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Modified NHE Distribution with Properties and Applications

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Abstract

We have defined a novel three- parametric model having bathtub- shaped hazard rate curve called a modified NHE distribution in this study. Several statistical features of recommended model are explicitly derived. Least square, maximum likelihood and Cramer-Von-Mises are three renowned estimations approaches which can be applied to derive model parameters. The estimators' asymptotic confidence intervals are also defined. The suggested distribution's goodness of fit is evaluated through a real data set. On real -world data set, we have discovered that the innovative suggested model outperforms other competitive models.

Keywords: Exponential extension distribution, Hazard function, MCMC, Modified NHE, Quantile function

Introduction

Probability models are frequently employed to the research of reliability analysis in several sectors of engineering, biological sciences, and applied statistics, according to nearly all of the literatures on probability and applied statistics. Existing models don't always produce a better fit when modeling reliability data. Consequently most of the researchers are drawn in towards adjusting traditional models and exploring their adaptability and pertinence. Extra parameter(s) is (are) added to the baseline distribution to develop these novel modified distributions, which usually provide a good fit when compared to traditional models.

In the last two decades, it has been discovered that a novel model family might be created through the exponential model as the baseline model. Several researchers were modified the exponential distribution to generate novel model. Some of novel models developed by modifying exponential model are the generalized inverted exponential distribution (Abouammoh & Alshingiti , 2009), gamma EE (Ristic & Balakrishnan, 2012), and generalized exponential (GE) (Gupta & Kundu ,2007) and exponential Extension (EE) model (Kumar, 2010). Lemonte (2013) recommended a novel exponential-type model with a failure rate function that is inverted bathtub, constant, declining, bathtub-shaped and growing. By combining the extended exponential and generalized exponential distributions (Gupta & Kundu, 2001), Rasekhi et al. (2017) created the modified exponential distribution whose CDF having four parameters $x > 0$ and (λ, θ, b, a) is

$$Q(x) = 1 - \left\{ 1 - \frac{ab}{a+b} \log\left[1 - \left(1 - e^{-\lambda x}\right)^{\theta} \right] \right\} \left\{1 - \left(1 - \exp(-\lambda x)^{\theta}\right\}^{a}$$

In this work, we have defined the novel model by revising the exponential model's extension (Nadarajah & Haghighi, 2011) which is also named as NHE distribution (Chaudhary & Kumar, 2020). The NHE distribution's CDF and PDF having α (shape parameter), λ (scale parameter) and $x > 0,\ (\alpha\ \lambda) > 0$ are shown below.

$$G(x) = 1 - \exp\left\{ 1 - \left(1 + \lambda x\right)^{\alpha} \right\} \tag{1}$$

$$g(x) = \lambda\alpha \exp\left\{ 1 - \left(1 + \lambda x\right)^{\alpha} \right\}\left(1 + \lambda x\right)^{\alpha-1} \tag{2}$$

Khan (2014) defined a two-parameter modified model known as the modified inverse Rayleigh distribution with $x > 0$, and $(\alpha, \beta > 0)$ whose CDF is

$$F(x) = \exp\left[-\beta x^{-2} - \alpha x^{-1} \right]. \tag{3}$$

(Khan, 2015) defined another five-parameter modified model known as the modified beta Weibull probability distribution with the inverted bathtub failure rate function, which is ideal for modeling positive data sets. Similarly, Iriarte et al.(2018) created the modified slashed-Rayleigh distribution by extending the ordinary Rayleigh and exponential distributions. Gillariose et al., (2020) defined new modified model termed as the Marshall-Olkin modified Lindley distribution. Many writers have also used the NHE distribution to create flexible models, such as the Burr-X Nadarajah Haghighi distribution, which was established by (Elsayed & Yousof, 2019). The Logistic NHE distribution was defined by (Chaudhary & Kumar, 2020).

This article's main goal is to make suggested model more feasible by introducing an extra parameter described by (Nadarajah & Haghighi, 2011). The modified NHE distribution's statistical and distributional properties are explored, as well as its usefulness. The residual portion of the suggested research is structured as follows. The modified NHE distribution and their numerous statistical and distributional properties are defined. We have used three renowned estimation approaches such as CVME, MLE and LSE to estimate the model parameters. We have investigated the suggested distribution's usefulness and potential using real data set. We have calculated the estimated parameters values as well as fit statistics such as log-likelihood, AIC, BIC, and CAIC criterion in this section. It is discovered that the proposed distribution outperforms some well-known distributions. Finally, we offer some concluding remarks.

