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Applications of Real Numbers (ℝ) in Economics and Financial Models

Sher Singh Raikhola
Department of Mathematics, Bhaktapur Multiple Campus, Tribhuvan University
Email: raikholasher@gmail.com
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Abstract

This article explores the significance of real number theory in economics and finance, focusing on its role in improving the accuracy and reliability of economic analyses. As applied mathematics continues to evolve, real number theory has become an essential tool for economists and financial analysts. The main objective is to demonstrate how real numbers are used in various financial calculations, including interest rates, present value, inflation, GDP growth, risk management, and portfolio analysis.

To achieve this, a comprehensive review of relevant mathematical and financial literature was conducted, accompanied by a theoretical analysis of the properties of real numbers and their applications in economic contexts. Key concepts such as the Completeness Property, Archimedean Property, and important theorems like the Bolzano-Weierstrass Theorem and the Mean Value Theorem were examined. Practical examples and case studies were provided to illustrate how these principles can address real-world economic and financial challenges.

The findings highlight that real number theory is crucial in financial modeling and economic forecasting. It aids in optimizing production levels and assessing risks in investment portfolios. Overall, this article emphasizes the importance of integrating real number theory into economic research to enhance analysis and decision-making in the financial sector.

Keywords: real numbers, economics, finance, financial calculations, economic indicators, risk management

Introduction

In the context of national priorities for economic development, research plays a critical role in addressing challenges such as poverty alleviation, employment generation, and sustainable economic growth. The integration of advanced mathematical concepts into economic research is essential for formulating effective policies and strategies that can lead to significant improvements in these areas. This article explores the significance of real number theory in economics and finance, focusing on its role in enhancing the accuracy and reliability of economic analyses.

As applied mathematics continues to evolve and economic research expands, the role of real number theory has become increasingly important in the field of economics and finance. This theory is instrumental in improving the accuracy and rigor of economic analysis, allowing for more precise and reliable insights into economic issues. The integration of real number theory into economic research enhances the scientific foundation of the field, making it a critical tool for developing more effective economic models. To further advance economic analysis, it is essential to deepen the exploration of real number theory within the context of applied mathematics in economics (Warren, 2013).

Real numbers **(**ℝ**)** serve as a fundamental mathematical framework that underpins numerous applications in economics and finance. As a comprehensive set of numbers that includes both rational and irrational numbers **(**ℝ**)** allows for the continuous representation of economic and financial variables such as prices, interest rates, and economic indices. The ability to represent these variables as real numbers is crucial for the precise calculation of financial metrics, the accurate modeling of economic indicators, and the effective assessment of risks (Zhou, 2010).

Real numbers encompass all points on the number line, **(**ℝ**)** = $\{x|x \text{ is a real number}\}$, which allows for continuous and precise representation of measurable quantities such as prices (P), interest rates (r), and economic indices (I). This continuity is essential for capturing the full range of possible values in these domains, ensuring that no potential outcomes are excluded. The properties of real numbers, such as density and completeness, make them indispensable in the mathematical modeling and analysis of economic and financial systems (Kumbhar & Shinde, 2022).

In economics, real numbers are used to represent critical data such as GDP (Y), inflation rates (π), and unemployment figures (U). The completeness property of real numbers guarantees that optimization problems, such as finding the maximum utility max U(x)) or profit (max π(x)), are well-defined within R, enabling economists to model economic behavior with greater precision.

In finance, real numbers facilitate the modeling of asset prices (S), interest rates (r), and other financial variables. The ability to express these quantities as real numbers allows for the application of mathematical tools like calculus and linear algebra. For example, the computation of present value ($PV = \frac{FV}{(1+r)^t}$) and future value (FV = $C(1 + r)^t$) is fundamental in discounting cash flows and evaluating investment opportunities (Karatzas & Shreve, 1998).

This paper explores the application of real numbers in various economic and financial contexts, highlighting their role in financial calculations, economic analysis, risk management, and forecasting.

Statement of Problem

The accurate application of real numbers in financial and economic contexts is essential for effective decision-making and analysis. However, without a clear understanding of how to apply real numbers correctly, there can be significant errors in financial calculations, economic forecasting, and risk management. This paper aims to address the gap in understanding by exploring the fundamental properties of real numbers and their specific applications in economics and financial problems.

Objectives

To illustrate the use of real numbers in financial calculations, including interest rates and present value, inflation and GDP growth, risk management, portfolio analysis, budgeting and financial forecasting.

