the Square of the Diagonal in Baudhāyana's formulation **(**Das, 2024).

Consider a right-angled triangle with sides of lengths a, b, and c (the hypotenuse). According to Baudhāyana's formulation, the sum of the squares of the two shorter sides
$c^2 = a^2 + b^2$
Let's demonstrate this with an example:

Let a=3 and b=4. We will verify whether $3^2+4^2= c^2$holds true using Baudhāyana's method.

Following the formulation:
$3^2 + 4^2 = 5^2$
Thus, $c^{\ 2}$=25
Therefore, the hypotenuse is 5, confirming Baudhāyana's result.

Mathematically, Baudhāyana's proof of the Pythagorean theorem shows that for a right-angled triangle with sides a, b, and hypotenuse c, the relationship holds as $a^2 + b^2 = c^2$

Using Baudhyayan method

$$C^2 = \left(a - \frac{a}{8}\right)^2 + \left(\frac{b}{2}\right)^2$$

For example, with a=3and b=4, this becomes:

$$C^2 = \left(3 - \frac{3}{8}\right)^2 + \left(\frac{4}{2}\right)^2$$

Simplifying gives c≈10.95confirming the theorem

4. Converting Rectangle into square

The method for transforming a rectangle into a square with an equivalent area, as described in Verse I.54 of the Baudhayana Śulba Sutra utilizes the preceding construction. Begin with a rectangle ABCD where AB>BC, as depicted in Figure 4. Construct a square AEFD such that the excess portion of the rectangle is divided into equal halves. One of these halves is then added to the side of the square.

This process results in two squares: a larger square AGJC′ and a smaller square FHJB′. The required square is the difference in area between these two squares. In Figure 4, the side length of the new square is GL, where L is determined by EL=EB′.

To verify this, let AB=a and AD=b represents the sides of the rectangle. The side GL of the resulting square satisfies the relationship:

$$(GL)^2 = (EL)^2 - (EG)^2 = (EB')^2 - (EG)^2$$

$$= \left(b + \frac{a-b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2 = ab$$

Thus, the area of the new square matches the area of the original rectangle.

This is illustrated in figure 8 (Price, 2001).

