Orchid Academia Siraha ISSN: 2976-1379 (Print)
Vol. 1, Issue 1, Dec. 2022: 53-66 ISSN: 2976-1387 (Online)

Differential Equations of Motion of the Particle

Asst. Prof. Uday Kumar Karn
Department of Mathematics, Patan Multiple Campus, Lalitpur
karnuday33@gmail.com
Assoc. Prof. Sushil Chandra Karna*, PhD
Department of Mathematics, MBMC, Rajbiraj, Saptari
karnasushil55@gmail.com
* Corresponding Author

Abstract

This paper deals with linearised and normalized differential equations of relative motion of the system under the influence of magnetic force air resistance and oblateness of the earth in Nechvill's co-ordinate system. We have linearised the problem keeping in view that the length of the string connecting the two satellites is infinitesimally small in comparison to the distance of centre of mass of the system from the centre of attracting force and introduction of rotating frame of reference eliminates the product terms as usual. We have obtained a system of six orders non-autonomous, non-linear equations of motion describe the motion of one of two satellites and the motion of satellite relative to the first may easily be obtained by $m_1\overrightarrow{\rho_1}+m_2\overrightarrow{\rho_2}=0$

Keywords: differential equations of motion, frame of reference, linearized, normalized, oblateness

Introduction

A light, flexible, and extensible wire is placed in the center of the earth's gravitational field to connect the effects of air resistance, magnetic force, and the earth's oblateness on the motion of the satellites.The system's motion is investigated in relation to its center of mass, which is thought to be travelling along a Keplerian orbit.The differential equations of motion of the system under the effect of air resistance, magnetic force, and earth's oblateness have been determined using the first kind of Lagrange's equations of motion.The normalized and linearized differential equations for the mass of the system's particle m_1 were developed under the

 Vol. 1, Issue 1, Dec. 2022 ISSN: 2976-1379 (Print)
ISSN: 2976-1387 (Online)

presumption that the connecting cable's length is very little in comparison to the satellites' distance from the planet's center Flectcher (1965). Then differential equations of motion in rotating frame of reference obtained. After using Nechvill's co-ordinate system, a set of non-autonomous, non-homogeneous differential equations of the particle of mass m_1 of the system derived (Elsgotts, 1973).

Equations of Motion of the centre of mass

Let's think about how a system of particles with masses m_1 and m_2, respectively, would move in the earth's gravitational field if they were connected by a thin, flexible, and extensible string. Let $\vec{r}_1$ and $\vec{r}_2$ be their radius vectors with respect to the centre of the earth (Etkin, 1964).

Then, using the first type of Lagrange's equations for motion, we formulate the differential equations for the motion of the particles with masses m_1 and m_2 as.

$$m_1 \ddot{\vec{r}}_1 + \frac{m_1 \mu \vec{r}_1}{r_1^3} + \lambda \left[\frac{\left| \vec{r}_1 - \vec{r}_2 \right| - l_0}{l_0} \right] \frac{\left(\vec{r}_1 - \vec{r}_2 \right)}{\left| \vec{r}_1 - \vec{r}_2 \right|} = 0$$

$$\text{and } m_2 \ddot{\vec{r}}_2 + \frac{m_2 \mu \vec{r}_2}{r_2^3} - \lambda \left[\frac{\left| \vec{r}_1 - \vec{r}_2 \right| - l_0}{l_0} \right] \frac{\left(\vec{r}_1 - \vec{r}_2 \right)}{\left| \vec{r}_1 - \vec{r}_2 \right|} = 0 \qquad (1)$$

Where λ is the Hook's modulus of elasticity and μ is the product of gravitational constant with the mass of attracting center.

The condition of constraint is given by

$$\left| \vec{r}_1 - \vec{r}_2 \right|^2 \le l_0^{\,2} \qquad (2)$$

where l_0 is the string's natural length between the two satellites with masses m_1 and m_2

If the equality signs in (2) holds; then $\lambda \neq 0$ and the motion takes place with tight string and consequently tension in the string comes into play.

