Applicability of Portfolio Theory in Nepali Stock Market

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Abstract

In the rapidly growing stock market of Nepal, this study tests the applicability of the portfolio creation model and attempts to aware investors about the potential portfolio alternatives they can make to achieve their peculiar risk-return need, through a robust optimization model. A portfolio model using Markowitz mean-variance method is applied to calculate the optimal portfolio and portfolios fitting the investor specific needs, from a sample of 20 Group "A" listed companies on NEPSE. The monthly stock prices between April 2010 and December 2014 of sample companies are used as training data. And, the applicability of the model is tested based on their prices on April 2015. From the analysis it is concluded that such mean-variance optimization is applicable in Nepal. Furthermore, most of the stocks, even from different sectors, are highly correlated to each other illustrating the lack of diversification opportunity at NEPSE. Additionally, the significantly high volatility even at global minimum variance level illustrated the risky nature of business environment in the country. There is an opportunity for high return, but the investor's willingness to gain this is tested through the high magnitude of minimum risk. These findings call for the policy makers' immediate attention in creating a favorable environment to bring the real sector companies in the public trading realm and enhancing the commodities and derivatives market in the country, thereby helping stimulate the investment environment in Nepal.

Key Words: Investment Decisions, Portfolio Choice, Portfolio Optimization, Markowitz Frontier

JEL Classification: G11

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I. INTRODUCTION

Nepali stock market has a relatively short history. The modern development of stock market began with the establishment of Securities Exchange Centre (SEC) in 1976, which aimed at facilitating and promoting the growth of capital market (Gurung, 2004). The floor opened for secondary trading of government bond in 1981 and corporate shares in 1984. The full-fledged stock trading operation began in Nepal only after the conversion of Securities Exchange Centre into Nepal Stock Exchange (NEPSE) in 1993. As of mid 2015, there are 408 scrips listed in NEPSE with market capitalization of over NRs. 1120.32 Billion.

The sharp increase in per capita income among urban middle class (CBS, 2011) has lured them towards the stock market. This can be illustrated from the fact that the one-day trading amount on 22 July 2015 was 1.2 Billion, equal to one sixth of the volume of annual trading in 2010. Despite this short popularity, a lot of research has been conducted in relation to stock price behaviors in NEPSE. A major finding that is consistently echoed is that stock prices in NEPSE show persistent and predictable behavior. Dangol (2010) also claims that the price changes in NEPSE are not random and can be predicted to gain desired returns.

The important question is what factors do investors need to consider in making investment decisions so that they may achieve the desired return within a stated risk level. Studies have shown that the relationship of one stock with the other in a portfolio, apart from individual stock characteristics, is largely responsible for determining the total return obtained from a portfolio by staying within the risk limit (Cochrane, 2003; Engels, 2004).

Every investor aims at selecting the best possible combination of securities that addresses his peculiar risk-return need. But, in a country like Nepal with a very low financial literacy, people often gamble while choosing stocks to invest. The individual stocks are chosen based on market psychology and heuristics, and such stocks are individually gathered to form a portfolio. Choosing securities based on their individual risk-return characteristics leads to creation of portfolio with risk and return highly deviated from what the investor is looking for. Thus, in order to get the utmost benefit from a collection of securities being held, it is very important to analyze them in terms of combination, rather than examining individually.

One of the most basic ways of creating a portfolio addressing the peculiar need of the investor was developed by Harry Markowitz (1952, 1959). Markowitz portfolio selection model attempts to maximize portfolio expected return for a given amount of portfolio risk or minimize risk for a given level of expected return, by sensibly choosing the assets (Kaplan, 1998). The theory models an asset's return-as mean, and the risk associated with the asset-as variance. By combining different assets, it seeks to reduce total variance of portfolio returns.

NEPSE index is at record high level beating the previous best mark of 1175 made in 2008. The latest political development, favorable monetary policy framework and the ample liquidity in market have lured new investors towards the security market. The prevailing context has made the scope of this research relevant and a necessity. Although investors attempt to create a portfolio yielding the desired return at the lowest possible risk, their effort often goes in vain due to the frequent fluctuations in the stock market. Not using an appropriate method for portfolio construction keeps these investors at a significant disadvantage (Hilsted, 2012).