The modified NHE (MNH) distribution

If X be a positive r.v. that follows MNH distribution with parameters (α, β, λ) having $(\alpha, \beta, \lambda) > 0$ and $x > 0$, then MNH model's CDF is

$$F(x) = 1 - \exp\left\{ 1 - \left(1 + \lambda x\, e^{\beta x} \right)^{\alpha} \right\} \tag{4}$$

Its corresponding probability density function is specified by

$$f\ (x) = \alpha\ \lambda\beta\ \ (1+\beta x)\ \exp\left\{ 1-\left(1+\lambda\ x\ \exp(\beta x)\right)^{\alpha} \right\}\left(1+\lambda\ xe^{\beta x}\ \right)^{\alpha-1} e^{\beta x} \tag{5}$$

MNH distribution's survival function

$$S(x;\ \alpha,\ \beta,\ \lambda)\ =\ \exp\ \left\{ 1-\left(1+\lambda xe^{\beta x}\ \right)^{\alpha} \right\} \tag{6}$$

The MNH distribution's failure rate function (HRF)

The HRF is defined by ,

$$h(\ x;\ \alpha,\beta,\lambda) = \lambda\ \alpha\ \beta\ \left(1+\lambda xe^{\beta x}\ \right)^{\alpha-1}\left(1+\beta x\ \right) e^{\beta x} \tag{7}$$

MNH distribution's reverse failure rate function

The reverse hazard function is,

$$\mathrm{Re}\, vHRF = \alpha\beta\lambda\left(1+\beta\ x\ \right)\ \left\{ \left[\exp\left\{ 1-\ \left(1+\lambda xe^{\beta x}\right)^{\alpha} \right\} \right]^{-1} -1 \right\}\left(1+\lambda\ x\ e^{\beta x}\ \right)^{\alpha-1} e^{\beta x} \tag{8}$$

Cumulative hazard function (CHF)

The CHF of $MNH(\alpha,\ \beta,\ \lambda)$ distribution can be expressed as

$$CHF = \left(1+\lambda xe^{\beta x}\right)^{\alpha} -1 \quad ;\ x>0,\ \ (\alpha\beta\lambda)>0 \tag{9}$$

Quantile function:

If T is a positive r.v. with $0<p<1$ as its distribution function, then p^th^-quantile of T such that $0<p<1$ can be defined as

$$Q_T(p) = F_T^{-1}(p)$$

$$\lambda xe^{\beta x} + 1 - \left\{1-\ln(1-p)\right\}^{1/\alpha} = 0 \tag{10}$$

The MNH's random deviation generation can be represented as,

$$\lambda xe^{\beta x} + 1 - \left\{1-\ln(1-v)\right\}^{1/\alpha} = 0 \quad ;0<v<1 \tag{11}$$

MNH distribution's skewness and kurtosis

Using quartiles, the skewness coefficient is,

$$S_k\left(\ Bowley\ \right) = \frac{Q\ (3/4)+Q(1/4)-2Q(\ 1/2\ )}{Q(3/4)-Q(1/4)},\ \text{ and}$$

Moors (1988) Coefficient of kurtosis is

$$K_u\left(\ Moors\ \right) = \frac{Q(0.875)-Q(0.625)+Q(0.375)-Q(0.125)}{Q(3/4)-Q(1/4)}$$

Figure 1 shows charts of $MNHE(\alpha,\ \beta,\ \lambda)$'s PDF and HRF with varying parameter values.

