

## Variation of Displacement Fields for Great Earthquakes due to Rectangular Fault System

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### Abstract

*The expressions for the displacement fields produced by a finite rectangular fault associated with great earthquakes are explained by elastic theory of dislocation. The variation of displacement fields has been estimated by considering the elastic half-space in contact with a traction free boundary and varying depth, distance from top to bottom of the fault. It is also carried out in an infinite elastic medium. Also, the variation of displacement fields with the distance from fault especially for strike slip fault shows that even if different fault length and approximately equal magnitudes, the variation of displacement fields with the distance from the fault is almost same in the case of great earthquakes viz. San Francisco (1906) and Bihar-Nepal (1934).*

**Keywords:** Lamé's constant, ring of fire, seismic waves, Burger vector or displacement vector

### Introduction

Earthquake is a form of energy of wave motion which originates in a limited region and spreads out in all direction from the source of disturbance and temporarily vibrates the surface of the earth. An earthquake is caused as a result of the fracture of weak zone inside the earth and it releases energy catastrophically which is experienced on the surface of the earth. The development in the field of seismology has been found to be accelerated at about the middle of the eighteenth century. John Michel of England related the earthquake phenomenon with the wave motion in the earth during 1970. Von Hoff, published an earthquake catalogue for the first time in 1840. Mallet used the word seismic for the first time in the last quarter of nineteenth century in order to describe the motion in the surface of the earth. The catastrophic effect of an earthquake had been related with the geological and tectonic processes up to early twentieth century.

Nowadays, seismology has been grown up as a branch of science that study of earthquake phenomenon and the earth's interior.

Earthquakes can be classified by various ways like on the basis of nature of source, order of magnitude, depth of focus, and distance between epicenter and station. On the basis of nature, earthquakes have been categorized as artificial and natural earthquake. Artificial earthquakes generate as a result of the disturbance caused by artificial process like bomb explosions, blasting for the roads construction, geophysical explorations, test of explosive items, etc. The collapse of caves or underground activities, slides and slumps or landslides may also cause earthquake which can be felt up to few kilometers. Sometimes, large earthquakes are attributed to great landslides. The energy release from rock bursts in mines may produce a true earthquake which sometimes might be recorded by distant seismograph. Earthquakes are also experienced as a result of the fall of meteorites (e.g. Richter, 1969). The natural disturbances caused inside the earth are the main sources of natural earthquakes. The causes may be either volcanic or tectonic. Volcanic activity or induce effects from geodynamic processes are responsible for volcanic earthquakes (Stacey, 1977). They lie in the well-known parts of the earth. One of such regions, the circumpacific belt, is also known as "ring of fire". Volcanic earthquakes caused as a result of volcanic eruptions are found to be relatively small. Sometimes major earthquakes have also originated near the plate of volcanic eruption. From field investigations, it has also been observed that earthquakes are not directly related with volcanic activities (e.g. Bullen and Bruce, 1985). Earthquakes are the direct evidence of the dynamics of the earth. Those caused by the release of elastic strain energy are called tectonic earthquakes. The outermost shell of the earth behaves as brittle material. The top portion is divided into six large and many other comparatively small thin layers called lithospheric plates. Below these plates, asthenosphere is composed of viscous materials and the conventional process is said to be responsible for the motion of these plates. The relative motion among them has been found capable to generate strain in the plate margins. The stresses are being generated in the boundary of the plate margins are being released in the form of seismic waves (Press, 1965). Such earthquake is known as interplate earthquake. The majority of earthquakes are of shallow type because earth's crust is heterogeneous according to depth of focus (Bott, 1982).

On the basis of Richter scale magnitude of the earthquake, it can be classified as e.g. Great earthquakes ( $M \geq 8$ ), Major earthquakes ( $7 \leq M < 8$ ), Moderate earthquakes ( $5 \leq M < 7$ ), Small earthquakes ( $3 \leq M < 5$ ), Micro earthquakes ( $1 \leq M < 3$ ) and Ultra micro earthquakes ( $M < 1$ ) (Lee and Stewart, 1981).