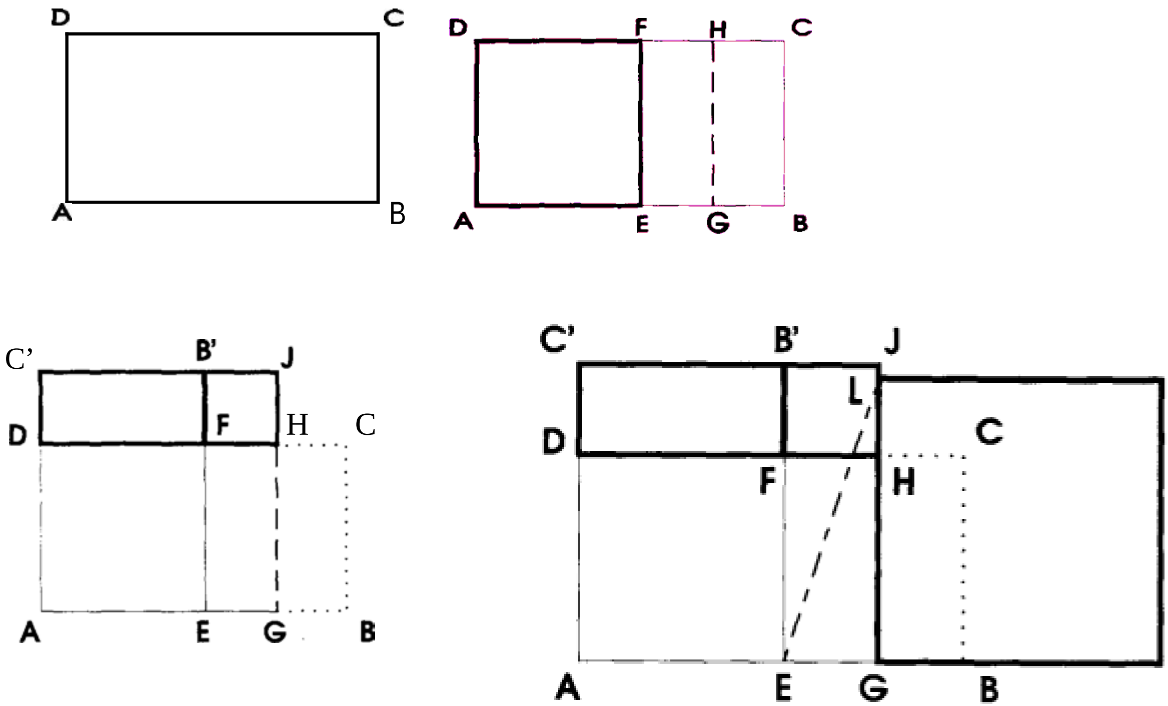


Figure 8: Converting rectangle into a square by Baudayan Sulba Sutra method

Datta suggests that the steps involved in this construction as marking off a square, dividing the excess, and rearranging the parts could form the basis for the method outlined later in Baudayan sulbasutra for approximating the square root of two. By applying this method repeatedly, approximations to $\sqrt{2}$ can be achieved (Price, 2001).

5. Converting a Square into a Circle

Verse I, 58 of the BaudhayanaŚulba Sutra (BSS) outlines a method for constructing a circle with an area approximately equal to that of a given square. Begin with a square ABCDas shown in Figure 9. Draw an arc DG with center O such that OF is parallel to AD. Let OF intersect DC at point F, and define H as a point one-third of the distance from F to G. The radius of the required circle is then OH.

This is shown diagrammatically in fig 9

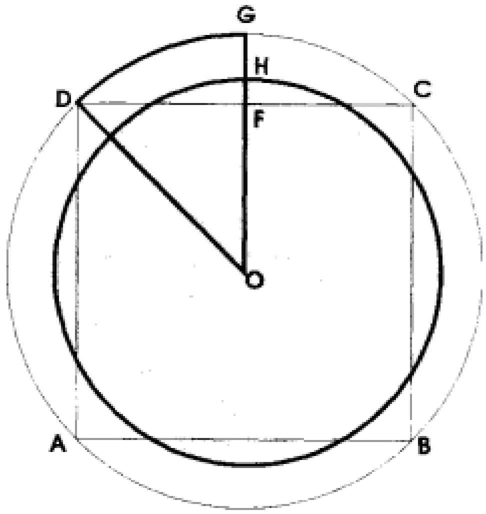


Figure 9. Converting a square into a circle (Price, 2001).

To understand this, let 2a be the side length of the square ABCD and r be the radius of the constructed circle. Then: r =OH=OF+FH

To approximate r, we use: r =OF+$\frac{1}{3}$(OG-OF)

=a+$\frac{1}{3}\left(a\sqrt{2}-a\right)$

Thus r = $\frac{a}{3}\left(2+\sqrt{2}\right)$

In modern mathematical terms, if d represents the diameter of the circle, the side length a of the corresponding square can be found using the following relation:

a =d$\times\frac{1}{8}\times\frac{29}{29}-\frac{1}{8}\times\frac{6}{8}$

Substituting d=3.141593and using the approximation 2≈1.414214, the area of the constructed circle can be computed as:

Area=πr^2=4.069011×a^2

This area is within approximately 1.7% of the correct value of a^2.

In the following verse, Baudhayana describes the reverse process converting a circle into a square.

6. Converting a Circle into a Square

Verse I, 59 of Baudayan Sulba Sutra describes converting a circle into a square. To achieve this:

- The BaudayanSulba Sutra method converts a square into a circle with equal area by using an ancient geometric approximation.