Researchers in Nepal have made great strides over the past couple of years in their attempt to identify the best combination of securities (e.g., Paudel & Koirala, 2006), but the public has known only little. In addition, many of these researches have not been able to accommodate the conditions and constraints in Nepali Stock Market; their efforts to practically help the investors make their decisions have gone in vain. Thus, a fresh approach is required to dissect the investor's mood in Nepali Stock Market. There is need for a study to state the advantage of creating an optimal portfolio. This study seeks to fill the gap by suggesting a suitable portfolio combination for varying needs of investors using the Markowitz mean-variance analysis model. The findings of this research can be valuable to prospective institutional and individual investors in making investment decisions. Furthermore, the research can pave path for future research works on portfolios with inclusion of other investment possibilities – commodities, real estate, currency and derivatives.

The rest of the paper flows as follows. The next section reviews the prominent literature of portfolio theory. Data and methodology are discussed in section three. Section four explains the results and findings, and finally section five concludes the paper with some implications for individual and institutional investors, and the policymakers.

II. REVIEW OF LITERATURE

2.1 Historical Review of Portfolio Theory

The area of portfolio management was explored much before the dawn of Modern Portfolio Theory. The concept of diversification can be found from Shakespeare's Merchant of Venice to the modeling done by the English and Scottish investment trusts in nineteenth century (Markowitz, 1999). Williams (1938), through his Dividend discount model, stated that the goal of investors was to find good stock and buy it at best price. Wiesenberger's 1941 annual report shows that the Investment Companies held large number of security (Markowitz, 1999). But, the consideration of risk-return tradeoff on the portfolio as a whole started only after the influential paper of Harry Markowitz on Portfolio Selection at the start of second half of the twentieth century.

Markowitz (1952, 1959) described the Modern Portfolio Theory for the first time. The portfolio problem was formulated as a choice of mean, representing expected returns, and variance, representing risk associated of a portfolio of assets. The theorems on holding constant variance, maximizing the expected return and holding constant return, minimizing variance led to the formation of an efficient frontier, which is used by the investor, based on the risk preference, to make the choice of desired portfolio. The Markowitz mean-variance formulation paved way for exploration of new dimensions in Portfolio research. Tobin (1958) in his Separation Theorem added a risk-free asset to the consideration enabling leverage and deleverage portfolios on the efficient frontier. The concept of super-efficient portfolio and capital market line thus developed was able to outperform the portfolios on the efficient frontier. The work was largely related to development of return distribution of assets and utility functions of investors that results in mean-variance theory being optimal (Markowitz, 1999). The Capital Asset Pricing Model developed in the 1960s proved an important achievement in the field of finance. Sharpe (1964), Lintner (1965) and Mossin (1966) are considered to have developed a similar security returns model. The model talks about the compensation made by the market to the investors taking systematic risk but not the asset specific risks. Sharpe (1964) conceptualized Beta encouraging investors to hold the market portfolio and leverage it with a position in a risk free asset. The model proved useful for predicting the equilibrium price of the asset, but the several unreasonable assumptions lying underneath hindered its practicality.

The essence of all these authors and portfolio finance experts can be summarized as – understanding the characteristics of securities helps to optimize or maximize the expected returns centered on the stated level of risk (Lai & Xing, 2011). Kaplan (1998) concludes that the mean variance approach of determining efficient frontier is powerful and the developments since its inception have made the model more robust thus, capable for practical application in the field of investment.

2.2 Empirical Findings

Investing in the global minimum variance portfolio with no short sale position constructed using block structure for covariance matrix of asset returns has an ability to outperform the naive portfolio that invests equally across all the risky assets (Disatnik & Katz, 2012). However, DeMiguel, Garlappi & Uppal (2009) favor the simple equal weighting rule in their empirical study of these portfolios in terms of Sharpe ratio. This argument falls within the realm of current research thus, is incorporated in the study with tests done in the case of Nepali market.

Clarke, De Silva & Thorley (2011) had 120 securities in their minimum variance set out of 1000 securities they considered for their study. DeMiguel et al (2009) mention that their long only portfolio assigned weight different from zero to only few assets. Similar findings have not been presented in context of Nepal. Maillard, Roncalli & Teiletche (2010) offer a new dimension to the portfolio analysis. The research devices an equally weighted risk contribution (ERC) strategy, which has volatility, located between minimum variance portfolio and equally weighted portfolio. This is an attempt to address the less diversification problem of minimum variance portfolio and make it more efficient.