A fault is slip surface in the earth across which discontinuous land movement takes place and its configuration under the ground cannot be observed (e.g. Kasahara, 1981) so that the construction of a model of the fault on the basis of the little surface evidence available (e.g. the length of the fault and its offset during an earthquake). A rectangular shape for the fault plane, with one pair of its sides parallel to the free surface is often assumed. In fact, the displacement field around a fault is explained reasonably well by this type of theoretical model. In this article, some of the earthquake parameters such as half of fault length, total slip or burger vector, distance between source and observer. Image point are denoted by  $L$ ,  $U_1$ ,  $R$  and  $Q$ . In order to understand the variation of the ratio of displacement field to the burger vector versus distance from the fault of great earthquake by taking the different  $D$ , the distance from the surface to the bottom and estimate the components of displacement field i.e. parallel displacement, perp. Displacement and vertical displacement.

### **Theory**

The expressions for the displacement fields caused by a finite rectangular fault placed in an elastic half space in welded contact with another elastic space have been obtained by integrating analytically over the fault surface (e.g. Singh et al, 1993). Rectangular fault can be defined as having known fault length of rectangular shape for the fault plane with one pair of its side parallel to its free surface. The expressions for the displacement fields caused by a finite rectangular fault for poisson solid which is lame's constant equals to rigidity of medium in a uniform elastic half space ( $m = \mu_2/\mu_1 = 0$ ) is given by Mansinha and Smylie (1967). This result can also be obtained by generalizing the result of Singh et al. (1993). The variation of displacement fields with distance from the fault in a uniform elastic half space ( $m = 0$ ) medium. The field point at which the displacement is determined is  $(x_1, x_2, x_3)$  and the variable of integration on the fault are  $(y_1, 0, y_3)$  as shown in Figure 1.

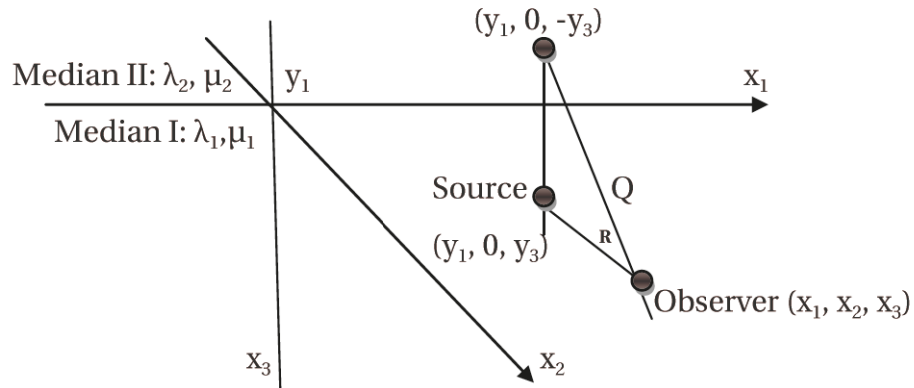


Figure 1. A source striking parallel to the  $x$ -axis lies at the point  $(y_1, 0, y_3)$ ,  $R$  is the distance between the observer at  $(x_1, x_2, x_3)$  and the source at  $(y_1, 0, y_3)$ ,  $Q$  is the distance between the observer at  $(x_1, x_2, x_3)$  and the image at  $(y_1, 0, -y_3)$

The displacement fields can be derived for a finite rectangular fault, extending horizontally over the range  $-L \leq x \leq L$  and vertically over the range  $d \leq x_3 \leq D$ , where  $x_1, x_3$  are coordinate in the fault system. The  $x_3$  axis is measured positive downward and the coordinate system is right-handed. The fault then lies in the plane  $x_2 = 0$ . It represents a uniform dislocation surface in the elastic half space  $x_3 \geq 0$ . The coordinate system and the fault parameters are shown in Figure 2.