- The procedure uses a combination of subtraction and proportionality to achieve this conversion with notable accuracy.

  To turn a circle into a square, the diameter has been divided into eight parts, one of those parts into twenty-nine parts, twenty-eight of which have been removed, followed by subtracting one-sixth and one-eighth of that sixth part.

  This is shown diagrammatically in fig 9

Explanation

The side length s of the square is adjusted to approximate the area of a circle. The rule applied is:

Side length of square$d = \mathrm{d} - \frac{\mathrm{d}}{8\mathrm{x}29}$

Where dis the diameter of the circle. This formula effectively reduces the square's side length, ensuring the square has the same area as a circle of diameter d. When d=3.141593, the accuracy of the result is approximately 1.7%.

Verification

To describe the BaudhayanaSulba Sutra procedure for converting a square into a circle with the same area: Let the side length of the square be s. The area of the square is Asquare=s^2

Acircle= $\frac{1}{4}\pi d^2$

To match the areas, set s^2=$\frac{1}{4}\pi d^2$and solve for d: d =s× $\sqrt{\frac{4}{\pi}}$

Using the approximation π≈3.1416, this simplifies to:

D =s× $\sqrt{\frac{4}{3.1416}}$

D≈s×1.128

Thus, the diameter of the circle that has the same area as the square is approximately.128×s. This method reflects the geometric transformation principles outlined in the Baudhayana Sulba Sutra.

Geometric Proof with Steps

We aim to geometrically construct a circle with the same area as a given square and derive the radius of that circle. This proof involves equating the area of the square to the area of the circle and finding the radius geometrically.

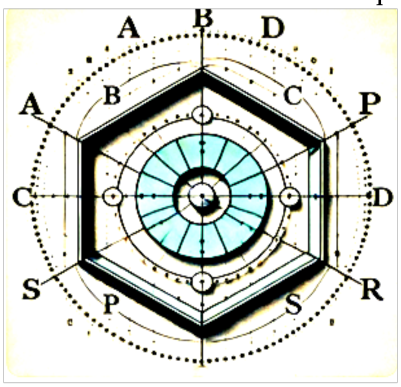
To prove the doubling of the square's area geometrically:

1. **Square Construction:**
   - Draw square ABCDwith side length s.
   - Area of square = $A = s^2$.

2. **Circle Area Setup:**
   - Area of circle $A = \pi r^2$
   - Set the circle's area equal to the square's $A = \pi r^2 = s^2$
3. **Find Radius r:**

Solve for r: r=$\frac{s}{\sqrt{\pi}}$

4. **Circle Construction:**
   - Draw the circle with radius r=$\frac{s}{\sqrt{\pi}}$ centered at P (center of square).
5. **Geometrical Result:**
   - The circle's area equals the square's area, demonstrating the relationship between the square's side length and the circle's radius.



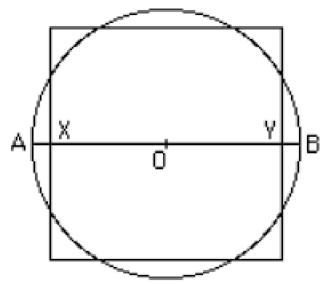


Figure 9: Baudhayana Sulba Sutra procedure for converting a square into a circle

Broader Implications of Baudhāyana's Work

Baudhāyana's understanding of geometry extended beyond the **Pythagorean theorem**. His methods for transforming geometric shapes, such as converting rectangles to squares or combining two squares into one, were foundational for altar construction.

Comparison: Modern Geometry vs. Sulba Sutra Methods

1. Pythagorean Theorem:

- **Modern Proof**: Uses algebraic methods and geometric arguments.
- **Sulba Sutra Approach**: Demonstrates the theorem through constructive methods, arranging physical shapes rather than using algebra.