Roncalli (2010) analyzes the impact of weight constraints in portfolio theory following the work of Jagannathan & Ma (2003). The study uses Dow Jones EURO STOXX 50 to show that weight constraints are useful for investors and portfolio managers to substantially modify the covariance matrix. This approach is very useful to obtain more robust portfolio with small concentration. Ang, Hodrick & Zhang (2006) attempted to argue that the safer investments also come with higher risk potential. But, Bali & Cakici

(2008) conclude that these findings lack robustness when exposed to numerous markets. Several researchers have concluded that low volatility stocks have low returns but investment horizon could have an impact on that relationship (Amenc et al, 2011). Engles (2004) studied different portfolio optimization models in mathematical way. The research illustrates the ability of Tesler models, based on Value at Risk, to work with not just normally distributed returns, but with each distribution from the elliptical family. This presents an opportunity for portfolio managers to address the lower tail risk associated with the returns.

Various studies have implemented the newer mathematical models thus assisting in the enhancement of portfolio theory in recent time. Portfolio rebalancing model based on fuzzy decision theory accommodates the uncertainty associated with the return, risk and the liquidity of portfolio thus, is useful particularly in unstable financial environments (Fang, Lai & Wang, 2006). Given that the developing countries are subject to unstable economic conditions, this model could be suitable for finding the optimal portfolios to invest (Fabozzi, Gupta & Markowitz, 2002) especially in the stock market like Nepal.

Cesarone, Scozzari & Tardella (2009) attempted to resolve the problem associated with high variables in traditional mean-variance model. By utilizing some recent theoretical results on quadratic programming the algorithm devised by the authors is able to handle more than 2000 variables. The favorable results obtained from the test conducted in some major stock markets meant that the investors could use it for their decision-making. Konno & Yamazaki (1991) suggest the use of mean-absolute deviation risk function leading to linear program instead of quadratic program, which is complex to solve. This gives an opportunity for investors to simplify the portfolio calculations in multi asset scenario. Soleiman, Golmakani & Salimi (2009) and Golmakani & Mehrshad (2011) included real world features of financial market in their models. Inclusion of minimum transaction lots and sector capitalization constraints among other constraints they have formed mixed-integer non-linear programming model. Some parts of constraints from this model have been incorporated in the current study.

The distribution of returns has been subject of many studies. The preference of a rational investor is to maximize the returns, meaning the fondness in lying at the upside of a tail. Konno, Tanaka & Yamamoto (2011) proposed an algorithm for solving optimization problem for the construction of portfolio with shorter downside tail and longer upside tail. Hu & Kercheval (2010) propose methods for portfolio optimization of returns data with Student ‘t’ and skewed ‘t’ distributions. These works have made the portfolio managers able to capture heavy tails and skewness in the returns data.

The current study is based on the assumption of normal distribution of returns. The consideration of merely mean and variance of returns makes the model simplistic compared to the ones with additional moments, which could be better at describing the return distributions of a portfolio. Some researchers (Kraus & Litzenberger, 1976) have added more moments such as skewness in their portfolio theories. Fama (1965), Elton & Gruber (1974) and Konno et al (2011) among others have given more realistic real world returns distribution. But, the Mean-Variance theory continues to remain the base of Modern Portfolio Theory. Elton & Gruber (1997) offer two reasons for this: Mean-

Variance theory itself places large data requirements while there is no evidence that adding additional moments improves desirability of portfolios and, the wide known intuitive appeal of the implications of Mean-Variance theory.

2.3 The Context of Developing Market

Despite massive advances made in the portfolio theory their application has been tested only in few financial markets of the developing countries (Puelz, 2002; Konno & Yamazaki, 1991). Financial markets in developing countries have different characteristics with the securities behaving differently (Konno & Yamazaki, 1991) to those of the western countries. Thus, there exists a need to examine the applicability of portfolio optimization models in stock markets of developing countries.

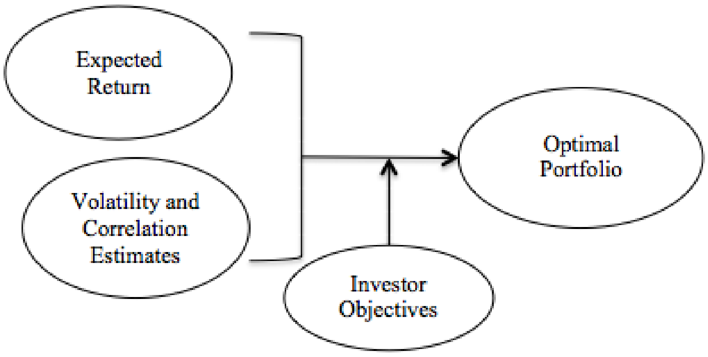
The developing markets face high uncertainty in calculating future expected returns due to their economic and political environment. In such markets historical performance might not seem to be a fair indicator of future performances of stock (Fabozzi et al 2002). Taghizadegan, Darvish & Bakhshayesh (2014) endeavored to overcome this through the use of fuzzy optimal model. The study was concentrated on stock mutual funds at Teheran Stock Exchange (TSE). This presents an ideal alternative for the investors and portfolio managers from the developing countries. Mokta (2013) attempted to reduce the uncertainty brought by inclusion of numerous input estimates through the use of single index models. The study conducted on Dhaka Stock Exchange (DSE) used 33 stocks, providing the optimal portfolio with 6.17% return at a risk of 8.76%.