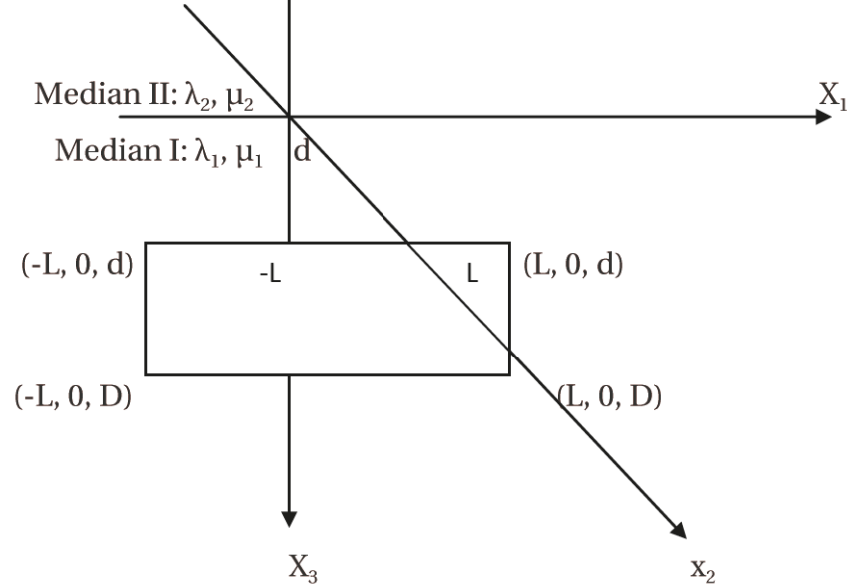


Figure 2. Geometry of a rectangular fault occupying the region  $-L \leq x \leq L, x_2=0, d \leq x_3 \leq D$  of medium ( $x_3 > 0$ ), which is welded contact with medium II ( $x_3 < 0$ )

The displacement field ( $u_1, u_2, u_3$ ) is found by integrating over the fault surface. For strike slip fault equation is given by

$$U_m = \frac{U_1}{4\pi(K+1)} \int_{-L}^D \int_{-L}^{+L} \left( \frac{\partial u_{m1}}{\partial y_2} + \frac{\partial u_{m2}}{\partial y_1} \right) dy_1 dy_3 \quad (1)$$

Here,  $U$  is the uniform dislocations over the fault surface. A Poisson solid has been assumed in such cases Lamé's constant ( $\lambda$ ) and rigidity of the medium ( $\mu$ ) are equal. The quantities ( $u_1, u_2, u_3$ ) are the displacement produced by a double couple forces in the half space located at ( $y, 0, y_3$ ) acting in the  $x_m = (m = 1.2.3)$  directions on taking  $m = \mu_2/\mu_1 = 0$  and the expressions for the displacement field caused by a finite rectangular fault in an elastic half-space with traction force boundary arrive at the corresponding equation, i.e.

$$u_1 = \frac{U_1}{12\pi} [x_2(x_1 - y_1) \left\{ \frac{2}{R(R+x_3-y_3)} - \frac{5Q+8y_3}{2Q(Q+x_3+y_3)^2} + \frac{4x_3y_3(2Q+x_3+y_3)}{Q^2(Q+x_3+y_3)^2} \right\} + 3 \tan^{-1} \left\{ \frac{(x_1-y_1)(x_3-y_3)}{Rx_2} \right\} - 3 \tan^{-1} \left\{ \frac{(x_1-y_1)(x_3-y_3)}{Qx_2} \right\}] \quad (2)$$

$$u_2 = \frac{U_1}{12\pi} \left[ \{-\log(R - x_3 - y_3)\} + \frac{1}{2} \log(Q + x_3 + y_3) - \frac{5x_3-3y_3}{2(Q+x_3+y_3)} - \frac{4x_3y_3}{Q(Q+x_3+y_3)} + x_2^2 \frac{2}{R(R+x_3-y_3)} - \frac{5Q+8y_3}{2Q(Q+x_3+y_3)^2} + \frac{4x_3y_3(2Q+x_3+y_3)}{Q^2(Q+x_3+y_3)^2} \right] \quad (3)$$

$$u_3 = \frac{U_1}{12\pi} x_2 \left[ \frac{2}{R} - \frac{2}{Q} + \frac{4x_3y_3}{Q^3} + \frac{3}{Q+x_3+y_3} + \frac{2(x_3+3y_3)}{Q(Q+x_3+y_3)} \right] \quad (4)$$

Notice that the symbol  $\parallel$  to indicate that the limits of the double integration.

The value of  $R$  and  $Q$  will be equal if the displacement field calculated at the free surface ( $x_1, x_2, 0$ ) then it can written as

$$R=Q= [(x_1-y_1)^2 + x_2^2 + y_3^2]^{1/2} \quad (5)$$



The fault parameters for two great earthquakes are mentioned in Table 1.