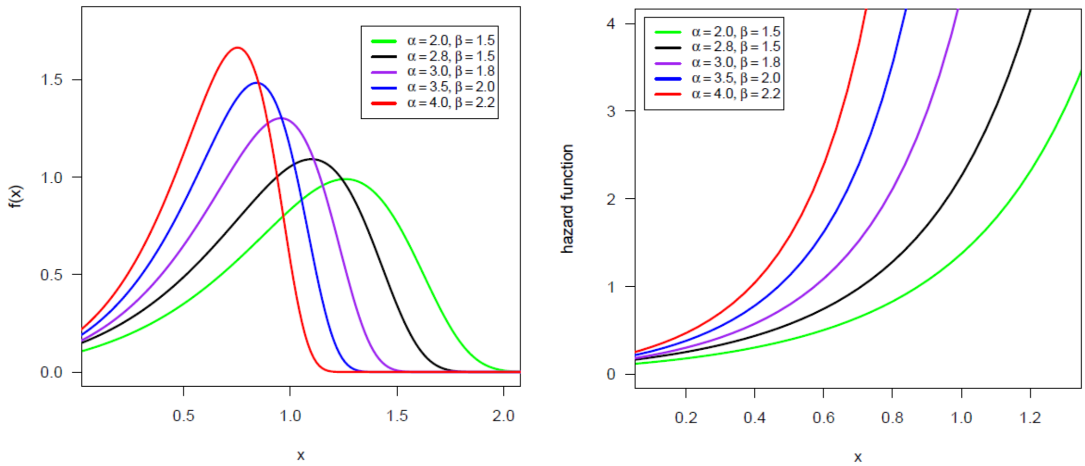


Fig. 1: Hazard function (right section) and PDF (left section) charts with fixed λ and various β and α values.

Estimation Methods

MNH'parameter values can be estimated in this part using the following renowned estimation methods:

MLE Method

MLE is the most often utilized approach for estimating a model's parameter. If a random sample $x_1, x_2, \ldots, x_n$ be drawn from $MNHE(\alpha,\ \beta,\ \lambda)$, then, the likelihood function, $L(\alpha, \beta, \lambda)$ is defined by

$$L(\phi;\ x_1, x_2, \ldots, x_n) = f(x_1, x_2, \ldots, x_n / \phi) = \prod_{i=1}^{n} f(x_i / \phi)$$

$$L(\alpha,\beta,\lambda) = (\alpha\beta\lambda)^n \prod_{i=1}^{n} e^{\beta x_i}(1+\beta x_i)\left(1+\lambda x_i e^{\beta x_i}\right)^{\alpha-1} \exp\left\{1-\left(1+\lambda x_i e^{\beta x_i}\right)^{\alpha}\right\};\quad x>0$$

The density of log-likelihood is

$$\ell(\alpha,\beta,\lambda \mid \underline{x}) = n\ln(\alpha\beta\lambda) - \sum_{i=1}^{n}\left(1+\lambda x_i e^{\beta x_i}\right)^{\alpha} + \sum_{i=1}^{n}\ln(1+\beta x_i) + \beta\sum_{i=1}^{n} x_i + n + (\alpha-1)\sum_{i=1}^{n}\ln\left(1+\lambda x_i e^{\beta x_i}\right. \quad (12)$$

Differentiating (12) with respect to alpha, beta & lambda, we have,

$$\frac{\partial\ell}{\partial\alpha} = \frac{n}{\alpha} + \sum_{i=1}^{n}\ln[1+\lambda x_i e^{\beta x_i}] + \sum_{i=1}^{n}\left[\left\{\ln[1+\lambda x_i e^{\beta x_i}]\right\}^{\alpha}[1+\lambda x_i e^{\beta x_i}]\right]$$

$$\frac{\partial\ell}{\partial\lambda} = \frac{n}{\lambda} + (\alpha-1)\sum_{i=1}^{n}\left\{x_i e^{\beta x_i}\left(1+\lambda x_i e^{\beta x_i}\right)^{-1}\right\} - \alpha\sum_{i=1}^{n}\left\{\left(1+\lambda x_i e^{\beta x_i}\right)^{\alpha-1} x e^{\beta x_i}\right\}$$

$$\frac{\partial \ell}{\partial \beta}= \frac{n}{\beta}+\sum_{i=1}^{n}\left\{\left(1+\beta\, x_i\right)^{-1} x_i\right\}-\alpha\sum_{i=1}^{n}\left(1+\lambda\, x_i\, e^{\beta\, x_i}\right)^{\alpha-1}+\sum_{i=1}^{n} x_i-\lambda\sum_{i=1}^{n} x_i^{2}\, e^{\beta x_i}$$
$$+\lambda\,(\alpha-1)\sum_{i=1}^{n}\left\{\left(1+\lambda\, x_i\, e^{\beta x_i}\right)^{-1} x_i^{2}\, e^{\beta x_i}\right\}$$