Despite numerous studies on portfolio analysis having been conducted in Nepal, very few are robust enough to accommodate the changing market pattern. Paudel & Koirala (2006) is among the popular portfolio study conducted in the country. It covers relatively old study period (1997-2006) and considers only combination of two-stock portfolios. The optimal portfolios from this study largely consist of financial institution stocks. But, the inclusion of only two stock portfolios seems irrelevant in current securities market context. Thus, there exists a need to examine the applicability of portfolio optimization models in stock markets of developing countries. This study therefore aims to explore the relevance and applicability of modern portfolio optimization theory in the Nepal Stock Exchange.

2.4 Conceptual Framework

Portfolio construction and evaluation is done on the basis of risk return characteristics of individual stocks and the mutual relationships between them. Using the Markowitz mean-variance analysis model (1952, 1959), the expected stock return and volatility, and correlation estimates associated with it are used to represent return of stocks and co-movement of stocks with each other. These primary parameters are used in combination in order to determine the optimal portfolio. The optimal portfolio to be built depends on the objectives of the investor. Thus, the peculiar need of the investor acts as a moderating variable in the model.

Figure 1: Relationship of stock parameters and optimal portfolio



The investor specific and market related constraints obtained from the literature review of Soleiman, Golmakani & Salimi (2009) and Golmakani & Mehrshad (2011) have been incorporated in the study. The risk measures proposed by Konno et al (2011) have been excluded since Markowitz (1952, 1959) states that the first two moments namely, mean and covariance are adequate measures for a normally distributed return curve.

III. METHODOLOGY

This research aims to create a stock-only portfolio with diverse classes of stocks in it. In order to limit the number of stocks for investor's contemplation the stocks classified as belonging to Group "A" companies (Sensitive Index) by NEPSE are considered as population. The Group "A" classified companies are supposed to have minimum paid up capital of Rs. 20 million and at least 1000 shareholders. The firm should have been making profit for minimum of last three years, with the market price of share higher or equal to the book price. The firm should have filed the transaction and income statements within six months of completion of the fiscal year.

The stratified sampling method was used to come up with a sample of 20 stocks to include in our portfolio, imitating the portfolio size of a random investor. The Group "A" list contains 130 companies divided into 8 sub-categories namely, Commercial Banks, Development Banks, Finance Companies, Insurance Companies, Hydropower, Hotel, Production and Refinery, and Others. The latter four sub sectors have very less representation, totaling to 6, in the Sensitive Index list. Therefore, these four sectors have been reclassified into others category. Thus the final strata for consideration consists of Commercial Banks, Development Banks, Finance Companies, Insurance Companies and Others.

A total of 20 stocks have been randomly drawn from the five strata developed, with each stratum getting a proportional representation. Although the objective was to include these randomly drawn stocks from each stratum into the portfolio, it could not be possible due to the difficulty in obtaining data. It is extremely important to have the data belonging to the same period for all the companies in order to accurately measure their covariance and correlation. Thus, the stocks that were traded on or prior to April 2010 have been included. Out of these 20 stocks nine are of Development Banks, four each of Commercial Banks and Finance Companies, two from the Insurance companies and one from the Others stratum.

The monthly stock return, variance, covariance and correlations between them have been calculated for 57 months, from April 2010 to December 2014. The stocks of a couple of financial institutions were not traded for some period while they were in the process of merger. In such cases, the recent trading price has been continued until the scrips were reopened for trading. The test check of mean-variance model in NEPSE has been done at April 2015 stock prices. It is unsuitable to check the applicability for the stocks whose trading has been halted in the check period. Thus, only the stocks that were open for trading as of April 2015 have been included in the sample.

3.1 Markowitz Mean-Variance Model

The Modern Portfolio Theory rests on certain assumptions made about the stock returns.

i. Investment horizon is one year.

ii. The volatility associated with the return is measured by risk.

iii. The gain properties are based on expected return on portfolio.