Table 1. *Magnitude and Fault Parameters*

Events	Magnitude	Half of fault length (L) Km.	Displacement vector ( $U_1$ ) Km
1. San Francisco April 18,1906	8.3	200	0.0061
2. Bihar-Nepal Jan 15, 1934	8.4	129	0.0275

In the case of San Francisco earthquake, the fault is in the plane  $x_2 = 0$  and extends horizontally and vertically over the range  $-L \leq x_1 \leq L$  and  $d \leq x_3 \leq D$  respectively. The value of  $D$  may be taken to  $2L$ ,  $L$ ,  $L/2$  but in this article the value of  $D = 2L$  is considered and assuming  $d = 0$ . Then the value of  $R$  and  $Q$  can be calculated from equation 5 and using assumption  $x_1$  is equal to zero for parallel displacement ( $u_1$ ) and using the assumption  $x_1$  is equal to the half of fault length  $L$  for perpendicular displacement ( $u_2$ ) as well as vertical displacement ( $u_3$ ). It should be noted that in both cases the same value of  $R$  and  $Q$  have been used. After putting all the values of fault parameters in equations (2), (3) and (4) yields the parallel, perpendicular and vertical displacement for the value of  $D = 2L$  which are mentioned in Table 2.

Table 2. *Displacement Fields with Displacement Vector and Distance from Fault for San Francisco Earthquake*

S.N.	( $x_2$ )	$u_1/U_1$	$u_2/U_1$	$u_3/U_1$
1	1	0.04980	-0.02885	0
2	200	0.01409	-0.08393	0.05275
3	400	$5.147 \cdot 10^{-3}$	-0.11459	0.07560
4	600	$2.655 \cdot 10^{-3}$	-0.13845	0.08324
5	800	$1.639 \cdot 10^{-3}$	-0.15534	0.08573
6	1000	$1.114 \cdot 10^{-3}$	-0.16754	0.08642

Where,  $x_2$  = Distance from fault,

$u_1$  = average value of a parallel displacement

$u_2$  = average value of perpendicular displacement

$u_3$  = average value of vertical displacement

For Bihar-Nepal earthquake event also follow the same process as above and the displacement fields for value of  $D = 2L$  are mentioned Table 3.

Table 3. Displacement Fields with Displacement Vector and Distance from Fault for Bihar-Nepal Earthquake

S.N.	( $x_2$ )	( $u_1/U_1$ )	( $u_2/U_1$ )	( $u_3/U_1$ )
1	1	0.04969	-0.02281	0
2	129	0.01408	-0.07229	0.05276
3	258	$5.105 \cdot 10^{-3}$	-0.10293	0.07561
4	387	$2.64 \cdot 10^{-3}$	-0.12682	0.08327
5	516	$1.723 \cdot 10^{-3}$	-0.14368	0.08575
6	645	$1.127 \cdot 10^{-3}$	-0.15595	0.08643

Variation of Displacement Fields to Displacement Vector with Distance from Fault for San Francisco earthquake that represented one event is presented in Figure 3, 4, and 5.