Equate overhead 3 equations to zero and solve concurrently for alpha, beta and lambda , we have corresponding ML estimate $\hat{\alpha},\ \hat{\beta}\ and\ \hat{\lambda}$. The software such as Matlab, R, Mathematica, etc. are also used to get the estimated values for optimization of (14). The observed information matrix (OIM) must be determined to estimate the confidence intervals for alpha, beta and lambda and the hypothesis testing. The OIM for alpha, beta and lambda might be calculated as follows:

$$T = \begin{bmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{bmatrix}$$

Where

$$T_{11}=\frac{\partial^2 l}{\partial\alpha^2}=\operatorname{var}(\alpha),\ T_{12}=\frac{\partial^2 l}{\partial\alpha\partial\beta}=\operatorname{cov}(\alpha,\beta),\ T_{13}=\frac{\partial^2 l}{\partial\alpha\partial\lambda}=\operatorname{cov}(\alpha,\lambda)$$
$$T_{21}=\frac{\partial^2 l}{\partial\beta\partial\alpha}=\operatorname{cov}(\beta,\alpha),\ T_{22}=\frac{\partial^2 l}{\partial\beta^2}=\operatorname{cov}(\beta),\ \mathrm{T}_{23}=\frac{\partial^2 l}{\partial\beta\partial\lambda}=\operatorname{cov}(\beta,\lambda)$$
$$T_{31}=\frac{\partial^2 l}{\partial\lambda\partial\alpha}=\operatorname{cov}(\lambda,\alpha),\ \mathrm{T}_{32}=\frac{\partial^2 l}{\partial\beta\partial\lambda}=\operatorname{cov}(\beta,\alpha),\ \mathrm{T}_{33}=\frac{\partial^2 l}{\partial\lambda^2}=\operatorname{var}(\lambda)$$

Assume, $\Omega=(\alpha,\beta,\lambda)$, then associated maximum likelihood estimator of Ω is $\hat{\Omega} = (\hat{\lambda}, \hat{\beta}, \hat{\alpha})$, then $\{\hat{\Omega}-\Omega\}\rightarrow N_3\left[0,\left(T(\Omega)\right)^{-1}\right]$ where the Fisher's information matrix (FIM) is $T(\Omega)$. To optimize the likelihood that creates OIM, the Newton-Raphson algorithm is used. As a result, the variance-covariance matrix is

$$\left[T(\Omega)\right]^{-1} = \begin{pmatrix} v(\hat{\alpha}) & cv(\hat{\beta},\hat{\alpha}) & cv(\hat{\alpha},\hat{\lambda}) \\ cv(\hat{\beta},\hat{\alpha}) & v(\hat{\beta}) & cv(\hat{\beta},\hat{\lambda}) \\ cv(\hat{\alpha},\hat{\lambda}) & cv(\hat{\beta},\hat{\lambda}) & v(\hat{\lambda}) \end{pmatrix} \qquad (13)$$

Where cv = covariance and v= variance

As a result of MLEs' asymptotic normality , then (1- γ)* 100 percent confidence intervals for beta, alpha and lambda might be formed as follows:

$\hat{\beta} \pm Z_{\gamma/2} * S.D.(\hat{\beta})$ $\hat{\lambda} \pm Z_{\gamma/2} * S.D.(\hat{\lambda})$, and $\hat{\alpha} \pm Z_{\gamma/2} * S.D.(\hat{\alpha})$

LSE method

For assessing the Beta distribution's parameters, Swain et al. (1988) suggested weighted and ordinary LS estimators. Through minimizing (14) with regard to unknown parameters alpha, beta and lambda of the MNH distribution yields the least-square estimators.

$$W(X;\alpha,\beta,\lambda)=\sum_{i=1}^{n}\left[F(X_i)-\frac{i}{n+1}\right]^2 \quad (14)$$

Consider the random variables that are arranged as $X_{(1)} < X_{(2)} < \ldots < X_{(n)}$ with their distribution functions $F(X_i)$, where random sample $\{X_1, X_2, \ldots, X_n\}$ formed from $F(X_i)$ having size n. The LS estimators $\hat{\alpha}$, $\hat{\beta}$, *and* $\hat{\lambda}$ of alpha, beta & lambda are determined respectively through minimizing (17) with regard to alpha, beta and lambda.