Methodology

This study utilizes a comprehensive review methodology to investigate the application of real number theory in economics and finance. The research process is structured in several key stages. Initially, the researcher conducted a thorough literature review, gathering a diverse range of mathematical and financial texts that focus on the application of real numbers in economic contexts. This involved sourcing relevant articles, books, and scholarly papers that examine both theoretical frameworks and practical implications of real number theory in finance. Following the literature collection, an extensive theoretical analysis was performed to explore the properties of real numbers. This analysis encompassed key concepts such as the Completeness Property, Archimedean Property, Bolzano-Weierstrass Theorem, Intermediate Value Theorem, Mean Value Theorem, and Cauchy-Schwarz Inequality. The focus was on understanding how these mathematical principles support various economic and financial models. To demonstrate the relevance of these theoretical concepts, the study incorporated illustrative examples derived from financial calculations and economic indicators. For instance, the analysis of interest rates, present value calculations, and risk management techniques showcased how real number theory can be applied to solve practical financial problems. Additionally; iterative methods were developed to address economic and financial challenges effectively. These methods aimed to enhance analytical precision, allowing for more informed decision-making processes in economic scenarios. The research further involved analyzing financial and economic data using real numbers to uncover trends and relationships. This analysis highlighted the practical implications of real number applications in real-world economic situations.

Finally, the findings were synthesized to emphasize the significance of real number theory in resolving economic and financial issues. This synthesis provided a comprehensive understanding of the critical role real numbers play in enhancing economic analysis.

Literature Review

The real number system is integral to both Markov processes and economic and financial modeling. It provides the framework needed to represent continuous variables, enabling precise analysis and the development of models that reflect the true nature of economic and financial systems. The application of real numbers in these contexts enhances the ability to make predictions, optimize strategies, and understand complex market dynamics (Fallahnezhad, Ranjbar, and Zahmatkesh Sredorahi, 2020)

A Markov chain is a sequence of random variables, denoted by X_j, where j represents the time steps. This process is distinguished by its memoryless property, indicating that the future state depends solely on the current state, with no regard to previous states:

$$P\,[X_j = l \mid X_0 = K_0,\ X_1 = K_1,\ \ldots,\ X_{j-1} = K_{j-1}\,] = P\,[X_j = l \mid X_{j-1} = K_{j-1}]$$

An example of a Markov chain is a simple random walk, where state transitions are based on equal probability to adjacent states, or the movement of a game piece in Monopoly, where states represent positions on the board and transitions are determined by dice rolls.

Markov chains are widely applied in various fields, including decision-making processes and signal processing (Fallahnezhad, Ranjbar, and Zahmatkesh Sredorahi, 2020). Markov processes, which extend the concept to continuous or discrete time indices, model phenomena like radioactive decay or population dynamics. Continuous state space processes, such as Gaussian processes for noise modeling or Brownian motion for financial asset prices, use real numbers or intervals of real numbers to describe the state space, crucial for modeling market prices.

Theoretical Analysis

The Completeness Property of Real Numbers

Statement: Any non-empty subset S of real numbers **(ℝ)** that is bounded above possesses a least upper bound (supremum) in **(ℝ)**. Formally, if S ⊆ **(ℝ)** and S is non-empty and bounded above, then there exists a number sup (S) ∈ **(ℝ)** such that:

Sup (S) ≥ x for all x ∈ S (it is an upper bound).

If y < sup (S), then there exists some x ∈ S such that y < x (it is the least upper bound).

(Rudin, 1976), (Bergé, 2008).

Proof:

Assume S is non-empty and bounded above. If S has no least upper bound, either M is not the smallest upper bound, or no smallest upper bound exists. This contradicts the completeness property of R. Hence, S must have a supremum.

Implication

In economics and finance, optimization problems frequently arise, such as maximizing profit, utility, or minimizing costs. The Intermediate Value Theorem (IVT) plays a crucial role in ensuring these problems are well-defined and solvable by guaranteeing the existence of solutions.

Archimedean Property

Statement: For any real numbers x and $y$with $x > 0$, there exists a positive integer n such that $nx > y$.

Proof:

Assume no such n exists, implying $x \leq \frac{y}{n}$ for all n. As $n$increases, $\frac{y}{n}$ approaches 0, contradicting the assumption that $x > 0$ (Bényei, Szeptycki & Van Vleck, 2000).

Implication

This ensures no infinitely small or large numbers, crucial for financial calculations like compounding interest. For instance, a 1% interest rate will eventually double an investment, demonstrating the practical application of the Archimedean property.

Bolzano-Weierstrass Theorem

Statement: Every bounded sequence in Real numbers **(ℝ)** has a convergent subsequence.

Proof:

Consider a sequence $\{x_n\}$ that is bounded in **(ℝ)**, meaning there exists real numbers m and M such that m≤ x_n ≤ M for all n.

Since the sequence is bounded, the set $\{x_n\}$ is contained within the closed interval [m, M].

The interval [m, M] is compact in **(ℝ)**, meaning any infinite sequence within this interval has a convergent subsequence by the Bolzano-Weierstrass Theorem.

