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Multiple Internal Rates of Return and Decision Criteria Among Mutually Exclusive and Independent Projects

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Abstract
This paper focuses on solving methods of unique and multiple internal rates of return of the series of cash flow when net present value is equal to zero, and decision criteria of accepting the independent project. When a project's cash flow has only two sign variations, the internal rate of return (IRR) rule is simple and for analysis, the case with an investment's cash flows with three or more sign variations to have multiple IRR solutions. The study, also, discussed selecting the best one among mutually exclusive projects on the basis of incremental internal rate of return (IRR) analysis. In the case of mutually exclusive projects, selecting projects based on the minimum attractive rate of return (MARR), accept a pure investment opportunity if IRR $\geq$ MARR, otherwise reject and accept a pure borrowing opportunity if IRR $\leq$ MARR, otherwise reject. In practice, the internal rate of return is a popular rule for the project accept/reject decisions.

Keywords: Cash-flow, IRR, Investment, MARR, Present value

Introduction:

The internal rate of return (*IRR*) is one of the fundamental criteria for financial decision making, which in its relative expression provides information to the investor on the return on his/her investment. It is a rate of return on investment in a project that respects the time value of cash flows throughout the life of the project. It measures a project's expected rate of return. The return does not depend on anything except the cash-flow of the project. Thus, the *IRR* provides a single number summarizing the merits of a project. Mathematically *IRR* is the discount rate that makes the net present value (*NPV*) of all the cash flows (both positive and negative) equal to zero for a specific investment, that is the internal rate of return for an investment is the interest rate that is equal to the annual compound rate earned on a saving account with the same cash flow. The term internal refers to the fact that the calculation excludes external factors, such as the risk-free rate, inflation or various financial risks. The *IRR* is designed to account for the time preference of money and investments. A given return on investment received at a given time is worth more than the same return received at a later time, so the later, would yield a lower IRR than the former. The internal rate of return is based only on a project's cash flow. For *IRR* to exists, both costs and benefits must be defined if only a project's benefits are defined then the present value is positive for all interest rates. If there are only costs, then the present value is always negative. Only if there costs and benefits can the *NPV* may equal to zero for some interest rate. In general, *IRR* is an average discount rate at which *NPV* is equal to zero, *IRR* tells how much the rate of returns are getting from the project.

More than one internal rate of return from the same project that makes the net present value of the project equal to zero. This situation arises when the internal rate of return method is used for a project in which negative cash-flows follow positive cash-flows. Multiple internal rates are not easy to explain or interpret properly but there are some techniques to remove multiple internal rates return (Blank & Tarquin, 1989), (White, Case, Pratt, & Agee, 1998), inaccurate (Park, 1997), ambiguous and contradictory (Steiner, 1979). When multiple rates of return are found there are no rational means for judging which of them is appropriate for determining economic desirability (Thuesen & Fabrycky, 1989).Multiple (or even complex-valued) internal rates, each has meaningful interpretation as a rate of return on its own underlying investment, but problems of using multiple internal rates of return (Hazen, 2003). Studying complex solution of the *IRR* equation and its applications should not be forbidden or neglected, but it should also be understood that the introduction of such advanced concepts into the industry, which by its nature has to be reasonably conservative, requires diligent and comprehensive research and a lot of qualified discussions(Yuri & Alexander, 2013). But if there are no real-valued internal rates of return then it is meaningless and complex-valued *IRR* cannot apply in the real situation. The main criticism of using *IRR* in project ranking is the inability to rank due to nonexistence of positive *IRR* and the other criticisms of IRR are the ranking inconsistency and the existence of multiple *IRR* (NG & Beruvides, 2015).

The use of the *IRR* as profit measure is very popular in the industrial and financial world. We shall define the *IRR* method as the method of economic evaluation which primarily uses *IRR* as the decision criteria that will include all possible conditions whether they occur often or rarely. One of the uses of *IRR* is by corporations that wish to compare capital projects. It can be a valuable aid in project acceptance and selection. The *IRR* method refers not only to the use of the *IRR* as a merit measure, but also the adoption of a set of decision rules based on the concept of *IRR* including incremental analysis of mutually exclusive proposals. *IRR* cannot be used to rank cash flows for mutually exclusive projects, excepts on an incremental basis. A mutually exclusive project prevents another project from being accepted. On the other hand, an independent project is one in which accepting or rejecting one project does not affect the acceptance or rejection of other project under consideration. Using the *IRR* method as a decision criterion may some time lead to selecting projects that do not maximize wealth if the projects are mutually exclusive.