$$W(X;\alpha,\beta,\lambda)=\sum_{i=1}^{n}\left[1-\exp\left\{1-\left(1+\lambda x_i e^{\beta x_i}\right)^{\alpha}\right\}-\frac{i}{n+1}\right]^2 ;(\alpha,\beta,\lambda)>0,\ x>0. \quad (15)$$

Differentiating (17) with regard to alpha , beta and lambda , we have

$$\frac{\partial W}{\partial \alpha}=2\sum_{i=1}^{n}\left[1-\exp\left\{1-D(x_i)^{\alpha}\right\}-\frac{i}{n+1}\right]\exp\left\{1-D(x_i)^{\alpha}\right\}D(x_i)^{\alpha}\ln\left\{D(x_i)\right\}$$

$$\frac{\partial W}{\partial \beta}=2\alpha\lambda\sum_{i=1}^{n}x_i^2 e^{\beta x_i}\left[1-\frac{i}{n+1}-\exp\left\{1-D(x_i)^{\alpha}\right\}\right]\exp\left\{1-D(x_i)^{\alpha}\right\}D(x_i)^{\alpha-1}$$

$$\frac{\partial W}{\partial \lambda}=2\alpha\sum_{i=1}^{n}x_i e^{\beta x_i}\left[1-\exp\left\{1-D(x_i)^{\alpha}\right\}-\frac{i}{n+1}\right]\exp\left\{1-D(x_i)^{\alpha}\right\}D(x_i)^{\alpha-1}$$

where $1+\lambda x_i e^{\beta x_i} = D(x_i)$

Similarly, through minimizing B with regard to β, α and λ, weighted LS estimators might be constructed.

$$B(X;\alpha,\beta,\lambda)=\sum_{i=1}^{n}w_i\left[F(X_{(i)})-\frac{i}{n+1}\right]^2$$

Where $w_i = weights = \frac{1}{Var(X_{(i)})}=\frac{(n+2)(n+1)^2}{i(n-i+1)}$

As a result, the estimators of weighted LS can be determined through minimizing (16) with regard to α, λ and β .

$$B(X;\alpha,\beta,\lambda)=\sum_{i=1}^{n}w_i\left[1-\exp\left\{1-\left(1+\lambda x_i e^{\beta x_i}\right)^{\alpha}-\frac{i}{n+1}\right\}\right]^2 \quad (16)$$

CVME method

By minimizing function (17), the CVM estimators of alpha, beta and lambda are derived.

$$M\left(X\right)=\frac{1}{12n}+\sum_{i=1}^{n}\left[F\left(x_{i:n}\mid\alpha,\beta,\lambda\right)-\frac{2i-1}{2n}\right]^{2}$$
$$=\frac{1}{12n}+\sum_{i=1}^{n}\left[1-\exp\left\{1-\left(1+\lambda x_i\, e^{\beta x_i}\right)^{\alpha}\right\}-\frac{2i-1}{2n}\right]^{2} \qquad (17)$$

Differentiating (19) with regard to alpha , beta and lambda ,we have

$$\frac{\partial M}{\partial\alpha}=2\sum_{i=1}^{n}D\left(x_i\right)^{\alpha}\ln\left\{D\left(x_i\right)\right\}\left[1-\exp\left\{1-D\left(x_i\right)^{\alpha}\right\}-\frac{2i-1}{2n}\right]\exp\left\{1-D\left(x_i\right)^{\alpha}\right\}$$

$$\frac{\partial M}{\partial\lambda}=2\alpha\sum_{i=1}^{n}\left[1-\exp\left\{1-D\left(x_i\right)^{\alpha}\right\}-\frac{2i-1}{2n}\right]x_i e^{\beta x_i}D\left(x_i\right)^{\alpha-1}\exp\left\{1-D\left(x_i\right)^{\alpha}\right\}$$

$$\frac{\partial M}{\partial\beta}=2\alpha\lambda\sum_{i=1}^{n}x_i^{2}D\left(x_i\right)^{\alpha-1}\left[1-\exp\left\{1-D\left(x_i\right)^{\alpha}\right\}-\frac{2i-1}{2n}\right]\exp\left\{1-D\left(x_i\right)^{\alpha}\right\}e^{\beta x_i}$$

where $1+\lambda x_i e^{\beta x_i}=D\left(x_i\right)$

CVM estimators can be obtained by solving $\frac{\partial M}{\partial\alpha}=0, \frac{\partial M}{\partial\beta}=0 \; and \; \frac{\partial M}{\partial\lambda}=0$ at the same time.