Therefore, there exists a subsequence $\{x_{n_k}\}$ that converges to some limit L [m, M].

(Ross, & Lopez, 2013), (Apostol, 1974).

Implication

This theorem is fundamental in financial modeling and economic forecasting, where bounded sequences often represent time series data like stock prices or economic indicators. The existence of convergent subsequences implies that trends or steady states can be identified within these data sets, aiding in the prediction and analysis of long-term behavior.

Mean Value Theorem

Statement: If f is continuous on $[a, b]$ and differentiable on (a, b), then there exists some $c \in (a, b)$ such that $f'(c) = \frac{f(b) - f(a)}{b - a}$ (Berrone, & Moro, 2000; Bartle and Sherbert, 2011).

Implication

Helps in optimization by finding points where the instantaneous rate of change equals the average rate over an interval. For instance, it can identify optimal production levels by equating the average and instantaneous rates of change in profit functions.

Cauchy-Schwarz Inequality (Scarlatti, 2024).

Statement: For any vectors $u$and $v$in an inner product space, the inequality $|\langle u, v\rangle| \leq \|u\| \, \|v\|$ holds.

Implication

This inequality is used in portfolio theory and risk management to measure the correlation between assets and assess the risk and return of investment portfolios.

Applications and Illustrations

Completeness Property of Real Numbers

Consider a set S of possible returns on an investment: $S = \{0.01, 0.05, 0.07, 0.10\}$. The set S is bounded above by 0.10. To find the maximum possible return, we look at the supremum of S. Here, the supremum is 0.10.

Calculation: If the return r is bounded such that $0 \leq r \leq 0.10$, the supremum sup $(S) = 0.10$. This confirms that the maximum return possible in this bounded set is 0.10.

Archimedean Property

It includes Interest Formula which states that the amount A after n years with an initial principal P and r is an annual interest given by:

$A = P\,(1 + r)^n$

Suppose if we invest NRS 1,000 at an annual interest rate of 1% (or 0.01 as a decimal). We want to know how many years it will take for us investment to grow to at least NRS 2,000.

Substitute Values:

Initial principal P =1000

Interest rate r = 0.01

Desired amount A ≥ 2000

Set up the inequality to find n:

$1000\,(1 + 0.01)^n \geq 2000$

Simplify the Inequality: Divide both sides by 1000:

$(1 + 0.01)^n \geq 2$

Solve for n: Take the natural logarithm of both sides to solve for n:

In $[(1+0.01)^n] \geq$ ln2

n· ln (1+ 0.01) ≥ ln2

n ≥ $\frac{\ln 2}{\ln(1+0.01)}$

Approximate values:

ln2 ≈ 0.693

ln (1+ 0.01) ≈ 0.01

n ≥ $\frac{0.693}{0.01}$ ≈ 69.3So, it will take approximately 70 years for the investment to grow from NRS 1,000 to at least NRS 2,000 at a 1% annual interest rate.

3. Bolzano-Weierstrass Theorem

Bounded Sequence: A sequence $\{x_n\}$ of stock prices is said to be bounded if there exist real numbers M and m such that: m ≤ x_n ≤ M for all n

This means the stock prices do not go beyond a certain upper and lower limit.

Convergent Subsequence: A subsequence $\{x_{n_k}\}$ of $\{x_n\}$ converges to a limit L if:

$\lim_{n\to\infty} x_{n_k} = L$

This implies that ask gets larger, the values of the subsequence get arbitrarily close to L.

Implication of Convergent Subsequences: If $\{x_n\}$ is a bounded sequence, the Bolzano-Weierstrass theorem guarantees that $\{x_n\}$ has at least one convergent subsequence. This is fundamental in financial modeling and forecasting as it helps identify steady states or trends within the data.

If the sequence is {60, 62, 65, 67, 70, …} and is bounded and the theorem shows a bounded sequence has a convergent subsequence **{ p_{n_k}} such that $\{p_{n_k}\} \to L$, where L is the limit of this subsequence. For practical use, this implies that despite fluctuations, trends will converge to some stable value within the given bounds.**

Suppose a stock price sequence $\{p_n\}$over a year is bounded between NRS 50 and NRS 150. To identify potential price trends, we need a convergent subsequence.

Suppose we have a time series data set of stock prices over 12 months:

x_1=100, x_2=102, x_3=101, x_4=105, x_5=110, x_6=115, x_7=120, x_8=118, x_9=121, x_{10}=123, x_{11}=125, x_{12}=126

Minimum price m =100

Maximum price M =126

Therefore, the sequence $\{x_n\}$ is bounded because all stock prices fall within the range [100,126].