Future value and present value:

Financial institutions are willing to pay interest on deposits because they can lend the money to the investors. The future value (*FV*) of a present amount ofmoney will be larger than the existing amount because of the accumulated interest over time. Conversely, the amount of money at the beginning of each period is computed backward by subtracting the interest for the period from the amount at the end of that period, the net effect is to discount or reduce the future sum to its present value (*PV*)(Tung & Thomas, 1992).

Then Future value (F) = $P(1+i)^n$ and present value (P) = $\frac{F}{(1+i)^n}$, where i = Interest rate per interest period (express in decimal), P is present value and F is the future value. For a series of disbursements $C_0, C_1, \dots \dots, C_n$ at time $t = 0,1, \dots \dots, n$ where $C_k\ (k = 0, 1, 2, \dots, n)$ are maybe positive, negative or zero which is designated as positive for receipts and negative for disbursement. Then

Net future value (NFV) = $\sum_{t=0}^{n} C_t(1+i)^{n-t}$ (1)

Net Present value (NPV) = $\sum_{t=0}^{n} C_t(1+i)^{-t}$ (2)

From (1) and (2), we obtained $NFV = (1+i)^n NPV$ is the relation between NPV and NFV.

Let the internal rate of interest (IRR) be r, where NPV =0. Then from equation (1) or (2), we obtained the series of cash flow is

$$C_0(1+r)^n + C_1(1+r)^{n-1} + C_2(1+r)^{n-2} + \dots + C_n = 0 \quad (3)$$

This is a polynomial equation of degree n in $(1 + r)$ and therefore it has exactly n solutions, including complex ones.

Solving for the *IRR*:

In the polynomial equation (3), the solution of r is generally quite complicated when n is large. Sometimes we use approximate numerical solutions by trial and error. If the unique value of r exists for an investment (or borrowing) cash flow profile whether the profile is conventional [one or more early periods of disbursement (or receipts), following by one or more early periods of receipts (or disbursements), otherwise unconventional], the case is referred to as pure investment (or pure borrowing) that is if the stream of payments consists of a single outflow followed by multiple inflows occurring in equal periods ($C_0 < 0, C_n \geq 0$ for $n \geq 1$), in this case, there is a unique solution of IRR. Mathematically, The IRR equation (3) is a generalized polynomial equation. For the unique value of r, we can analyze using Descarts' (Rene Descartes, 1596-1650) rule of sign. Descartes' rule of sign for determining the maximum number of positive real roots of a polynomial $p(x)$ in one variable based on the number of times that the signs of its real number coefficients change when the terms are arranged in the canonical order (from highest power to lowest power). It is noted that zero is a root when $p(x)$ has no constant term. It is a technique for getting information on the number of positive real roots of a polynomial. Descartes' rule of signs, first described by the French philosopher and mathematician Rene Descartes in 1637(Bajracharya, Basnet, & Phulara, 2014).

Theorem-1 (The IRR Uniqueness):(David, Marilous, & Larry, 2007)

If there exists an integer m for which $C_0, C_1, C_2, \dots, C_m$ are of the same sign or zero (but not all zero) and for which $C_{m+1}, C_{m+2}, \dots, C_n$ are all opposite sign or zero (but not all zero), then $C_0(1+r)^n + C_1(1+r)^{n-1} + C_2(1+r)^{n-2} + \dots + C_n = 0$ has at least one positive solution of $(1 + r)$.

Proof: We have,

$C_0(1+r)^n + C_1(1+r)^{n-1} + C_2(1+r)^{n-2} + \dots + C_n = 0$ is the polynomial

equation of degree n in $(1 + r)$. Using Descarts' rule of sign, there is exactly one change of sign. So, there is at most one positive root of the polynomial.
However, this theorem does not guarantee that r is positive.
Theorem-2 (The IRR Uniqueness)**:** (David, Marilous, & Larry, 2007)
If there exists an r for which

α) $1 + r > 0$

β) $\sum_{k=0}^{m} C_k (1+r)^{m-k} > 0$ for all integer m satisfying $0 \leq m \leq n-1$ and

χ) $\sum_{k=0}^{n} C_k (1+r)^{n-k} = 0$.