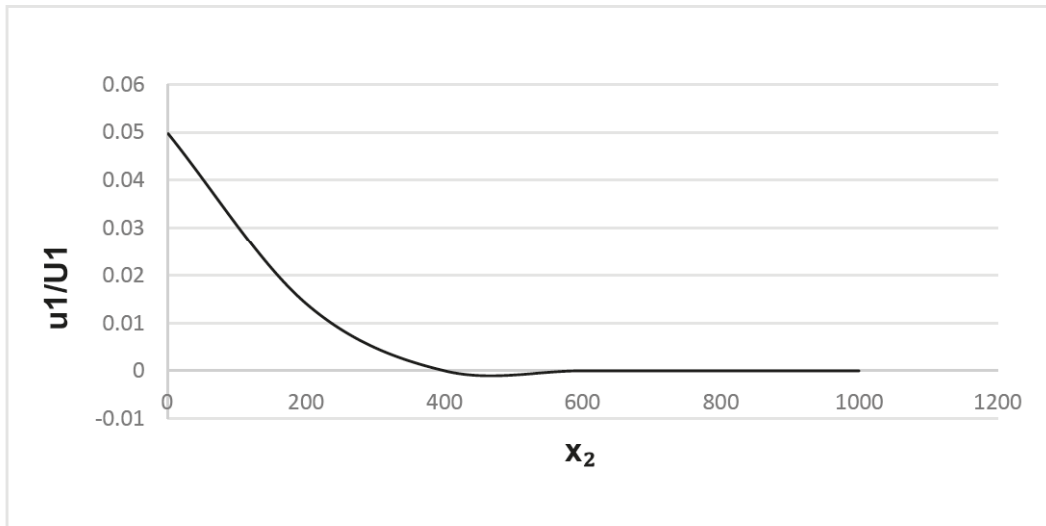


Figure 3. Variation of ratio of parallel displacement field and displacement vector with  $x_2$

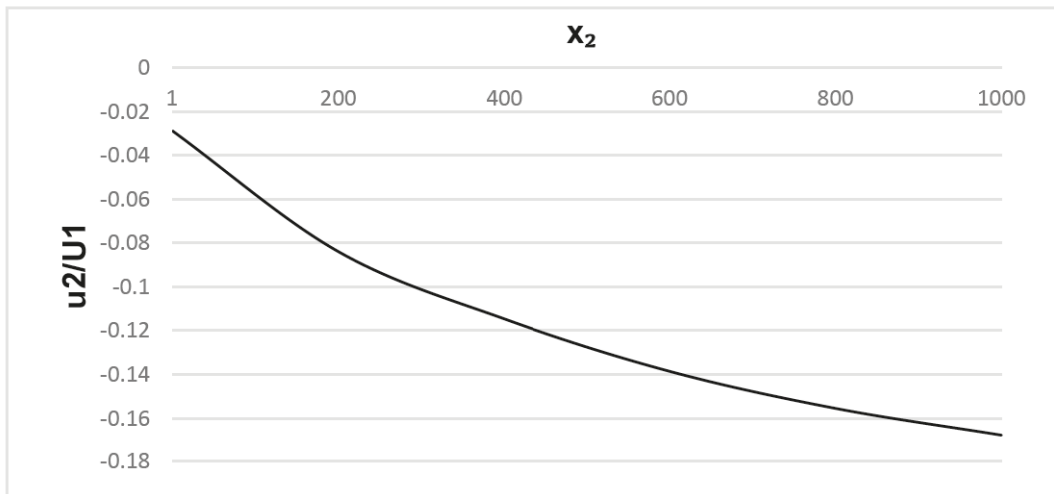


Figure 4. Variation of ratio of perpendicular displacement field and displacement vector with  $x_2$

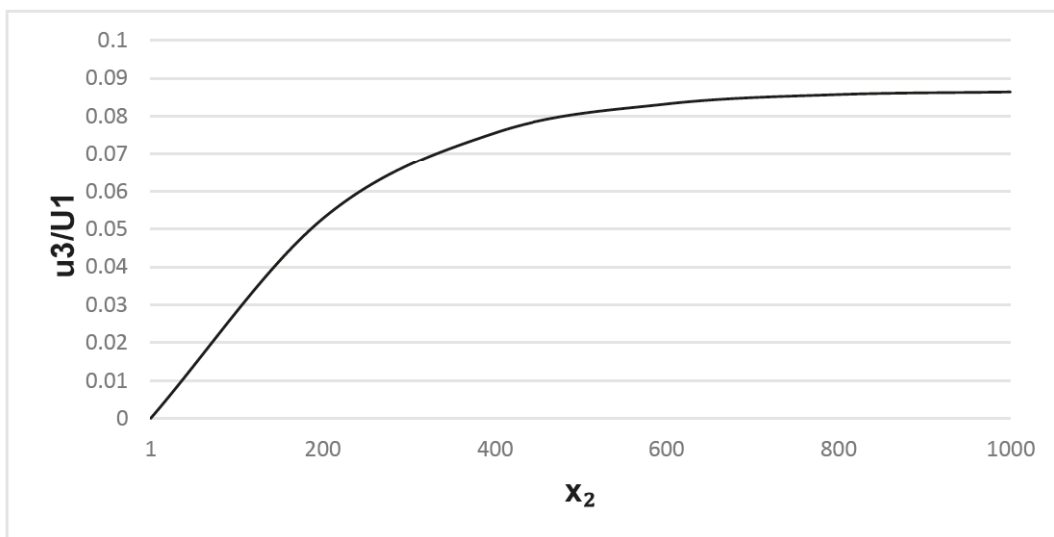


Figure 5. Variation of ratio of vertical displacement field & displacement vector with  $x_2$

Variation of displacement fields to displacement vector with distance from fault for Bihar- Nepal earthquake that represented another event is presented in Figure 6, 7, and 8.