To find a convergent subsequence, we look for subsequences that seem to approach a particular value. For instance:

Consider the subsequence $\{x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}\}$:

Subsequence: 115,120,118,121,123,125,126

As we observe the subsequence, the stock prices appear to be increasing steadily. To formalize this, we can compute limits of certain subsequences.

If we take the subsequence $\{x_7, x_8, x_9, x_{10}, x_{11}\}$: (with prices 120, 118, 121, 123, 125), we see that it appears to converge towards a limit around 123 or 125.

We can approximate the limit of a subsequence if needed. For instance, the average of the last few terms might give an estimate:

Average of last terms = $\frac{120 + 118 + 121 + 123 + 125}{5}$ = 121.4. This suggests that the stock prices tend to stabilize around this average value in this subsequence.

The concept of convergent subsequences in bounded sequences is essential for financial modeling and economic forecasting. It helps analysts identify trends and steady states, improving predictions and decision-making for long-term behavior in finance and economics.

Intermediate Value Theorem

Example: Suppose we model profit $P(x)$ as a function of production levels x, where

$P(100) = 5000$ and $P(150) = 8000$

P (150) = 8000. We find out if there is a production level x where the profit is NRS 6000.

Calculation:

Identify the given values:

$P(100) = 5000$

$P(150) = 8000$

We check if there is some c such that $P(c) = 6000$.

Apply the Intermediate Value Theorem (IVT): The IVT states that if $P(x)$ is continuous on the interval [100, 150]

[100,150] and if $P(100) = 5000$ and $P(150) = 8000$, then for any value y between 5000 and 8000, there exists some c in the interval [100, 150], such that $(c) = y$.

In this case, since 6000 is between 5000 and 8000, the IVT guarantees that there exists some $c \in [100,150]$ where

$(c) = 6000$.

Therefore, there is a production level c within the range [100,150] where the profit is exactly NRS 6000.

The break-even point of a company, where total revenue equals total cost, can be determined by solving the equation $R(x) - C(x) = 0$, where R(x) represents the revenue function and C(x) represents the cost function. Assuming both functions are continuous, this equation identifies the production level at which profit is zero.

Let the revenue function be $R(x) = 50x$ and let the cost function be

$C(x) = 10x + 500$, where x represents the quantity of units produced and sold.

The profit function P(x) is given by:

$P(x) = R(x) - C(x) = 50x - (10x + 500)$

Simplifying the profit function, we get:

$P(x) = 40x - 500$

Application of the Intermediate Value Theorem:

We want to find the break-even point, where P(x) = 0. This means solving the equation:

$40x - 500 = 0$

This can be rearranged to find x: $40x = 500$, $x = \frac{500}{40} = 12.5$

This solution tells us that the break-even point occurs when 12.5 units are produced and sold. However, because x must be a whole number in practical scenarios, we check the values at x =12 and x=13 to apply the IVT.

At $x = 12$: $P(12) = 40(12) - 500 = 480 - 500 = -20$

At x=13: P (13) = 40 (13) – 500 = 520 – 500 = 20

Since P (12) = − 20 and P (13) = 20, and the function is continuous, according to the Intermediate Value Theorem, there must be some c in the interval (12, 13) where P(c) = 0. This means the break-even point occurs somewhere between 12 and 13 units.

In this example, the break-even point lies between 12 and 13 units. The Intermediate Value Theorem ensures a solution exists within this range, confirming the use of numerical methods to find precise solutions in real-world financial models.

Mean Value Theorem

The Mean Value Theorem states that if $P(x)$is continuous on the closed interval $[a,\ b]$and differentiable on the open interval $(a,b\ )$, then there exists a point $c\in(a,b)$ such that the derivative $P'(c)$ equals the average rate of change: $P'(c) = \frac{P(b) - P(a)}{b - a}$

Assume the profit function $P\ (x)$ is continuous and differentiable. For production levels a = 100 and b = 150, where P (100) = 5000 and P (150) = 8000, find the point where the rate of change equals the average rate.

The average rate of change of profit between production levels 100 and 150 is calculated as:

$\frac{P(150) - P(100)}{150 - 100} = \frac{(8000 - 5000)}{50} = \frac{(3000)}{50}$ = 60 units per production level 150 − 100

Apply the Mean Value Theorem (MVT):

In this case, the theorem guarantees that there exists a point c within the interval (100, 150), where $P'(c)$ = 60.

Therefore, there is a production level c in the interval (100, 150) where the instantaneous rate of profit change $P'(c)$is exactly 60 units per production level, matching the average rate of change over the interval.

Suppose we have a function f(x) that represents the revenue R in thousands of rupees generated by a company, where f(x) is defined as:

f(x) = x^2 + 2x + 1.

Let's find the value of c where the Mean Value Theorem applies, over the interval [1, 4].