Application

To demonstrate the applicability of suggested model, we use the data given below which pertains to carbon fibres's breaking stress with a length of 50 mm (GPa) as described by (Nichols & Padgett, 2006). [4.90, 2.50, 2.53, 2.55, 2.55, 2.56, 2.59, 2.67, 2.73, 2.74, 2.79, 2.81, 2.82, 2.85, 2.87, 2.88, 2.93, 2.95, 2.96, 2.97, 3.09, 3.11, 3.11, 3.15, 3.15, 3.19, 3.22, 3.22, 3.27, 3.28, 3.31, 3.31, 3.33, 3.39, 3.39, 3.56, 3.60, 3.65, 3.68, 3.70, 0.39, 0.85, 1.08, 1.25, 1.47, 1.57, 1.61, 1.61, 1.69, 1.80, 1.84, 1.87, 1.89, 2.03, 2.03, 2.05, 2.12, 2.35, 2.41, 2.43, 3.75, 4.20, 4.38, 4.42, 4.70, 2.48]

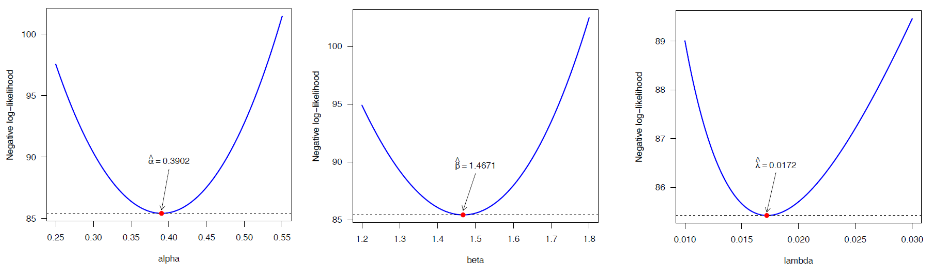
The MLEs of the MNH distribution are evaluated by applying optim () function in R software described by (R Core Team , 2020) through maximizing (12) .The Log-Likelihood's value obtained is l = -85.42196. Table 1 shows the MLEs for alpha, beta, and lambda together with their standard errors (S.E.).

Table 1

S.E and. MLE for alpha, beta, and lamda of MNH

Parameter	MLE	SE
alpha	0.3902	0.15553
beta	1.4671	0.44938
lambda	0.0172	0.01153

The ML estimates can be computed independently using profile log-likelihood function plots in Fig. 2 for model parameters.



Fig, 2: Charts of Profile for log-likelihood function for λ, β and α parameters.

The Q-Q chart and the P-P chart are shown in Fig. 3, and the recommended model matches the data rather pretty well.

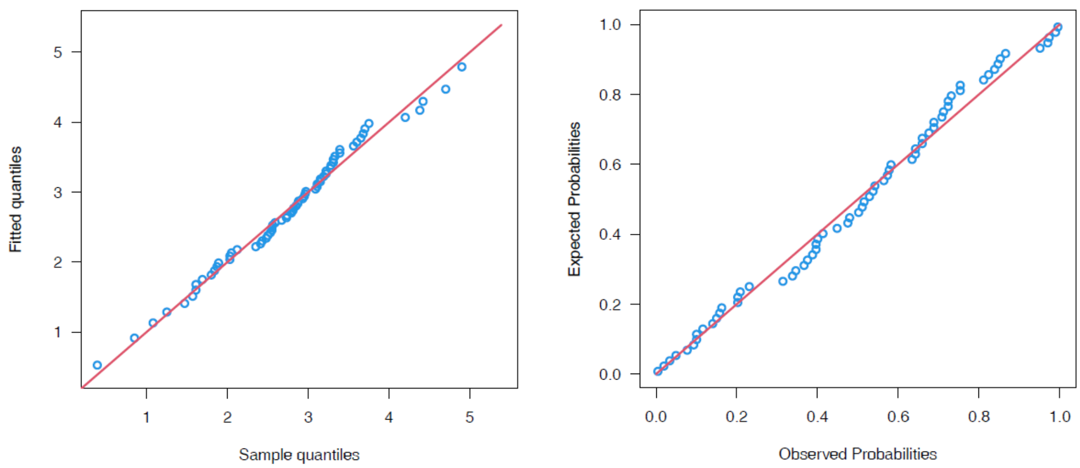


Fig. 3. The Q-Q chart (leftward section) & P-P chart (rightward section) for MNH model.