Proof: If possible, there is a next solution R of (c), that is $\sum_{k=0}^{n} C_k (1+R)^{n-k} = 0$ satisfying (a) and (b). Without loss of generality, we may assume that $R > r$. At first, by induction on m that

$$\sum_{k=0}^{m} C_k (1+R)^{m-k} > \sum_{k=0}^{m} C_k (1+r)^{m-k} \text{ or } m = 1, 2, \ldots, n.$$

For $m = 1$, $C_0(1+R) + C_1 > C_0(1+r) + C_1$, which is true because from condition (b) with $m = 0$, we have $C_0 > 0$.

We assume that for $m = t$, $\sum_{k=0}^{t} C_k (1+R)^{t-k} > \sum_{k=0}^{t} C_k (1+r)^{t-k}$ is true.

Show that $\sum_{k=0}^{t+1} C_k (1+R)^{t+1-k} > \sum_{k=0}^{t+1} C_k (1+r)^{t+1-k}$.

Now, $\sum_{k=0}^{t+1} C_k (1+R)^{t+1-k} = \sum_{k=0}^{t} C_k (1+R)^{t-k}(1+R) + C_{t+1}$

$$> \sum_{k=0}^{t} C_k (1+r)^{t-k}(1+r) + C_{t+1}$$

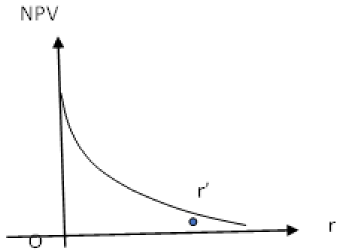
$$= \sum_{k=0}^{t+1} C_k (1+r)^{t+1-k}$$

Which proves the inequality by induction.

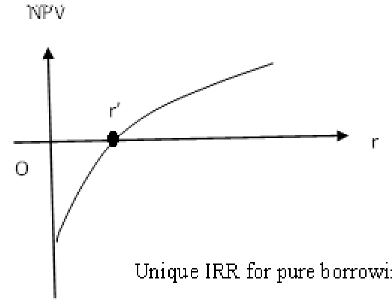
Thus, $\sum_{k=0}^{t+1} C_k (1+R)^{t+1-k} > \sum_{k=0}^{t+1} C_k (1+r)^{t+1-k}$.

By condition (c) $\sum_{k=0}^{n} C_k (1+r)^{n-k} = 0$ which is contradiction. Hence $r = R$. Therefore, r is unique.

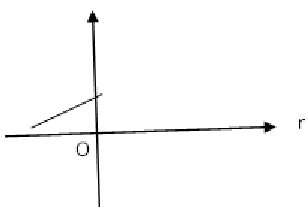
When the value of *IRR* is not unique, it means that either no value of *IRR* exists in the positive range of r or that multiple values of *IRR* are present in the positive range of r. If there are multiple sign changes of *NPV* in *NPV* = 0, in this case, when r-axis (horizontal line) is the tangent to the curve representing *NPV*, the root at the tangent point is referred to as a repeated root. Graphically, *NPV* versus r as shown in the following figure,



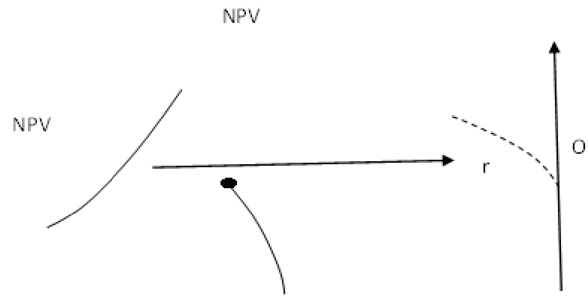
Unique IRR for pure investment



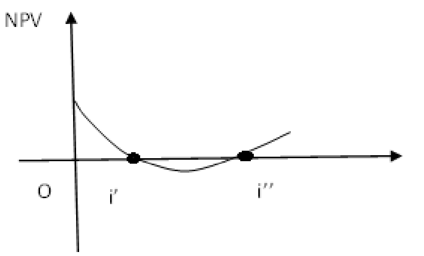
Unique IRR for pure borrowing



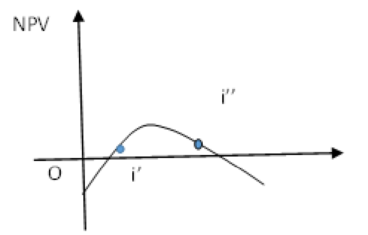
No positive IRR for positive NPV at r = 0