Calculate the average rate of change over [1, 4]:

f (1) = 1^2 + 2·1 + 1 = 4

f (4) = 4^2 + 2·4 + 1 = 25

The average rate of change is:

$\frac{f(4) - f(1)}{4 - 1} = \frac{25 - 4}{3} = \frac{21}{3} = 7$

Find the derivative f ′ (x): $f'(x) = 2x + 2$

Set the derivative equal to the average rate of change:

f ′(c) = 2.5 is a point within the interval (1,4) where the instantaneous rate of change of the function f(x) is equivalent to the average rate of change over the interval [1, 4].

Maximizing Profit Using the Mean Value Theorem

To illustrate how the Mean Value Theorem (MVT) can be applied to maximize profit, let's illustrate by the following example.

Profit Function: Suppose a company's profit function is:

P(x) = $-x^2$ +10x + 15

where x represents the number of units sold. The goal is to find the optimal production level that maximizes profit.

Define the Interval [a, b]:

Let's consider the interval [2, 6] as the range of units sold.

First, we need to determine the profit at the endpoints of the interval.

$P(2) = -2^2 + 10\cdot2 + 15 = -4 + 20 + 15 = 31$

$P(6) = -6^2 + 10\cdot6 + 15 = -36 + 60 + 15 = 39$

Now, estimate the average rate of change of profit over the interval [2, 6]:

Average rate of change $= \frac{P(6) - P(2)}{6-2} = \frac{39-31}{4} = \frac{8}{4} = 2$

Find the Derivative P′(x):

To find where the instantaneous rate of change of profit equals the average rate of change, we need the derivative of P(x):

$P'(x) = \frac{d}{dx}(-x^2 + 10x + 15)$

$= -2x + 10$

Apply the Mean Value Theorem:

According to the MVT, there exists some point c in the interval (2, 6) where the instantaneous rate of change of profit equals the average rate of change:

$P'(c) = 2, -2c + 10 = 2, -2c = 2 - 10, -2c = -8, c = \frac{8}{2} = 4$

So, at x = 4, the instantaneous rate of change of profit is equal to the average rate of change over the interval [2, 6].

Interpret the Result:

The result $c = 4$ tells us that at x = 4 units sold, the rate of change of profit matches the average rate of change of profit over the interval [2,6]. This is a key point in understanding the behavior of the profit function.

Since the profit function $P(x) = -x^2 + 10x + 15$is a downward-facing parabola (as indicated by the negative coefficient of x^2), the maximum profit occurs at the vertex of this parabola. For this quadratic function, the vertex is indeed at x = 5, which lies in the interval [2, 6].

Minimizing Costs: The theorem helps identify intervals where the average cost per unit changes significantly, allowing companies to adjust strategies to minimize overall costs.

Minimizing Costs Using the Mean Value Theorem

To demonstrate how the Mean Value Theorem (MVT) can be applied to minimize costs, let's work through following examples

Cost Function:

Consider a company's cost function given by:

$C(x) = x^3 - 6x^2 + 9x + 2$

where x denotes the quantity of units produced. We want to find an interval where the average cost per unit changes significantly, which helps in identifying strategies to minimize overall costs.

Define the Interval [a, b]:

Let's analyze the interval [1,4] for production levels.

Determine the average cost per unit: First, find the cost at the endpoints of the interval.

$C(1) = 1^3 - 6\cdot1^3 + 9\cdot1 + 2$

=1 – 6 + 9 + 2 = 6

C (4) = 4^3 – 6· 4^3+ 9·12

= 64 – 96 + 36 + 2 = 6

Calculate the average cost per unit over the interval [1, 4]:

Average cost per unit: $= \frac{C(4) - C(1)}{4 - 1} = \frac{6 - 6}{3} = \frac{0}{3} = 0$

In this case, the average cost per unit over the interval [1, 4] is 0, which indicates that the cost function does not change on average over this interval.

Find the Derivative C ′ (x):

To apply the Mean Value Theorem, we need the derivative of C(x):

$C'(x) = \frac{d}{dx}(x^3 - 6x^2 + 9x + 2) = 3x^2 - 12x + 9$

Apply the Mean Value Theorem:

The Mean Value Theorem states that there exists a point ccc within the interval (1, 4) where the instantaneous rate of change of cost equals the average rate of change of cost. Since the average cost per unit is 0, we set the derivative equal to 0:

$C'(c) = 0,\ 3c^2 - 12c + 9 = 0$

Solve this quadratic equation: $3c^2 - 12c + 9 = 0$

$$c^2 - 4c + 3 = 0$$

Factorize the quadratic equation: $(c-1)(c-3) = 0$, $c = 1$ or $c = 3$

Both solutions c = 1 and c = 3 fall within the interval (1, 4). These are the points where the instantaneous rate of change of cost matches the average rate of change, which is 0 in this example.