   In order to characterize the Risk-Return properties of portfolios, some assumptions were made about the probability distribution of the security returns as well.

iv. $R_i \sim$ iid N (μ, σ_i^2), i=A, B…. , where R_i denote the simple return on security i

   The simple return on security was considered, as the portfolio calculations is based on weighted average of simple returns. Each simple return is independent and identically distributed random variable with mean μ and variance σ.

v. $Cov(R_A, R_B) = \sigma_{AB}$

vi. $Cor(R_A, R_B) = \rho_{AB}$

vii. Investors prefer high expected returns $E[R_i]= \mu_i$

viii. Investors loathe high variance $(Ri)= \sigma^2$

The case of two risky assets A and B, with simple returns denoted by R_A and R_B respectively, is considered for simplicity. The share of total wealth W_0 on asset A and B can be written as,

$$x_A = (\text{Rs. in A}) / W_0 \qquad \text{and} \qquad x_B = (\text{Rs. in B}) / W_0$$

Two types of portfolios were considered for investor's contemplation. The long-only portfolio, where the asset weights are always positive ($x_{A,}$ $x_B > 0$). The short-allowed portfolio has one of the asset weight negative ($x_A < 0$ or $x_B < 0$). These allocations held under the assumption that all wealth is allocated between these two assets:

$$x_A + x_B = 1$$

This means to form a portfolio with short sales allowed, one of the assets had to be sold short with the proceeds from the sell used to purchase more of the other stock.

Then, the return on portfolio was measured by the expected return,

$$\mu_p = E[R_p] = \mu_A x_A + \mu_B x_B$$

and the risk of the portfolio was measured by the variance.

$$\sigma_p^2 = Var(R_p) = x_A^2\, \sigma_A^2 + x_B^2\, \sigma_B^2 + 2x_A\, x_B \sigma_{AB}$$

As the asset returns were normally distributed, the portfolio return was also considered to be normally distributed.

$$R_p \sim \text{iid } N\, (\mu_p, \sigma_p^2)$$

This is the main focus of the mean variance model. If the asset returns distribution is normal then the probability distribution is completely characterized by the mean and variance.

Thus, the end of period wealth of the Investor can be written as,

$$W_1 = W_0\,(1+R_p) = W_0\,(\,1 + R_A x_A + R_B x_B)$$

and the distribution of end of period wealth,

$$W_1 \sim N\,(\,W_0\,(1+\mu_p)\,,\, \sigma_p^2 W_0^2)$$

Since the portfolio return was normal, the end of period wealth was also normally distributed with mean of $W_0\,(1+\mu_p)$ and volatility of $\sigma_p^2 W_0^2$.

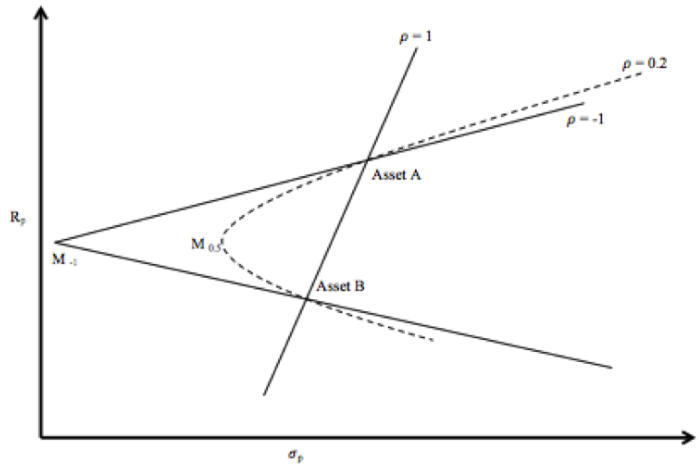
Portfolio Frontier

The risk return trade-off between portfolios is graphically illustrated through the two-dimensional plot of expected return and volatility known as the portfolio frontier. All possible portfolios i.e. the values of x_A and x_B are considered with mean and variance for each of them being plotted in the two dimensional space. The shape of the frontier largely depends on the correlation between two assets. If $\rho_{AB} = -1$, then there exists portfolio that has no risk, $\sigma_p^2 = 0$. If $\rho_{AB} = 1$, then the assets are perfectly correlated and there is no benefit from diversification in terms of risk reduction. The benefit of having negative correlation is quite evident but the diversification is beneficial even if assets are positively correlated ($0 < \rho_{AB} < 1$).

Efficient Portfolio

Efficient portfolios are the portfolios with highest expected return for a given level of risk as measured by the portfolio standard deviation. The tip of Markowitz bullet (points M-1 and M0.5 in Figure 2) separates the efficient and inefficient portfolios. This is particularly helpful as it narrows down the potential portfolios investors can invest into a subset.