Motion of the System Relative to their Centre of Mass

In order to study the relative motion of the system, we must predict about the motion of the centre of mass in the beginning itself (Chernouske, 1963, 1964).

Let $\vec{R}$ be the radius vector of the centre of mass of the system w.r.to the attracting center (Earth). Then we have

$$\vec{R} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2} \tag{3}$$

Adding the two equations of (1) we get

$$M\ddot{\vec{R}} + \mu\left(\frac{m_1\vec{r}_1}{r_2^3} + \frac{m_2\vec{r}_2}{r_2^3}\right) = 0 \tag{4}$$

Where $M = m_1 + m_2$

Now we shall make use of the assumption that the maximum extended length l_E of the string is infinitesimally small compared to the distances r_1 and r_2 of the particles from the centre of force.

i.e. $$\frac{l_E}{r_1} << 1 \text{ and } \frac{l_E}{r_2} << 1 \tag{5}$$

Let $\vec{\rho}_1$ and $\vec{\rho}_2$ denote the radius vector of the particles of mass m_1 and m_2 respectively with origin at the centre of mass of the system (Sharma, 1974).

Then, $\vec{r}_1 = \vec{R} + \vec{\rho}_1$

$$\vec{r}_2 = \vec{R} + \rho_2 \tag{6}$$

Then clearly $\rho_1 < l_E$

$\rho_2 < l_E$

Thus $\rho_1 << r_1$

and $\rho_2 << r_2$

 Vol. 1, Issue 1, Dec. 2022 ISSN: 2976-1379 (Print)
ISSN: 2976-1387 (Online)

But $r_1 \approx r_2 \approx R$

Hence $\frac{\rho_1}{R} << 1 \text{ and } \frac{\rho_2}{R} << 1$ (7)

Eliminating $\vec{r}_1$ and $\vec{r}_2$ from (4) with the help of (6) and then expanding in ascending particles of small equalities.

$\frac{\rho_1}{R}$ and $\frac{\rho_2}{R}$ we get

$$M\ddot{\vec{R}} + \frac{\mu M \vec{R}}{R^3} = \vec{F}_1 + \vec{F}_2 + 0(3) \tag{8}$$

Where $\vec{F}_1 = \frac{3\mu}{R^3}\left(m_1\vec{\rho}_1 + m_2\vec{\rho}_2\right) - \frac{3\mu}{R^5}\left[\vec{R}\left(m_1\vec{\rho}\rho + m_2\rho_2\right)\right]\vec{R}$

$$\vec{F}_2 = -\frac{3}{2}\frac{\mu}{R^3}\left[m_1\left\{\left(\frac{\rho_1}{R}\right)^2 - 5\left(\frac{\vec{R}}{R}\frac{\vec{\rho}_1}{R}\right)\right\}\right] + m_2\left\{\left(\frac{\rho_2}{R}\right)^2 - 5\left(\frac{\vec{R}}{R}\frac{\rho_2}{R}\right)^2\right\}\vec{R} \tag{9}$$

(3) stands for third and higher order terms in infinitesimals $\frac{\rho_1}{R}$ and $\frac{\rho_2}{R}$

Equations of Motion of the System under the Influence of AIR Resistance Magnet

Force. In the Central Gravitational of Oblate Earth

Let's imagine that the two satellites of the system are particles with masses of m_1 and m_2 and their radius vectors $\vec{r}_1$ and $\vec{r}_2$ respectively, in relation to the attractive center (Beletaky & Novicova, 1969). The length of the string connecting the two particles of mass m_1 and m_2 be denoted by l_0. Suppose that '1' be the string length at any time (Karn, 2002).