Table. 2 shows MNH model's expected parameters values using LSE, MLE, and CVME methods, along with the associated AIC criterion and negative log-likelihood .

Table 2

[AIC, log-likelihood and Estimated parameters]

Estimation Method	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\lambda}$	LL	AIC
LSE	0.8787	1.0843	0.0137	-89.8009	185.6017
MLE	0.3902	1.4671	0.0172	-85.4220	176.8440
CVME	0.6776	1.2539	0.0122	-88.6614	183.3229

Table 3 shows the statistics of A^2, W, and KS, as well as the MLE, LSE, and CVE estimates' related p-values. Fitted distributions' histogram and density function, as well as Q-Q chart of LSE ,MLE and CVM, are shown in Fig.4

Table 3

[The statistics A^2 ، KS & W with their p-values]

Estimation Method	*A^2 (p-value)*	*KS (p-value)*	*W(p-value)*
LSE	0.6782(0.5767)	0.0601(0.9710)	0.0333(0.9651)
MLE	0.3961(0.8521)	0.0799(0.7929)	0.0640(0.7906)
CVME	0.6143(0.6339)	0.0584(0.9780)	0.0335(0.9642)

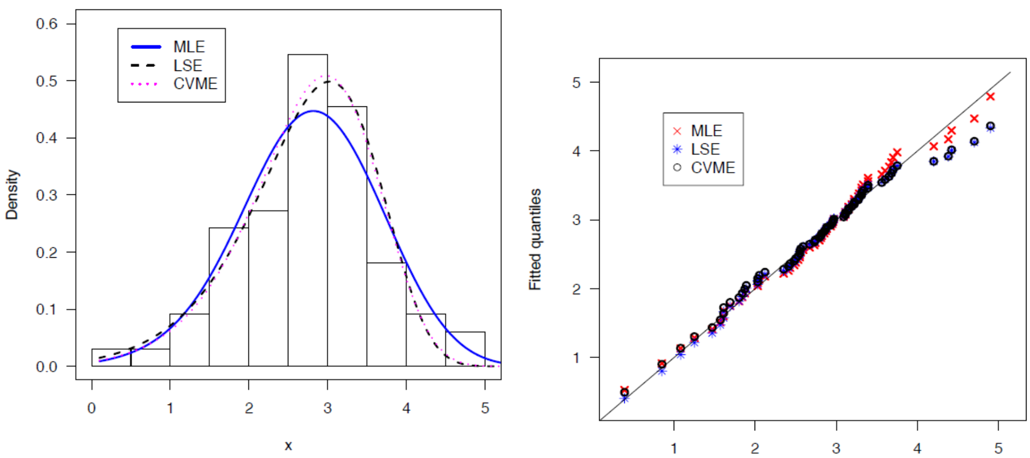


Fig.4: The density function & the Histogram of fitted distributions (left-hand section) & Q-Q chart for LSE, MLE and CVM (right section).

We've demonstrated the flexibility of the MNH model through a real dataset that has already been used by other researchers in this part. We have used competitive four distributions such as Generalized Exponential (GE) (Gupta & Kundu, 1999), Generalized Exponential Extension (GEE) , (Lemonte, 2013), Exponentiated Exponential Poisson (EEP) (Ristić & Nadarajah, 2014), Chen distribution (Chen, 2000), Generalized Gompertz (GGZ) distributions (El-Gohary et al., 2013), Weighted Lindley (WL) distribution (Ghitany et al., 2011) and to compare the suggested model's potential.

In Table 4, Corrected Akaike information (CAIC), Hannan-Quinn information (HQIC), Akaike information (AIC), and Bayesian information criterions (BIC) are applied to judge the potentiality of the suggested distribution, and the results are exhibited.