Figure 2: The Polar Cases of Correlation



The shape of the frontier parabola depends on the correlation between the assets. It does depend on the expected return and standard deviation, but even by holding these two parameters constant and changing the correlation, the shape of the frontier can be changed. If the correlation is positive then risk-return tradeoff is completely linear. If the correlation is perfectly negative (ρ = -1, Figure 2) then it is a perfect hedge. The tip of frontier ρ = -1 namely, M-1 is the portfolio of A and B with least possible volatility.

The choice of efficient portfolio the investor holds will completely depend on the individual risk preference. A risk averse investor chooses portfolio towards the global minimum variance portfolio sacrificing some upside potential gain for safety of low volatility while the risk tolerant investor holds portfolio with high volatility compensated with higher expected gain. The best portfolio starts upward the curve from point M and the investor chooses the exact location in the curve based on risk preference.

Global Minimum Variance Portfolio

The edge of Markowitz bullet has the portfolio with smallest possible variance and is also known as global minimum variance portfolio. It has a nice intuitive appeal as safe investment and is chosen by the most risk-averse investors.

This was found by minimizing the variance equation subject to given constraint.

$$\min_{xA,\, xB} \sigma_p^2 = Var(R_p) = x_A^2\, \sigma_A^2 + x_B^2\, \sigma_B^2 + 2x_A\, x_B \sigma_{AB}$$

$$\text{subject to, } x_A + x_B = 1$$

Using substitution method with,

$$x_B = 1 - x_A$$

To get the univariate minimization,

$$\min_{xA} \sigma_p^2 = Var(R_p) = x_A^2\, \sigma_A^2 + (1- x_A)^2 \sigma_B^2 + 2x_A (1- x_A)\, \sigma_{AB}$$

The first order derivative,

$$\partial(\sigma_p^2)/ \partial x_A = \partial\, (x_A^2\, \sigma_A^2 + (1- x_A)^2 \sigma_B^2 + 2x_A (1- x_A)\, \sigma_{AB}) / \partial x_A$$

$$= 2\, x_A\, \sigma_A^2 - 2\, (1- x_A)\, \sigma_B^2 + 2\, \sigma_{AB}\, (1- 2\, x_A)$$

Setting the derivative to 0, in order to solve for $x_{A,}$

$$\min_{xA} = (\sigma_B^2 - \sigma_{AB}) / (\sigma_A^2 + \sigma_B^2 - 2\, \sigma_{AB})$$

$$\text{and, } \min_{xB} = 1- \min_{xA}$$

This gives the analytic solution for minimum variance portfolio. This shows that the minimum variance portfolio depended on variance of assets and covariance between them. In this study the above calculations for the multi assets case have been done using the solve.QP() in R under the quadprog package, which is generally used for quadratic optimization problems.

3.2 Real world restrictions

The previous section discusses the solution methods of the unrestricted mean-variance analysis. The Markowitz model in its earliest form assumes that investment is unrestricted (Mayanja, 2011). There is restriction of short sales of stocks in Nepal. This restriction has been incorporated in the study by adding the constraint $x_i \geq 0$.

IV. RESULTS

The simple returns on stocks were calculated because the portfolio analysis holds for simple returns rather than the continuously compounded returns. The mean of monthly returns was obtained for each of the twenty stocks. Since an assumption is made that the returns on stocks are normally distributed, the mean of monthly returns gives the expected return of the stocks. The correlations ranged from -0.23 (CBBL and CIT) to 0.76 (EBL and NABIL) for the data set. This showed that most of the stocks are highly correlated to each other. This is particularly common for the stocks belonging to the same sub-sector. The dependent variables were then fed into a model for mean-variance optimization developed in R program. The characteristics of global minimum variance portfolio, equally weighted portfolio, portfolio subject to target returns and the tangency portfolio were calculated through the use of this model. The test check of the model showed that it provides 71.86% of the actual return desired by the investor during the test period.

4.1 The Power of Portfolio

An investor with no portfolio knowledge and looking to invest in stock with low volatility would have chosen the EBL stock, which has one of the lowest standard deviations (0.4239) in the sample. But, a heavy investment in the stock with low individual variability does not necessarily make the portfolio the least risky of all possible portfolios. This has been shown from the fact that the minimum variance portfolio has a standard deviation of 0.2171, almost half that of the least risky stock. It is worth noting that the minimum variance portfolio has allocated zero weight to the EBL stock. This is due to the high positive correlation coefficient of EBL stock with majority of other stocks in the portfolio. The average correlation coefficient of EBL with remaining 19 stocks is 0.4392, underscoring its low portfolio risk reducing potential.