Then the constraint of system is given by

$$\left|\vec{r}_1 - \vec{r}_2\right|^2 \le l_0^{\ 2} \tag{10}$$

Under the inspiration of air resistance, magnetic force, and earth's oblateness, the equation of motion of two particles of mass m_1 and m_2 associated by an extendable string of natural length l_0 can be stated using Lagrange's equation of motion of first kind as follows:

$$m_1\ddot{\vec{r}}_1 + \frac{m_1\mu\vec{r}_1}{r_1^3} + \frac{3m_1\mu k_2}{r_1^5}\vec{r}_1 + \lambda\left[\frac{\left|\vec{r}_1-\vec{r}_2\right|-l_0}{l_0}\right]$$

$$\frac{\left(\vec{r}_1-\vec{r}_2\right)}{\left|\vec{r}_1-\vec{r}_2\right|} = \vec{F}_{a_1} + Q_1\left(\vec{r}_1\times\vec{H}\right)$$

and $m_2\ddot{\vec{r}}_2 + \frac{m_2\mu\vec{r}_2}{r_2^3} + \frac{3m_2\mu k_2\vec{r}_2}{r_2^5} - \lambda\left[\frac{\left|\vec{r}_1-\vec{r}_2\right|-l_0}{l_0}\right]\frac{\left(\vec{r}_1-\vec{r}_2\right)}{\left|\vec{r}_1-r_2\right|} = \vec{F}_{a_2} + Q_2\left(\vec{r}_2\times\vec{H}\right)$

Where $\vec{F}_{ai}$ (i = 1, 2, ...) is the aerodynamic force

$$k_2 = \overline{\in}R_e^2/3$$

$$\alpha R = \frac{R_e - R_P}{R_e} = \text{Earth's of oblateness}$$

We have

and
$$\left.\begin{aligned}\vec{\rho}_1 &= \frac{m_2}{m_1+m_2}\left(\vec{r}_1-\vec{r}_2\right)\\ \vec{\rho}_2 &= \frac{m_1}{m_1+m_2}\left(\vec{r}_2-\vec{r}_1\right)\end{aligned}\right\} \quad (11)$$

multiplying first equation of (11) say m_1 and equation by m_2 and adding we get.

$$m_1\vec{\rho}_1 + m_2\vec{\rho}_2 = 0 \quad (12)$$

Hence $\vec{F}_1$ given in (9) vanishes identically on using (12). Therefore, neglecting the 2nd and higher order agitation relations in (9), the centre of

 ISSN: 2976-1379 (Print)
ISSN: 2976-1387 (Online)

mass of the system’s equation given by (8) takes the system (Chetayev, 1966).

$$M\ddot{\vec{r}} + \frac{\mu M \vec{R}}{R^3} = 0 \tag{13}$$

Clearly (13) demonstrate that the system's center of mass can be anticipated to move with more precision up to and including order-infinitesimal along a Keplerian elliptical orbit (Dubeshin, 1952) i.e. $\frac{\rho_1}{R}$ and $\frac{\rho_2}{R}$.

The center of mass of the system of two satellites connected by an extendable string in the center of the gravitational field of attraction has thus been demonstrated to travel along a specific Keplerian elliptical orbit (Singh, 1971, 1973). $\bar{\in} = \alpha_R - \frac{m}{2}$ where $m = \frac{\Omega^2 R_e}{g_e}$

$\Omega =$ Rotational angular velocity of the earth.

$R_e =$ Earth's equatorial radius.

$R_p =$ Earth's polar radius.

$g_e =$ Earth's gravitational field as a function.

$Q_i = \frac{charge \quad q_i \quad on the \quad i^{th} \quad particle}{Velocity \quad of \quad light \quad C}$; (i = 1, 2)

$\vec{F} =$ The earth's magnetic field intensity for equatorial satellites

$= \frac{-\vec{\nabla}(\vec{m}.\vec{r})}{r^3}$

$\vec{m} =$ Earth’s magnetic moment.

Since it is expected that air drag in this situation varies as the square of the speed of the moving particles (Solue, 1960), it can be expressed as follows:

$$\vec{F}_{a_i} = -\rho_a C_i \left|\dot{\vec{r}}_0\right| \dot{\vec{r}}_i \quad (14)$$

Where ρ_a is the density of the air, which will be assumed constant throughout the study.