Interpreting the results

At $c = 1$: The instantaneous rate of change of cost is 0, meaning that at x=1, the cost function has a local minimum or maximum. Since C (1) = 6, this point is where the cost is not changing, but it's also the endpoint of the interval.

At c = 3: The instantaneous rate of change of cost is also 0. Therefore, at x=3, the cost function has a local extremum. To determine whether it is a minimum or maximum, we can examine the second derivative:

$C''(x) = \frac{d}{dx}(3x^2 - 12x + 9)$

$= 6x - 12$

C ″ (3) = 6·3 – 12 = 18 – 12 = 6 > 0 Since $C''(3) > 0$, x = 3 is a local minimum.

Hence, the cost function C(x) has a local minimum at x = 3. This is a critical point where the cost is minimized within the given interval. By using the Mean Value Theorem, we identified that x=3 is where the average cost per unit changes significantly, guiding the company to optimize production levels to minimize costs effectively.

Cauchy-Schwarz Inequality

The Cauchy-Schwarz Inequality states: $|\langle u, v\rangle| \leq \|u\|\ \|v\|$

Example: For two investment portfolios, u = (0.1, 0.2, 0.3) and v = (0.2, 0.1, 0.4), calculate the dot product and apply the Cauchy-Schwarz Inequality.

Calculation:

Calculate the dot product: $\langle u, v\rangle = 0.1 \times 0.2 + 0.2 \times 0.1 + 0.3 \times 0.4 = 0.02 + 0.02 + 0.12 = 0.16$

Calculate the norms of u and v:

For u: $\|u\| = \sqrt{0.1^2 + 0.2^2 + 0.3^2} = \sqrt{0.01 + 0.04 + 0.09} = \sqrt{0.14} \approx 0.374$

For v:

$\|v\| = \sqrt{0.2^2 + 0.1^2 + 0.4^2} = \sqrt{0.04 + 0.01 + 0.16} = \sqrt{0.21} \approx 0.458$

Apply the Cauchy-Schwarz Inequality:

Substitute the calculated values: $0.16 \leq 0.374 \times 0.458 \approx 0.171$

The inequality $0.16 \leq 0.171$holds true.

This confirms that the correlation between the two portfolios u and v is within the expected bounds, as validated by the Cauchy-Schwarz Inequality.

Financial examples

Present Value:

Discounting future cash flows to their present value is essential in finance (Varian ,1993). The present value formula is

$$PV = \frac{FV}{(1 + r)t}$$

Where,

PV = Present value

FV = Future value

r = Discount rate

t = Number of periods

Example: If FV = NRS 2000, r = 6% annually, and t = 5years:

PV=2000 / $(1 + 0.06)^5 \approx$ NRS 1490.53

Properties: Demonstrates the time value of money by discounting future cash flows.

Economic Indicators

Inflation Rate

Formula:

Inflation Rate = $\frac{CPIt - CPIt - 1}{CPIt - 1} \times 100\%$

Where,

$CPIt$ = Consumer Price Index at time

$CPIt - 1$ = Consumer Price Index at time

Understanding inflation is crucial for monetary policy and economic planning.

Example: If $CPIt$ = 250and $CPIt - 1$ = 240:

Inflation Rate = 250 – 240/ 240 × 100% ≈ 4.17%

Properties: Reflects changes in the purchasing power of money.

GDP Growth Rate

Formula:

GDP Growth Rate = $\frac{GDPt - GDPt - 1}{GDPt - 1} \times 100\%$

Where,

$GDPt$ = GDP at time t

$GDPt - 1$ = GDP at time $t - 1$

This metric helps gauge the economic health and development of a country.

Example: If $GDPt$ = NRS 1.2 trillion and $GDPt-1$ = NRS 1.1 GDP trillion:

GDP Growth Rate =1.2−1.1/ 1.1 × 100% ≈ 9.09%

Properties: Indicates the rate of economic growth over time.

Economic indicators like GDP, inflation, and unemployment are expressed using real numbers. For example, a GDP of NRS 2,00,000 crore represents the economic state quantitatively.

Risk Management

Formula: $(Rp) = E\ (R_p) = \sum_{i=1}^{n} w_i\, E(R_i)$

Where,

(Rp) = Expected return of the portfolio

wi = Weight of asset

(Ri) = Expected return of asset

Application: Portfolio Expected Return

This helps investors optimize their portfolios based on expected returns and risks.

For a portfolio with 50% in Asset A (expected return 8%) and 50% in Asset B (expected return 12%):

(Rp) = 0.5 × 0.08 + 0.5 × 0.12 = 0.10 or 10%

Properties: Helps in optimizing portfolio returns based on asset weights.

Precise Mathematical Modeling (Braverman, 2013).