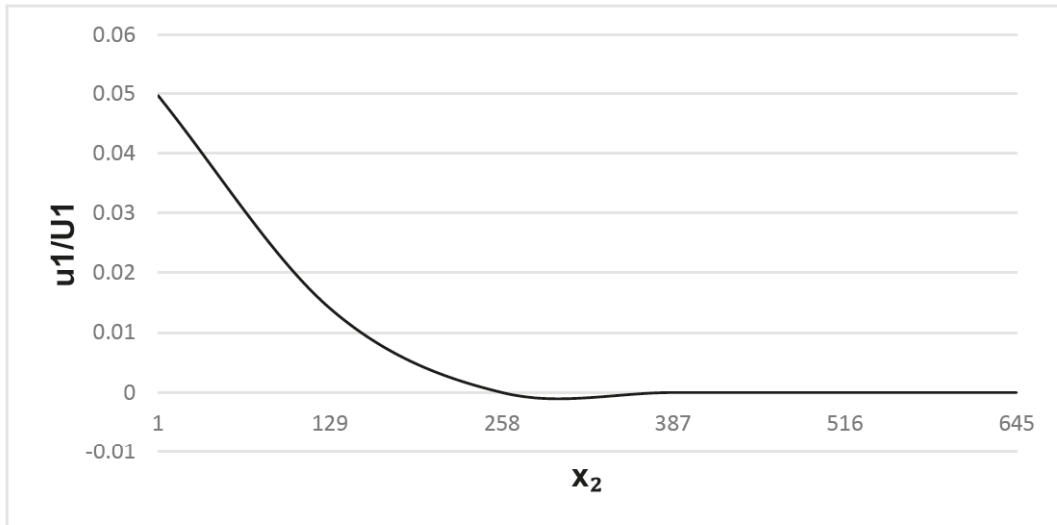


Figure 6. Variation of ratio of parallel displacement field and displacement vector with  $x_2$

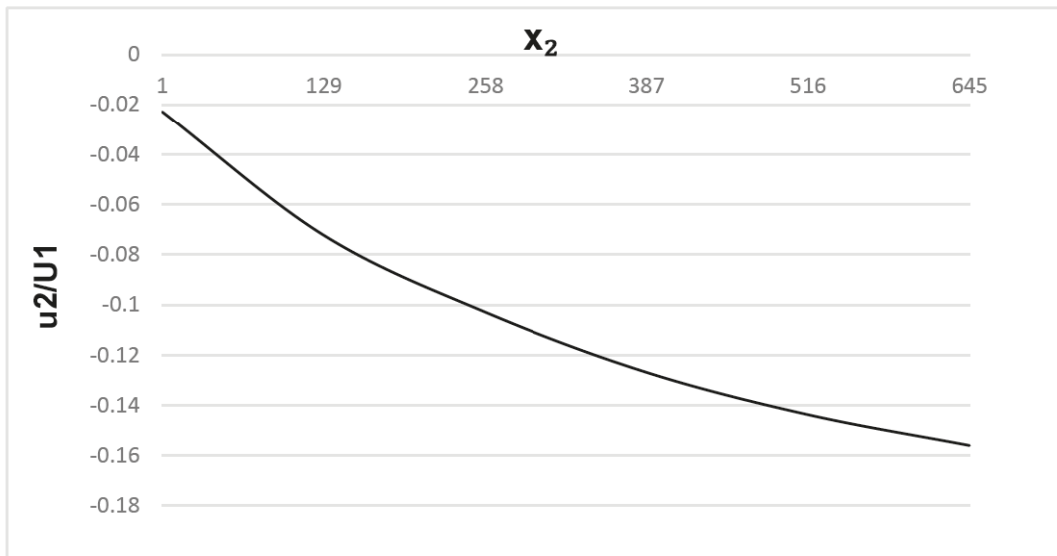
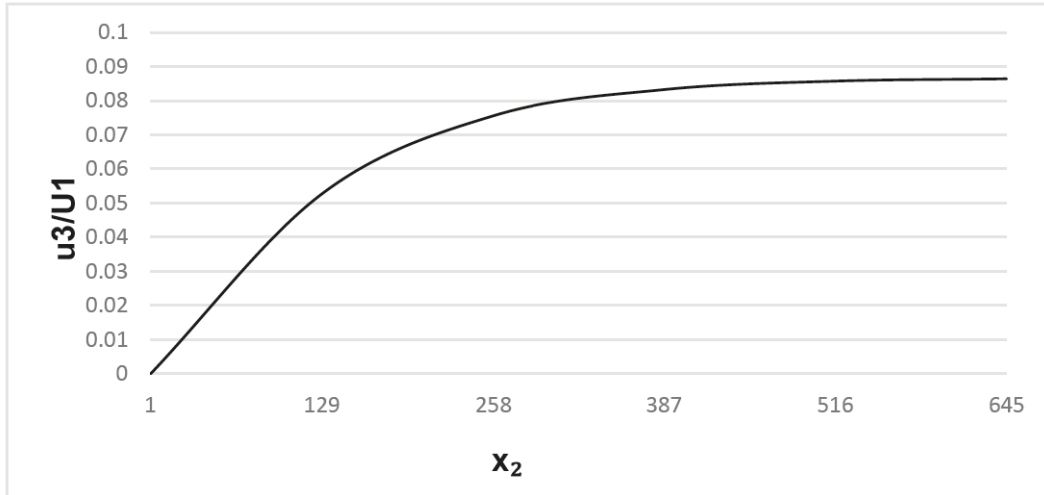