C_i = Ballistic Coeff[H]

Now the equations of motion (2) can be written in the form

$$m_1\ddot{\vec{r}}_1 + \frac{m_1\mu\vec{r}_1}{r_1^3} + \frac{3\mu m_1 k_2 \vec{r}_1}{r_1^5} + \lambda\left[\frac{\left|\vec{r}_1 - \vec{r}_2\right| - l_0}{l_0}\right]$$

$$\frac{(r_1 - r_2)}{\left|\vec{r}_1 - \vec{r}_2\right|} = -\rho_a C_1 \left|\dot{\vec{r}}_1\right| \dot{\vec{r}}_1 + Q_1\left(\dot{\vec{r}}_1 \times H\right)$$

and

$$m_2\ddot{\vec{r}}_2 + \frac{m_2\mu\vec{r}_2}{r_2^3} + \frac{3m_3\mu k_2 \vec{r}_2}{r_2^5} - \lambda\left[\frac{\left|\vec{r}_1 - \vec{r}_2\right| - l_0}{l_0}\right]\frac{\left(\vec{r}_1 - \vec{r}_2\right)}{\left|\vec{r}_1 - \vec{r}_2\right|} =$$

$$-\rho_0 C_2 \left|\dot{\vec{r}}_2\right| \dot{\vec{r}}_2 + Q_2\left(\vec{r}_2 \times \vec{H}\right) \quad (15)$$

Linearised and Normalized Differential Equations of Relative motion of the system. We divide the two equations of (4) by m_1 and m_2 respectively and then on subtraction; we get

$$\ddot{\vec{r}}_1 - \ddot{\vec{r}}_2 + \mu\left(\frac{\vec{r}_1}{r_1^3} - \frac{\vec{r}_2}{r_2^3}\right) + \lambda\frac{(m_1 + m_2)}{m_1 m_2}\left[\frac{\left|\vec{r}_1 - \vec{r}_2\right| - l_0}{l_0}\right]$$

$$\frac{\left(\vec{r}_1 - \vec{r}_2\right)}{\left|\vec{r}_1 - \vec{r}_2\right|} + 3\mu k_2\left(\frac{\vec{r}_1}{r_1^5} - \frac{\vec{r}_2}{r_2^5}\right) + \rho_a\left[C_1\left|\dot{\vec{r}}_1\right|\dot{\vec{r}}_1 - C_2\left|\ddot{\vec{r}}_2\right|\dot{\vec{r}}_2\right] = \frac{Q_1}{m_1}\left(\dot{\vec{r}}_1 \times \vec{H}\right) - \frac{Q_2}{m_2}\left(\dot{\vec{r}}_2 \times \vec{H}\right) \quad (16)$$

 Vol. 1, Issue 1, Dec. 2022 ISSN: 2976-1379 (Print)
ISSN: 2976-1387 (Online)

using (6) and ignoring minuscules of 2nd and higher order we get

$$u\left(\frac{\vec{r}_1}{r_1^3}-\frac{\vec{r}_2}{r_2^3}\right)=\frac{\mu}{R^3}(\vec{r}_1-\vec{r}_2)-\frac{3\mu\vec{R}}{R^5}\left[\vec{R}.(\vec{r}_1-\vec{r}_2)\right] \tag{17}$$

$$3\mu k_2\left(\frac{\vec{r}_1}{r_1^5}-\frac{\vec{r}_2}{r_2^5}\right)=\frac{3\mu k_2}{R^5}(\vec{r}_1-\vec{r}_2)-\frac{15\mu k_2}{R^7}\left[\vec{R}(\vec{r}_1-\vec{r}_2)\right]\vec{R} \tag{18}$$

and $\rho_a\left[C_1|\dot{\vec{r}}_1|\dot{\vec{r}}_1-C_2|\dot{\vec{r}}_2|\dot{\vec{r}}_2\right]=\rho_a\dot{R}\dot{\vec{R}}(C_1-C_2)+\rho_a R$