Real numbers allow precise modeling of continuous phenomena because they offer an infinitely divisible scale. For example:

Here:

t is time (a real number)

$s\ (t)$ is the position of the object at time t

Trajectory of a Moving Object: Real numbers represent time and position accurately. For a position function $(t) = 2t^2 + 3t + 1$, the position at any time t can be precisely calculated.

Example Calculation:

Find the position at $t = 2$:

s (2) = 2(2^2) + 3 (2) + 1 = 2 (4) + 6 + 1 = 8 + 6 + 1 = 15

At $t = 2$ seconds, the position of the object is 15 units.

Population Growth:

The growth of a population can be modeled using continuous functions.

Here:

$P\ (t)$ is the population at time t

P_0 is the initial population

r is the growth rate

t is time

$P\ (t) = P_0\, e^{rt}$, real numbers enable exact representation of population changes over time.

Example Calculation:

If the initial population

P_0 is 1000, the growth rate r is 0.05, and we want the population at $t = 3$ years:

P (3) =1000 $e^{0.05 \times 3}$ ≈ 1000$e^{0.15}$ ≈ 1000 × 1.1618 = 1161.80

After 3 years, the population is approximately 1162.

Change in Temperature Over Time

Temperature changes can be modeled using continuous functions as well.

Temperature Function:

T(t) = 20 + 5 Sin (t), real numbers provide accurate temperature readings at any time.

Here:

$T(t)$ is the temperature at time t

t is time

Example Calculation:

Find the temperature at $t = \pi$:

(π) = 20 + 5 Sin (π) = 20 + 5 × 0 = 20

At $t = \pi$, the temperature is 20 degrees.

Results

The above illustration presented applies key real numbers concept in various economic and financial scenarios, offering practical insights into investment returns, profit maximization, cost minimization, and financial modeling. Below, we explore several key properties and theorems of real numbers, along with their applications in economic and financial calculations.

Completeness Property of Real Numbers

For a bounded set S, the supremum Sup (S) = max (S).

Here, Sup (S) is the supremum (least upper bound) of the set S.

Archimedean Property

Given the equation

$A = P\ (1 + r)^n$

where $A \geq$ desired amount, solve for n using logarithms:

$n \geq \frac{\ln\text{ (desired amount / P)}}{In\ (1 + r)}\ x_n x_{n_k}$

Bolzano-Weierstrass Theorem

If $\{x_n\}$is bounded, then there exists a convergent subsequence $\{x_{n_k}\}$ such that $\lim_{n\to\infty} x_{n_k} = L$ where L is the limit of the convergent subsequence.

Intermediate Value Theorem

If $f$is continuous on $[a, b]$ and $(a) \leq y \leq f(b)$, then there exists $c \in [a, b]$ such that $f(c) = y$.

Mean Value Theorem

∃ c ∈ (a, b) such that $f'(c) = \frac{f(b) - f(a)}{b - a}$

Where,

$f'(c)$is the derivative of $f$at c.

Cauchy-Schwarz Inequality

$|\langle u, v\rangle| \leq \|u\|\ \|v\|$ where $\langle u, v\rangle$is the dot product, and $\|u\|$ and $\|v\|$are the norms of vectors $u$and v.

Real number in financial calculations (Lusardi & Mitchell, 2011**)**

Real numbers are integral to financial and economic calculations, enabling precise measurement and analysis of various factors such as investment growth, present value, inflation, economic performance, and risk management. followings are some of the results derived from the above illustrations. Real numbers are fundamental in finance for quantifying and analyzing economic phenomena. They allow for precise calculations and comparisons of financial metrics. Below is a summary of the formulas and their mathematical relations:

Compound Interest: $A = P\,(1 + r^n)^{nt}$

It calculates the future value A of an investment based on the principal P, interest rate r, compounding frequency n, and time t. This formula demonstrates exponential growth and illustrates how real numbers are used to assess investment returns over time.

Present Value: $PV = \frac{FV}{(1+r)^t}$

Above formula determines the current worth of future cash flows FV discounted at rate r overtime t. This highlights how real numbers are employed to evaluate the present value of future financial benefits.

Inflation Rate: $= \frac{\text{CPIt - CPIt} - 1}{\text{CPIt - 1}} \times 100\%$

Inflation Rate measures the percentage change in the Consumer Price Index (CPI), reflecting changes in purchasing power. This uses real numbers to quantify inflation's impact on the economy.

GDP Growth Rate: $= \frac{\text{GDPt - GDPt} - 1}{\text{GDPt} - 1} \times 100\%$

GDP Growth Rate indicates economic growth by comparing GDP values over time. Real numbers are used to assess economic performance and trends.

Portfolio Expected Return $E\,(R_p) = \sum_{i=1}^{n} w_i\, E(R_i)$

$E\,(R_p)$ calculates the expected return of a portfolio based on the weights w_i and returns $E\,(R_i)$of individual assets. Real numbers help in optimizing investment strategies.