The CIT stock, which is different from the banking stocks that dominate the stocks on NEPSE, has the volatility of about 50%. Investors looking to invest in less risky asset would hesitate to invest in CIT stock. But, the global minimum variance portfolio has 21.93% of the investor wealth allocated to CIT stock. This is due to the low correlation coefficient of the CIT stock with other stocks in the sample. Its correlation coefficients are negative with majority of assets (as low as -0.23 with CBBL) and the average correlation with all the stocks is merely 0.0508, stating the risk reducing capability of the CIT stock.

The analysis of data shows that majority of stocks are highly correlated to each other, illustrating the difficulty in achieving a diversified portfolio from the stocks available for trading at NEPSE. This finding fits well with the study by Paudel (2002). Even the hydropower stocks and stocks of the insurance companies tend to move together with the banking institution stocks. The PLIC stock has average correlation of 0.5033 with the commercial banks taken for consideration in the study. This highlights the need for real sector companies in securities market of Nepal. The public trading of real sector firms could provide an investment alternative and diversification opportunity for Nepali investors. Apart from that, the existing business firms should be encouraged to trade publicly. Public flotation of stocks of these business firms not only helps the general investors get alternatives to invest but also helps these companies raise the funds easily from market, thus assisting in fulfilling their growth potential (Ferreira, Manso & Silva, 2012).

4.2 Global Minimum Variance Portfolio

Figure 3: Efficient frontier and global minimum variance portfolio

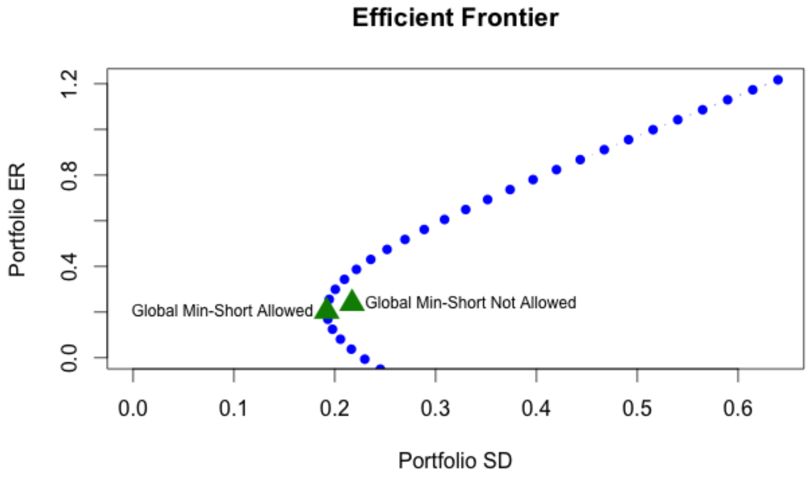


Table 1: Global Minimum Variance Portfolio

	Short Sales allowed	Short Sales not allowed
Portfolio Exp. Ret.	0.2036	0.2379
Portfolio Standard Dev.	0.1919	0.2171

The portfolio expected return with least possible standard deviation was calculated both without any constraints and with the constraints of not allowing for short sales. The least possible standard deviation for the portfolio was 19.19%. The figure rose by 2% when the stocks were not allowed to trade short (Refer to Appendix for weights). The corresponding expected returns were 20.36% and 23.79%. Jackson & Staunton (2001) state that addition of the constraints into the model is negatively associated with the returns yielded by the portfolio. This statement held true for NEPSE.

The number of securities with non-zero weight in a long-only global minimum variance portfolio is 10. The finding aligns with those of Clarke et al (2011) and DeMiguel et al (2009). This is due to the high concentration of low volatility stocks in the minimum variance portfolio. In practice, the minimum variance portfolio is used as a standard to measure the investment alternatives. Investors seldom make their decisions to invest in this portfolio. The objective of global minimum variance portfolio is to lower the risk rather than to optimize the risk to reward ratio. The choice of this portfolio leads to pronounced concentration in low volatility stocks with less correlation coefficient. However, this stock may be suitable for investors who opt for extremely defensive strategy.

The existence of risk close to 19.19% even at the minimum variance level means that Nepali stock market is subject to high level of fluctuations. The NEPSE index has closely followed the crests and troughs of political developments in the country, one of the major reasons for the high fluctuation. But, a 20.36% return even from the least risky portfolio underlines the high level of business potential in the country.