$$\left[\frac{\dot{\vec{R}}(\dot{\vec{R}}.\vec{\rho}_1}{R^2}+\dot{\vec{\rho}}\right]\frac{(C_1m_2+C_2m_1)}{m_2} \tag{19}$$

with the help of (17), (18) and (15) in (16) we get on using

$$\vec{H}=-\frac{\vec{\nabla}\vec{M}.\vec{r}_1}{r_1^3}\text{ for (i = 1,2)}$$

$$\ddot{\vec{r}}_1-\ddot{\vec{r}}_2+\frac{\mu}{R^3}(\vec{r}_1-\vec{r}_2)-\frac{3\mu\vec{R}}{R^5}\left[\vec{R}.(\vec{r}_1-\vec{r}_2)\right]$$

$$+\lambda\left(\frac{m_1+m_2}{m_1\times m_2}\right)\left[\frac{|\vec{r}_1-\vec{r}_2|-l_0}{l_0}\right]\frac{(\vec{r}_1-\vec{r}_2)}{|\vec{r}_1-\vec{r}_2|}$$

$$+\frac{3\mu\kappa_2}{R^5}(\vec{r}_1-\vec{r}_2)-\frac{15\mu k_2}{R^7}\left\{\vec{R}(\vec{r}_1-\vec{r}_2)\right\}\vec{R}$$

$$+\rho_a\dot{R}\dot{\vec{R}}(C_1-C_2)+\rho a\dot{R}\frac{\left[\dot{\vec{R}}\left(\dot{\vec{R}}.\dot{\vec{\rho}}_1\right)\right]}{R^2}+\vec{\rho}$$

$$\frac{c_1m_2+c_2m_1}{m_2}=-\left[\frac{Q_1}{m_1}\left\{\dot{\vec{r}}_1\times\vec{\nabla}\times\nabla\frac{\vec{M}.\vec{r}_1}{r_1^2}-\frac{Q_2}{m_2}\left\{\dot{\vec{r}}_2\times\vec{\nabla}\left(\frac{\vec{m}\vec{r}_2}{r_2^2}\right)\right\}\right\}\right] \tag{20}$$

But, $\vec{r}_1-\vec{r}_2=\vec{\rho}_1-\vec{\rho}_2=\left(\dfrac{m_1+m_2}{m_2}\right)\vec{\rho}$ (21)

dividing throughout by $\dfrac{m_1 + m_2}{m_2}$

$$\ddot{\vec{\rho}} + \frac{\mu\vec{\rho}_1}{R^3} - \frac{3\mu\vec{R}\left(\vec{R}.\vec{\rho}_1\right)}{R^5} + \frac{3\mu\kappa_2\vec{\rho}_1}{R^5} -$$

$$\frac{15\mu k_2\vec{R}}{R^7}\left(\vec{R}.\rho_1\right) + \lambda_\alpha\left[1 - \frac{\upsilon}{|\vec{\rho}_1|}\right]\vec{\rho}_1$$

$$+ \rho_a\dot{R}\dot{\vec{R}}\left(c_1 - c_2\right)\frac{m_2}{m_1 + m_2} + \rho a\dot{R}\left[\dot{\vec{R}}\frac{\left(\dot{\vec{R}}.\dot{\vec{\rho}}_1\right)}{R^2} + \vec{\rho}_1\right]\frac{c_1 m_2 + c_2 m_1}{m_1 + m_2}$$

$$= \frac{-m_2}{m_1 + m_2}\left[\frac{Q}{m_1}\dot{\vec{r}}_1 \times \vec{\tau}\left\{\frac{\vec{M}.\vec{r}_1}{r_1^{\,3}}\right) - \frac{Q_2}{m_2}\vec{r}_2 \times \vec{\tau}\left(\frac{\vec{M}.\vec{r}_2}{r_2^{\,3}}\right)\right] \tag{22}$$