Table 4

HQIC, CAIC, BIC, AIC, and Log-likelihood [LL]

Models	LL	AIC	BIC.	CAIC	HQIC.
MNH	-85.4220	176.8440	183.4130	177.2311	179.4397
EEP	-86.6899	179.3798	185.9488	179.7669	181.9755
GGZ	-85.6858	177.3716	183.9406	177.7587	179.9673
GEE	-87.2704	180.5408	187.1098	180.9279	183.1365
Chen	-88.1530	180.3060	184.6853	180.4965	182.0365
WL	-90.9291	185.8582	190.2375	186.0487	187.5887
GE	-95.3724	194.7447	199.1240	194.9352	196.4752

The density function and histogram of fitted distributions, as well as the empirical and the estimated distribution functions of the MNH distribution and a few competitive models, are depicted in Fig. 5.

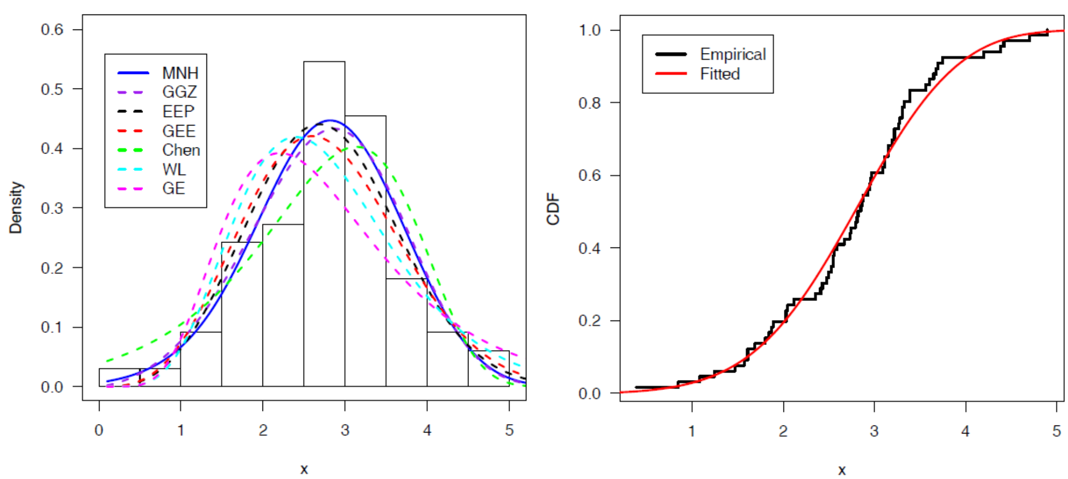


Fig. 5. Fitted distributions' density function & histogram (left-hand section) & Empirical distribution with estimated distribution functions (right-hand section).

The statistics for Anderson-Darling (AD), Cramer-Von Mises (CVM) and Kolmogorov-Simnorov (KS), to evaluate the MNH model's goodness-of-fit with other competing models are found in Table 5. We can accomplish that the MNH model has a lot better fitting and more consistent and trustworthy outcomes than other distributions because it has the lowest test statistic value and a higher p-value.

Table 5

Statistics and with their related p-values for goodness-of-fit

Models	*CVM [p-value]*	*AD [p-value]*	*KS [p-value]*
MNH	0.3961(0.8521)	0.0640(0.7906)	0.0799(0.7929)
[EEP]	0.5657(0.6804)	0.1014(0.5796)	0.0895(0.6662)
GGZ	0.4457(0.8020)	0.0715(0.7443)	0.0833(0.7498)
[GEE]	0.1096(0.4065)	0.1530(0.3812)	0.7816(0.4940)
Chen	0.9854(0.3647)	0.1460(0.4027)	0.1115(0.3849)
WL	1.2857(0.2370)	0.2419(0.1992)	0.1318(0.2016)
GE	2.0934(0.0818)	0.3730(0.0850)	0.1549(0.0842)

Conclusion

The modified NHE distribution is suggested and considered. The statistical and distributional features of the suggested distribution have been reported. MNH distribution's PDF curve can be positively skewed and, unimodal and the failure rate function can have monotone failure rates that are increasing, constant, or a wide range of monotonous failure rates. The purposed distribution fits the real dataset much better, as seen by the P-P and Q-Q charts. We have used a real data set to test three renowned estimating approaches, such as MLE, LSE, and CVME, and discovered that ML estimates outperform LSE and CVME estimates. An asymptotic confidence interval for MLEs has also been constructed. The usefulness demonstrates that the MNH model routinely outperforms rival distributions in terms of fit and flexibility. In the area of probability and statistics, we expect that suggested model will be an alternative.

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