Demand Function: $Q = a - bP$

Demand Function models the relationship between price $P$and quantity demanded Q, using real numbers to analyze market dynamics and price effects.

Discussion

Real numbers are central to financial and economic analysis, offering precise measurement and clarity in various scenarios. Their properties and associated theorems enable comprehensive evaluation and decision-making. Below, we explore how key real number concepts apply to practical financial contexts:

The exploration of real numbers in both mathematics and finance underscores their fundamental importance in diverse fields

The completeness property of real numbers ensures every bounded set has a least upper bound, crucial for determining maximum investment values. The Archimedean property helps solve for the number of compounding periods needed for financial goals. The Bolzano-Weierstrass theorem ensures any bounded sequence has a convergent subsequence, aiding long-term investment analysis. The intermediate value theorem helps identify key values like break-even points, while the mean value theorem provides insights into average rates of change, essential for understanding returns and growth rates. The Cauchy-Schwarz inequality aids in optimizing portfolios by assessing asset correlations and managing risks. Formulas like compound interest and present value are vital for understanding investment growth and the time value of money. Additionally, the inflation rate and GDP growth rate offer critical insights into economic conditions, influencing financial decisions and policy-making.

Overall, real numbers provide the mathematical foundation necessary for accurate financial analysis, strategic planning, and informed decision-making.

Conclusion

Real numbers form the backbone of financial and economic analysis, providing the essential mathematical framework for accurate measurement, projection, and optimization of various financial metrics and economic indicators. Through properties such as completeness, theorems like Bolzano-Weierstrass and the intermediate value theorem, and key concepts like compound interest and present value, real numbers enable precise calculations and informed decision-making. These mathematical tools support the effective analysis of investment growth, risk management, inflation, GDP performance, and other critical financial variables, ultimately guiding strategic planning and policy decisions. Their application ensures that financial strategies are grounded in rigorous analysis, leading to more reliable and actionable insights in both personal finance and broader economic contexts.

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Conflict of Interest:

The author declares that there are no conflicts of interest related to the publication of this paper.

References

Apostol, T.M. (1974). *Mathematical Analysis,* 2nd ed.; Addison-Wesley: Boston, MA, USA, Exercise 12.19.

Bartle, R. G., & Sherbert, D. R. (2011). *Introduction to real analysis.* Retrieved from https://docs.google.com/file/d/0B_jV8ojQ-nf-VlAzeW...

Bényei, Á., Szeptycki, P., & Van Vleck, F. (2000). Archimedean properties of parabolas. *American Mathematical Monthly, 107*(11), 945–949.

Bergé, Analía. (2008). The completeness property of the set of real numbers in the transition from calculus to analysis. *Educational Studies in Mathematics, 68*(2), 145–162. https://doi.org/10.1007/s10649-007-9101-5

Berrone, L. R., & Moro, J. (2000). On means generated through the Cauchy mean value theorem. *Aequationes Mathematicae, 60*, 1–14.

Braverman, M. (2013). Computing with real numbers, from Archimedes to Turing and beyond. *Communications of the ACM, 56*(9), 40–48. https://doi.org/10.1145/2500890

Fallahnezhad, M. S., Ranjbar, A., & Zahmatkesh Sredorahi, F. (2020). A Markov model for production and maintenance decision. *Macro Management & Public Policies, 2*(1). https://doi.org/10.30564/mmpp.v2i1.655

Karatzas, I. (1998). *Methods of mathematical finance.* Springer.

Kumbhar, A., & Shinde, A. (2022). *Real numbers and their interesting proofs*. JBNB: A Multidisciplinary Journal, Vol. 12, 2022. ISSN 2454-2776.

Lusardi, A., & Mitchell, O. S. (2011). Financial literacy and planning: Implications for retirement wellbeing. *National Bureau of Economic Research Working Paper Series*, (17078). https://doi.org/10.3386/w17078

Ross, K., & Lopez, J. M. (2013). *Elementary analysis: The theory of calculus* (2nd ed.). Undergraduate Texts in Mathematics. Springer. New York,

Rudin, W. (1976). "*Principles of Mathematical Analysis."* McGraw-Hill.

Scarlatti, S. (2024). Enhanced Cauchy–Schwarz inequality and some of its statistical applications. *Statistical Papers.* Advance online publication. https://doi.org/10.1007/s00362-024-01600-x

Varian, H. R. (1993). *Economic and Financial Modeling with Mathematica*. Telos Springer-Verlag.

Warren, P. (2013). Applications of mathematics in economics. Mathematical Association of America.

Zhou, X. (2010). Latest theory and modern development of financial mathematics. Popular Business (Second Half), 2, 165.