Where $\lambda_\alpha = \dfrac{m_1 + m_2}{m_1 m_2}\dfrac{\lambda}{l_0}$ (23)

$$\gamma = \frac{m_2 l_0}{m_1 + m_2}$$

Since $\dfrac{1}{r_i^3} = \dfrac{1}{\left(r_i^{\,2}\right)^3} = \dfrac{1}{\left(r_i^{\,2}\right)^3 L} = \dfrac{1}{\left[\left(\vec{R} + \vec{\rho}_2\right)^2\right]^{3/2}}; i = 1, \quad 2$

$$= \frac{1}{R^3} - \frac{3\vec{R}.\vec{\rho}_i}{R^5}$$

$$\frac{\vec{r}_i}{r_i^{\,3}} = \left(\vec{R} + \vec{\rho}_i\right)\left[\frac{1}{R^3} - \frac{3\vec{R}.\vec{\rho}_1}{R^5}\right]; i = 1,2$$

$$= \frac{\vec{R} + \vec{\rho}_i}{R^3} - \frac{3\vec{R}.\vec{\rho}_i}{R^5}\left(\vec{R} + \vec{\rho}_i\right) i = 1,2 \tag{24}$$

Hence, we have

$$\frac{Q_1}{m_1}\left\{\dot{\vec{r}}_1 \times \vec{\nabla}\left(\frac{\vec{M}.\vec{r}_1}{r_1^{\,3}}\right)\right\}\frac{Q_2}{m_2}\left\{\dot{\vec{r}}_2 \times \vec{\nabla}\left(\frac{\vec{m}.\vec{r}_2}{r_2^{\,3}}\right)\right\} = \vec{R} \times \vec{\nabla}\left(\frac{\vec{M}.\vec{R}}{R^3}\right)\left(\frac{Q_1}{m_1} - \frac{Q_2}{m_2}\right) \tag{25}$$

 Vol. 1, Issue 1, Dec. 2022 ISSN: 2976-1379 (Print)
ISSN: 2976-1387 (Online)

Using (25) in (22) we obtain the linearized vector equation of motion for the particle of mass m_1 comparative to the centre of mass of the structure in the procedure.

$$\ddot{\vec{\rho}}_1 + \frac{\mu\vec{\rho}_1}{R^3} - \frac{3\mu\vec{R}(\vec{R}.\vec{\rho}_1)}{R^5} + a_i\vec{R} + a_2{}^1$$

$$\left[\frac{\dot{\vec{R}}\left(\vec{R}.\vec{\rho}_1\right)}{R^2}\dot{\vec{\rho}}_1\right] + \frac{3\kappa_2\mu\vec{\rho}_1}{R^5} - \frac{15\mu\kappa_2\vec{R}\left(\vec{R}.\vec{\rho}_1\right)}{R^7} + \lambda_\alpha\left[1 - \frac{r^3}{\left|\vec{\rho}_1\right|}\right]\vec{\rho}_1$$

$$\frac{-m_2}{m_1+m_2}\left[\vec{R}\times\vec{\nabla}\left(\frac{\vec{M}.\vec{R}}{R^3}\right)\left(\frac{Q_1}{m_1} - \frac{Q_2}{m_2}\right)\right] \tag{26}$$

The condition of constraint given by (1) reduces to

$$\left|\vec{\rho}_1\right|^2 \le \upsilon^2 \tag{27}$$

If in (27) the system will be moving with moveable string if the inequality sign is true (Thakur, 1959). In this case, the motion is free from constraint and hence $\lambda_\alpha = 0$. If the equality signs in (27) holds then the motion is constrained motion and hence $\lambda_\alpha \ne 0$. (Etkin, 1964).