No positive IRR for negative NPV at r= 0



Multiple IRR for positive NPV at r = 0



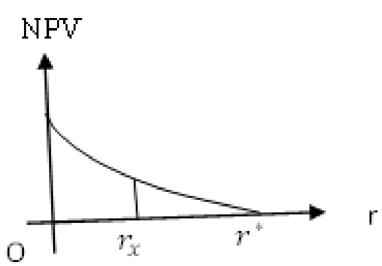
Multiple IRR for negative NPV at r = 0

We can find the multiple roots of *IRR* i.e. r by trial solution. If we start the trial solution r = 0. When two or more positive roots of r exist, each root may be obtained successfully from the remaining equation after the smallest one has been found and factored out. The approximate numerical solution may also be obtained by trial solution with the help of the discrete compound interest tables. Hence, the value of IRR always cannot be found analytically, in this case, numerical methods (Newton-Raphson's method or secant method or bisection method), trial-error solution methods, or graphical methods must be used.

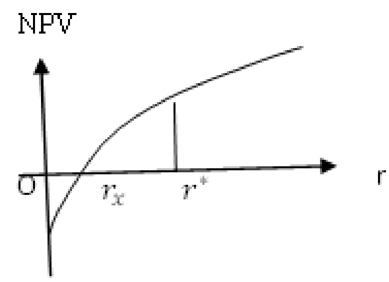
Decision criteria of accepting the mutually exclusive or independent projects:

We shall define the *IRR* method as the method of economic evaluation which primarily uses the *IRR* as the decision criteria for accepting mutually exclusive and independent projects that will include all possible conditions whether they occur often or rarely. The *IRR* method determines the acceptability

of independent projects. The minimum attractive rate of return (*MARR*) is a reasonable rate of return established for the evaluation and selection of alternatives by the top management in a firm reflects the opportunity cost of capital of the firm. Th MARR specified for the economic evaluation of investment proposal is worthwhile from the standpoint of the organization, but it cannot measure accurately (Tung & Thomas, 1992) (Martin, 2015). Let r be the interest rate for the time value of money if
(a) a loan's *IRR* is less than or equal to r, then the loan is attractive.
(b) an investment's *IRR* is greater than or equal to r, then the investment is attractive.
The *IRR* is independent of *MARR* and must be compared to the *MARR* to evaluate the acceptability of an independent project. We accept a pure investment opportunity if *IRR* $\geq$ *MARR*, otherwise rejects and accept a pure borrowing opportunity if *IRR* $\leq$ *MARR*, otherwise reject.
Let the *IRR* for an independent investment project be r_x, the criterion for accepting project x is $r_x \geq r^*$, otherwise, it should be rejected. The same result is obtained on the basis of the net present value criterion that is $r_x \geq r^*$. In the case of borrowing or financing, the criterion for accepting a loan *x* by the borrower is $r_x \leq r^*$, where r^* is the $MARR$ of the project.

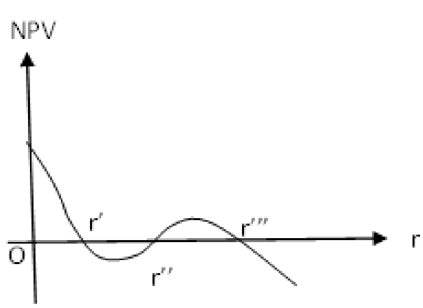


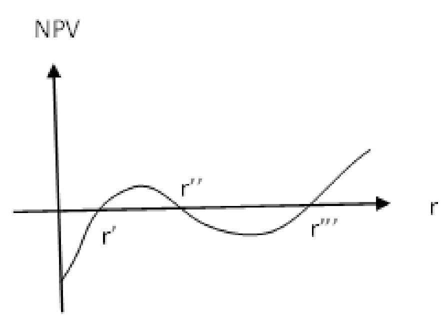
Conventional investment project



Conventional loan plan

When multiple values of IRR occur for an unconventional investment opportunity or unconventional borrowing opportunity, we should be able to evaluate them on the basis of the NPV decision criterion(Tung & Thomas, 1992). Because of the possibility of multiple IRR for investment and financing projects, as shown in figures of NPV versus$r$ graph