Figure 7. Variation of ratio of perpendicular displacement field and displacement vector with  $x_2$



*Figure 8.* Variation of ratio of vertical displacement field and displacement vector with  $x_2$

### Results

In parallel displacement  $x_2$  is taken to be zero while in the case of perpendicular and vertical displacement,  $x$ , is taken to be semi-fault length  $L$ . The variation of the interfacial displacement caused by a strike slip fault with the horizontal distance.  $X_2$  from the fault for the value of  $D = 2L$  for two great earthquakes such as Sanfrancisco and Bihar-Nepal earthquake. The distance is measured in units of semi fault length ( $L$ ) and the displacement is measured in units of the slip ( $U$ ). It is noted that in all the calculations assuming  $d = 0$ . The variation of the displacement caused by a strike slip fault with the distance from the fault for  $m = 0$ , shown in Figures (3, 4, 5, 6, 7, 8).

Figure 3 shows that in spite of being different value of the parallel displacement decreases asymptotically but not vanish with the distance from the fault and this result is in complete agreement with the corresponding result of Singh, et al. (1993). They have been used the two welded contact medium and the method of image. In the case of perpendicular displacement, the value is found to be negative but the nature of observed curve is same as the theoretical model, one developed by Singh et al. (1993). The vertical displacement is different from the parallel and perpendicular displacement as shown in the above figures. In the case,

initially the displacement  $u_3$  is zero with the distance from the fault increases. This result is in complete agreement with theoretical model for two welded contact medium. It is seen that the variation of displacement fields in a uniform elastic half space is similar to that of the variation of displacement fields caused by a finite rectangular fault in an infinite elastic medium. However, there is only difference in sign in case of perpendicular displacement.

### **Conclusions**

It is concluded that the variation of displacements with the half of the fault length is significant. The parallel and perpendicular displacement near the middle of the fault is maximum while in the case of vertical displacement is maximum near the end of fault and beyond this it seems to be almost constant with the distance from the fault while paralleled and perpendicular displacement at the end of the fault is minimum and beyond this it seems to be almost constant. The results are particularly useful for studying the variation of displacement fields caused by a finite rectangular strike slip fault associates with the help of same process it may be study the variation of surface displacement and shear stress caused by a finite rectangular strike slip fault in a uniform elastic half space. Lastly, from the observation of Figures (3, 4, 5, 6, 7, 8) of two events, it is concluded that even if different fault length gives approximately equal magnitudes, the variation of displacement fields with the distance from the fault is almost same.

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