Let us stabilize the vector $\vec{\rho}_1$ by familiarizing.

$$\vec{\rho}_1 = \frac{\upsilon\vec{\rho}_1 *}{l_0}$$

(28)

Then the vector equation (26) of the particle of mass m_1 takes the form

$$\ddot{\vec{\rho}}_1 * + \frac{\mu}{R^3}\vec{\rho}_1 * - \frac{3\mu\vec{R}\left(\vec{R}.\vec{\rho}_1 *\right)}{R^5} + a_1\dot{\vec{R}} + a_2\left[\frac{\dot{\vec{R}}\left(\vec{R}.\dot{\vec{\rho}}_1\right)}{R^2} + \dot{\vec{\rho}}_1 *\right]$$

$$+3\mu\kappa_2\vec{\rho}_1^{\,*} - \frac{15\,\mu\kappa_2\vec{R}}{R^7}\left(\vec{R}.\vec{\rho}_1^{\,*}\right)+$$

$$\lambda_\alpha\left[1-\frac{l_0}{\left|\vec{\rho}_1^{\,*}\right|}\right]\vec{\rho}_1^{\,*} = -\vec{R}\times\vec{\nabla}\left(\frac{\vec{M}.\vec{R}}{R^3}\right)\left(\frac{Q_1}{m_1}-\frac{Q_2}{m_2}\right) \tag{29}$$

Where $a_1 = \frac{a_1'(m_1+m_2)}{m_2} = \rho a R(c_1-c_2)$

$$a_2 = \frac{a_2^{\,1}(m_1+m_2)}{m_2} = \rho a R\frac{(c_1m_2+c_2m_1)}{m_2} \tag{30}$$

The system's mass center and the stabilized vector equation of the relative motion of the particle of mass m_1 (Hagihara, 1957).

Now the condition of constraint given by (28) takes the from

$$\left|\vec{\rho}_1^{\,*}\right|^2 \le l_0^{\,2} \tag{31}$$

Again, let us write

$\vec{\kappa}E$ = unit vector along the axes of the magnetic dipole of the earth

$= \frac{\vec{M}}{\left|\vec{M}\right|}$

u_E = The volume of the magnetic moment of the earth dipole M

$\vec{\rho}_r$ = unit vector along the radius vector $\vec{R}$

$= \frac{\vec{R}}{\left|\vec{R}\right|}$ where $\vec{R}$ is the radius vector of the centre of mass of the system with respect to the appealing center of force.

Hence, equation of motion given by (29) takes the form

$$\vec{\rho}_1^{\,*} + \frac{\mu}{R^3}\vec{\rho}_1^{\,*} - \frac{3\mu\vec{R}}{R^5}\left(\vec{R}.\vec{\rho}_1^{\,*}\right) + a_1\vec{R} + a_2\left[\frac{\dot{\vec{R}}\left(\vec{R}.\vec{\rho}_1^{\,*}\right)}{R^2} + \dot{\vec{\rho}}^{\,*}\right] + \frac{3\mu\kappa_2}{R^5}\vec{\rho}_1^{\,*} -$$

$$\frac{15\mu\kappa_2}{R^7}\vec{R}\left(\vec{R}.\vec{\rho}_1^{\,*}\right)+\lambda_\alpha\left[1-\frac{l_0}{\left|\rho^{2}{}_{1}^{*}\right.}\right]\vec{\rho}_1^{\,*}$$

$$=-\left(\frac{Q_1}{m_1}-\frac{Q_2}{m_2}\right)\left[\frac{\mu E^R}{R^3}\times\vec{\kappa}E-3\left(\vec{\kappa}E.\vec{\rho}_1\right)\vec{R}\times\vec{\rho}_r\right] \tag{32}$$

Our consideration of equation (32) leads us to the conclusion that there are two terms that make up atmospheric drag: one with coefficient as and the other with coefficient a_2 (Leipholz, 1980). The dissipative force, which is a component of atmospheric drag resulting from air friction, is shown by the phrase with coefficient a_2 with $\vec{\rho}_1^{\,*}$ as a factor. We may disregard these terms as the coefficient a_2 a parameter of the aerodynamic force, is a small quantity that is multiplied by a small quantity $\vec{\rho}_1^{\,*}$ since we are just examining first-order turbulence. Furthermore, if we continue using this dissipatine phrase, it will be hard to solve this topic analyically