Let NPV at r = 0% is denoted by $[NPV]_{0\%}$, use this as the basis for classification condition with a single value of IRR and with multiple values of IRR separately, starting from the lower to the highest value and zero value will not be counted. Let r_x be the internal rate(s) of return and r^* be $MARR$ on the project x. If there are multiple *IRR*, arrange them in ascending order i.e. $r_x' < r_x'' < r_x''' < \cdots$; exclude all non-positive rates. Then we may observe the following conditions based on the NPV criterion(Tung & Thomas, 1992)(Martin, 2015),

α) When $[NPV]_{0\%} > 0$ and there is only one value of IRR_x that is r_x', the project x is acceptable if $r_x' \geq r^*$,

β) When $[NPV]_{0\%} > 0$ and there are multiple values of IRR_x, the project x is acceptable if $r_x' \geq r^*$, unacceptable if $r_x' < r^* < r_x''$ and acceptable again if $r_x'' < r^* < r_x'''$and so on,

χ) When $[NPV]_{0\%} <$ 0 and there is only one value of IRR_x that is r_x', the project x is acceptable if $r_x' \leq r^*$,

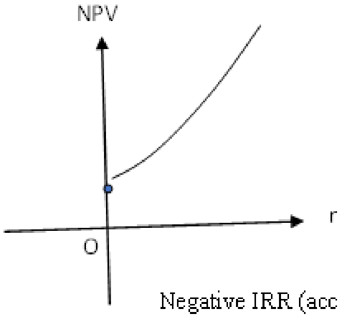
δ) When $[NPV]_{0\%}$ <0 and there are multiple values of IRR_x, the project x is unacceptable if $r_x' > r^*$, acceptable if $r_x' \leq r^* \leq r_x''$ and unacceptable again if $r_x'' < r^* < r_x'''$ and so on,

ε) When $[NPV]_{0\%}$ = 0 and slope $[\frac{dNPV_x}{dr}]_{0\%} > 0$, the project x is acceptable if $r_x' \geq r^*$, unacceptable if $r_x' < r^* < r_x''$, acceptable again if $r_x' \leq r^* \leq r_x''$ and so on,

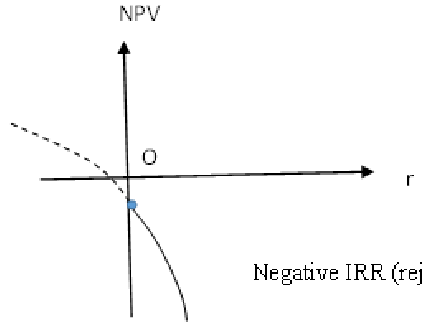
ϕ) When $[NPV]_{0\%}$ = 0 and slope $[\frac{dNPV_x}{dr}]_{0\%} < 0$, the project x is unacceptable if $r_x' > r^*$, acceptable if $r_x' \leq r^* \leq r_x''$, unacceptable again if $r_x' < r^* < r_x''$ and so on,

γ) When $[NPV]_{0\%}$ = 0 and slope $[\frac{dNPV_x}{dr}]_{0\%} = 0$, then the set of criteria can be extended as $[\frac{d^2NPV_x}{dr^2}]_{0\%} > 0$ or $[\frac{d^2NPV_x}{dr^2}]_{0\%} = 0$ or $[\frac{d^2NPV_x}{dr^2}]_{0\%} < 0$. If $[\frac{d^2NPV_x}{dr^2}]_{0\%} = 0$, we should test the third derivative.