2. Transformation of Rectilinear Figures:

- **Modern Geometry**: Uses transformation matrices to convert shapes.
- **Sulba Sutra**: Involves physically rearranging parts of geometric figures, such as dividing a square and rearranging the pieces to form a rectangle.

Example: Transforming a Square into a Rectangle

- **Modern Approach**: Scaling along one axis using a transformation matrix.
- **Sulba Sutra Method**: Dividing and rearranging the square geometrically to form a rectangle.

Solution Using Sulba Sutras Methods:

Original Area Calculation:

Original length: l

Original width: w

Original area: $A_1 = l \cdot w$

New Field Dimensions:

New length: $l+d$

New width: $w+k$

New Area Calculation:

New area: $A_2 = (l+d) \cdot (w+k)$

Requirement:

New area A_2 should be twice the original area:

$(l+d) \cdot (w+k) = 2 \cdot (l \cdot w)$

Expand and Simplify:

Expanding the left-hand side:

$(l+d) \cdot (w+k) = l \cdot w + l \cdot k + d \cdot w + d \cdot k$

Setting this equal to twice the original area:

$l \cdot w + l \cdot k + d \cdot w + d \cdot k = 2 \cdot (l \cdot w)$

Rearranging:$\cdot k + d \cdot w + d \cdot k = l \cdot w$

Solving for the Increment Values:

Rearranging to isolate terms involving d and k:

$d \cdot w + d \cdot k = l \cdot w - l \cdot k$

Solving for d: $d = \frac{l \cdot w - l \cdot k}{w + k}$

Verification and Example:

Let's say you start with $l=10$ units and $w=5$units.

Suppose you want the new dimensions to result in twice the area, and the increase in length $d$and width k needs to be calculated:

Original area:

$10 \cdot 5 = 50$square units

New area: $2 \cdot 50 = 100$ square units

New length: $10+d$

New width: $5+k$

Thus:$(10+d) \cdot (5+k) = 100$

Expanding and simplifying:

$50 + 10k + 5d + d \cdot k = 100$

$10k + 5d + d \cdot k = 50$

Solving for $d$and k involves choosing specific values that satisfy the equation. For instance, if k=5:

50+5d+5d=50

10d=0

d=0Thus, if$k$=5, d=0, and the new dimensions are

l+$d$and w+k, meeting the area requirement.

Other Mathematical Theorems in the Sulba Sutras

- **Equal Division of a Line**: Dividing a line segment into equal parts using geometric constructions.
- **Circle Division**: Dividing a circle into equal parts using ropes and diameters.
- **Pythagorean Theorem**: Geometrically proven by constructing squares on the sides of a right triangle.
- **Maximum Square in a Circle**: Describes the largest square that can be inscribed in a circle.
- **Parallelogram and Rectangle on the Same Base**: A parallelogram and rectangle with the same base and height have equal areas.

Conclusion

The Sulba Sutras, attributed to ancient Vedic mathematicians like Baudhāyana, represent a seminal contribution to early geometry. These texts not only document the Pythagorean theorem predating Pythagoras but also encompass a range of sophisticated geometric principles used in Vedic rituals. Baudhāyana's theorem and practical methods, such as those for constructing Pythagorean triples and transforming geometric shapes, illustrate an advanced understanding of geometry that was both theoretical and applied.

Baudhāyana's contributions, including the early formulation of the Pythagorean theorem and techniques for transforming shapes, reveal a deep mathematical knowledge that predates and complements later Greek and modern mathematical developments. His work, with its emphasis on geometric constructions and practical applications, underscores the rich and diverse history of mathematical thought, bridging ancient practices with foundational geometric principles still relevant today. The comparison between Sulba Sutras methods and modern geometry highlights the evolution of mathematical techniques, from geometric constructions to algebraic proofs, demonstrating the enduring significance of Baudhāyana's insights in the broader context of mathematical history.

Acknowledgment

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Conflict of Interest

The author declares no conflict of interest regarding the publication of this research.

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