Thus, the equation of motion (32) with only non-dissipative part of the aerodynamic force (Austin, 1965) takes the form

$$\ddot{\vec{\rho}}_1^{\,*}+\frac{\mu}{R^3}\vec{\rho}_1^{\,*}-\frac{3\mu}{R^5}\vec{R}\left(\vec{R}.\vec{\rho}_1\right)+a_1\dot{\vec{R}}+\frac{3\mu\kappa_2}{R^5}\vec{\rho}_1^{\,*}-\frac{15\mu\kappa_2}{R^7}\vec{R}\left(\vec{R}.\vec{\rho}_1^{\,*}\right)+\lambda_\alpha\left[1-\frac{l_0}{\left|\vec{\rho}_1^{\,*}\right|}\right]\vec{\rho}_1^{\,*}$$

$$=-\left(\frac{Q_1}{m_1}-\frac{Q_2}{m_2}\right)\left[\frac{\mu_E\vec{R}}{R^3}\times\vec{\kappa}_E-3\left(\kappa_E.\vec{\rho}_r\right)\vec{R}\times\vec{\rho}_r\right] \tag{33}$$

Conclusion

This equation (33) is the basic equation of motion of the particle of mass m_1 of the system. The motion of the other particle of mass m_2 can be easily obtained with the help of

$$m_1\vec{\rho}_1+m_2\vec{\rho}_2=0$$

References

Austin, F. (1965). Non-linear dynamics of free rotating flexibly connected double mass space station. *Journal of spacecraft and Rockets, 2*(6).

Beletaky, V.V. & Novikova, E.T. (1969). About the relative motion of two connected bodies in orbit. *Kosmicheskiye Issledovania, 7*(3), 377-384.

Chernouske, F. L. (1964). About the motion of the satellite to its centre of mass under the action of gravitational moments. *Applied Mathematics and Mechanics, 27*(3), 474-483.

Chernouske, F. L. (1963). Resonance problem in the motion of the satellite relative to its centre of mass. *Journal of Computer Mathematics and Mathematical Physics, 3*, 528-533.

Chetayev, N.G. (1966). *Stability of motion* (3rd ed.). "Nauka" Moscow, (Russian)

Das, S. K. (1969). *Effect of magnetic force on the motion of a system of two cable connected satellites in orbit.*

Dubeshin, G. N. (1952). *Basic theory of stability of motion.* Moscow State University.

Elsgotts, L. (1973). *Differential equations and the calculus of* variations. Mir Publishurs Moscow.

Etkin, B. (1964). Dynamics of gravity oriented orbiting systems with applications to passive stabilization. *A.I.A.A. Journal, 2*(6), 1008-104.

Flectcher, H. N. (1965). Dynamics analysis of two-body gravitationally oriented satellite Bell system technical. J*ournal, 42*, 2239-2266.

Hagihara, Y. (1957). *Stability incelestial mechanics* Kasai, Tokyo. In the plane of an elliptical orbit. *Cosmic Research, 15*, 698.

Karna, S. C. (2002). *Effect of three perturbative force namely air resistance, magneticforce of the earth on the motion and stabilities connected by elastic cable with central gravitational field of oblate* earth [Unpublished doctoral dissertation]. University of Bihar.

Vol. 1, Issue 1, Dec. 2022 ISSN: 2976-1379 (Print)
ISSN: 2976-1387 (Online)

Leipholz. (1980). *Stablility theory*. New York & London: Academic Press.

Sharma, B. (1974). *The motion of a system of two cable connected satellites in the atmosphere* [Unpublished doctoral dissertation]. University of Bihar.

Singh, R. B. (1973). The motion of two connected bodies in an elliptical orbit. *Bulletin of the Moscow State University Mathematics-Mechanics,* No. 3, 82-86.

Singh, R. B. (1971). Three-dimensional motion of two-connected bodies in the central gravitational field of force. *Russia Collections Problem of Guided Motion in Mechanics,* 210-215.

Soule, P. M. (1960). Rendezvous with satellites in elliptical orbits of low eccentricities. *AAS,* 60-71, Seattle, August 3-11.

Thakur, H. K. (1959). *The motion of a system of two satellites connected by an elastic calbe* [Unpublished doctoral dissertation]. University of Bihar.