η) When cash flow profile has no sign change, theIRR is either indeterminate or negative, as shown in the following figure of NPV versus r graph;



Negative IRR (accept)



Negative IRR (reject)

When all IRR are either indeterminate or negative, the decision criterion is to accept the project x if $[NPV]_{0\%} > 0$ and to reject project x if $[NPV]_{0\%} < 0$.

Analytically, the decision criteria for evaluation of independent projects can be computed of NPV and its derivatives at zero discount rate. The NPV of a series of cash flow $A_{t,x}$ (for $t = 0, 1, 2, \dots, n,\ \ t = 0$ is present time) for project x, where $x = 1, 2, 3, \dots$ denotes the projects 1, 2, 3, … respectively, can be obtained

$$NPV_x = \sum_{t=0}^{n} A_{t,x}(1+r)^{-t} \qquad (4)$$

Let $B_{t,x}$ and $C_{t,x}$ (for $t = 0, 1, 2, \dots, n,\ \ t = 0$ is present time) be the annual benefit and annual cost at the end of the year t for the same investment project x. Then $A_{t,x} = B_{t,x} - C_{t,x}$. Now,

$NPV_x = \sum_{t=0}^{n}(B_{t,x} - C_{t,x})(1+r)^{-t}$ and

$$[NPV]_{0\%} = \sum_{t=0}^{n} A_{t,x}$$

$$= \sum_{t=0}^{n}(B_{t,x} - C_{t,x}). \qquad (5)$$

Again, $[\frac{dNPV_x}{dr}] = -\sum_{t=0}^{n} t\, A_{t,x}\,(1+r)^{-(1+t)}$

$$= -\sum_{t=0}^{n} t\,(B_{t,x} - C_{t,x})(1+r)^{-(1+t)}.$$

Then slope at $r = 0\%$ is

$$[\frac{dNPV_x}{dr}]_{0\%} = -\sum_{t=0}^{n} t\, A_{t,x}$$

$$= \sum_{t=0}^{n} -t\,(B_{t,x} - C_{t,x}) \qquad (6)$$

If $[NPV]_{0\%} = 0$ and $[\frac{dNPV_x}{dr}]_{0\%} = 0$,then find

$$\frac{d^2 NPV_x}{dr^2} = \sum_{t=0}^{n} t\,(t+1)\, A_{t,x}\,(1+r)^{-(2+t)}$$

= $\sum_{t=0}^{n} t\,(t+1)(B_{t,x} - C_{t,x})(1+r)^{-(2+t)}$ and

$[\frac{d^2NPV_x}{dr^2}]_{0\%} = \sum_{t=0}^{n} t\,(t+1)\,A_{t,x}$

$$= \sum_{t=0}^{n} t\,(t+1)(B_{t,x} - C_{t,x}) \quad (7)$$

Summary of the decision criteria for accepting independent projects,

Range of r^*	$[NPV]_{0\%} > 0$	$[NPV]_{0\%} < 0$	$[NPV]_{0\%} = 0$	
			$[\frac{dNPV_x}{dr}]_{0\%} > 0$	$[\frac{dNPV_x}{dr}]_{0\%} < 0$
$0 \le r^* < r_x{}'$	Accept	Reject	Accept	Reject
$r_x{}' < r^* < r_x{}''$	Reject	Accept	Reject	Accept
$r_x{}'' < r^* < r_x{}'''$	Accept	Reject	Accept	Reject
$r_x{}''' < r^* < r_x{}^{iv}$	Reject	Accept	Reject	Accept
Negative or Indeterminate	Accept	Reject	–	–

Example: The flow profiles of three independent projects are given below. Using $MARR$ of 10%, apply the decision criteria based on the IRR to determine if each of these projects is acceptable; cash flow is Rs. 1000,

Year(t)	$A_{t,1}$	$A_{t,2}$	$A_{t,2}$
0	–545	– 340	– 340
1 - 7	150	150	150
8	0	– 830	– 710

For $x = 1$, there is a unique IRR because of one sign change in the cash flow profile. Now, $[NPV_1]_{0\%}$ =–545 + 7×150 + 0 = 505.

For the IRR, r_1, interpolating from 15% to 20%,

$$[NPV_1]_{15\%} = \sum_{t=0}^{8} A_{t,1}(1+r)^{-t} = A_{0,1} + \frac{A_{1,1}}{1+r} + \frac{A_{2,1}}{(1+r)^2} + \cdots + \frac{A_{8,1}}{(1+r)^8}$$

$$= -545 + \frac{150}{1+0.15} + \frac{150}{(1+0.15)^2} + \cdots + \frac{150}{(1+0.15)^7} + \frac{0}{(1+0.15)^8} = 79.1$$

Similarly, $[NPV_1]_{20\%}$ = –4.3.

Secant method: Assume that x_0 and x_1 are two initial estimates of the root α. Approximate the graph of $y = f(x)$ by the secant line determined by $(x_0, f(x_0))$ and $(x_1, f(x_1))$. Let its root be denoted by x_2. Using the slope formula with the secant line, we have

$$\frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Solving for x_2, $x_2 = x_1 - f(x_1).\frac{x_1 - x_0}{f(x_1) - f(x_0)}$ and so on.

General formula is $x_{n+1} = x_n - f(x_n).\frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})}$, $n \geq 1$.

This is secant method (Atkinson, 1989).
Thus, by using the secant method,

$$r_1 = 15\% + (20\% - 15\%)\frac{79.1}{79.1 + 4.3} = 19.7\% > 10\% \ (MARR).$$

Hence, proposed projects are acceptable.
For $x = 2$, there are two rates of returns because of two times sign changes in the cash flow profile.
Now, $[NPV_1]_{0\%}$ =−340+ 7×150 − 830 = −120.
For the IRR, r_2, interpolating from 5% to 10%,
$[NPV_2]_{5\%} = -340 + \frac{150}{1+0.05} + \frac{150}{(1+0.05)^2} + \cdots + \frac{150}{(1+0.05)^7} - \frac{830}{(1+0.05)^8}$ = −34.95

Similarly, $[NPV_1]_{10\%}$ = 3.3.
Thus, by using the secant method,

$$r'_2 = 5\% + (10\% - 5\%)\frac{(-34.95)}{(-34.95 - 3.3} = 9.57\ \%.$$

Again, interpolating $15\%\ to\ 25\%$, we obtained

$[NPV_2]_{15\%} = 12.67\ and\ [NPV_2]_{25\%} = -5$, using secant method,

$$r''_2 = 15\% + (25\% - 15\%)\frac{(12.67)}{(12.67 + 5} = 22.17\ \%.$$

We obtained, 9.57 % < 10% < 22.17 % $(r_2' < r^* < r_2'')$
Hence, proposed projects are acceptable.
For $x = 3$, there are two rates of returns because of two times sign changes in the cash flow profile.
Now, $[NPV_3]_{0\%}$ =−340 + 7×150 − 710 = 0.,
one IRR is $r'_3 = 0$ (zero value is ignored).
Now, $[\frac{dNPV_x}{dr}]_{0\%}$ =

$$-\sum_{t=0}^{8} t\, A_{t,x} = -[-0 \times 340 + 150\,(1 + 2 + 3 + \cdots + 7) - 8 \times 710] = 1480 > 0$$

(accept the project)
Interpolating from 25% to 30%,
$[NPV_3]_{25\%} = 14.87\ and\ [NPV_3]_{30\%} = -6.5$, using secant method,

$r''_3 = 25\% + (30\% - 25\%)\frac{(14.87)}{(14.87 + 6.5} = 28.48\ \%$ > 10% $(MARR)$.

Hence, the project is acceptable.

Conclusion:

The multiple internal rates of return as an evaluation criterion of mutually exclusive and independent projects. It is a widely used criterion for evaluating the independent projects, however, it presents disadvantages like reinvesting the intermediate revenue, late cost, the existence of many roots during solving out the respective mathematical equation, so that financial analysists are suggested to use it with more carefully. It is not a measure of investment attractiveness until a $MARR$ (minimum attractive rate of return) becomes available as a comparator, because the decision criteria of multiple IRR and conditions of accepting or rejecting the mutually exclusive and independent projects on the basis of $MARR$. Above discussed decision rules will correctly identify acceptable alternative and the best among mutually exclusive ones for borrowing or investment situation, for single or multiple internal rate(s) of return for conventional or unconventional cash-flows.

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