4.3 Equal Weighted Portfolio

Figure 4: Equally weighted portfolio

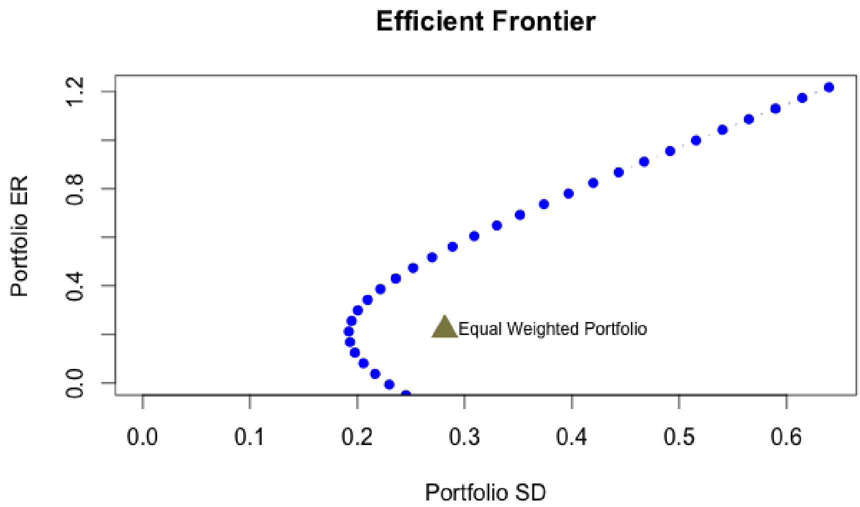


Table 2: Equally Weighted Portfolio

Each stock weight	0.5
Portfolio Exp. Ret.	0.2194
Portfolio Standard Dev.	0.2813

An alternative for the investors not willing to engage in processing of information is to invest equal fraction of their total wealth in all the stocks. Investing 0.05% of the wealth in the each of the 20 stocks in the sample yielded a return of 21.94% with the standard deviation of 28.13%. A possibility of earning significantly higher return with the same standard deviation is apparent from the Figure 4. Even the minimum variance portfolio, which emphasizes only on lowering the risk, earns more than this equal weight portfolio. Thus, investors are better off getting some information from the market and choosing an appropriate portfolio than investing equal fraction of their wealth in all the stocks. This fits well with research findings of Urban & Ormos (2012) and DeMigues et al (2009) but contradicts with the conclusions made by Plyakha (2012).

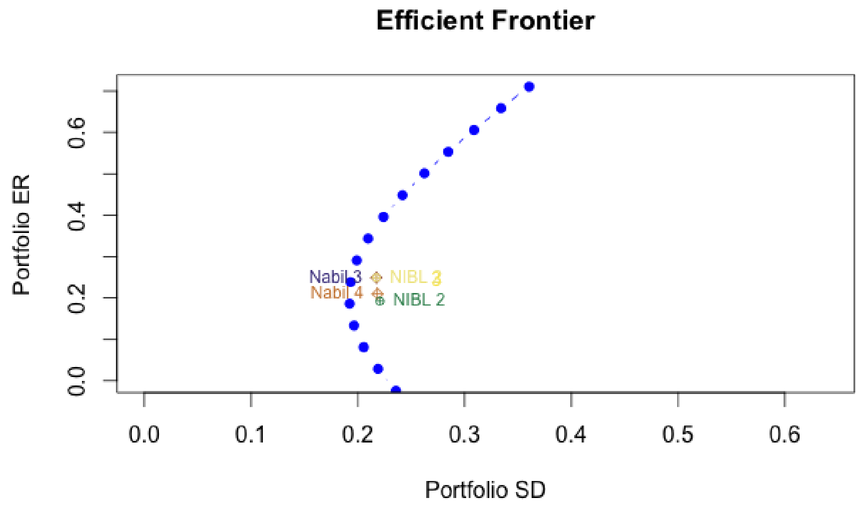
4.4 Efficient Portfolio Subject to Target Returns

As per the financial statements published by majority of mutual fund schemes, the highest chunk of investment goes into buying shares of group A companies. Thus, the mean-variance model created in this research could be ideal for these mutual funds to allocate the resources to ensure the attainment of proposed rate of return.

Nabil Investment Banking Limited launched its first mutual fund scheme named Nabil Balanced fund on March 2013. The scheme is worth 600 millions and has maturity of five years. The major objective of the scheme is to balance the risk of portfolio by investing in a mix of securities. The projected Return on Investment (ROI) is 18% and 21% for the third and fourth year. NIBL Capital launched the NIBL Samriddhi Fund – I with maturity of 7 years on July 2015. The scheme is worth 800 millions and aims at investing in mixed securities as well. The projected ROI is 19.23% and 24.92% for year 2 and year 3 respectively.

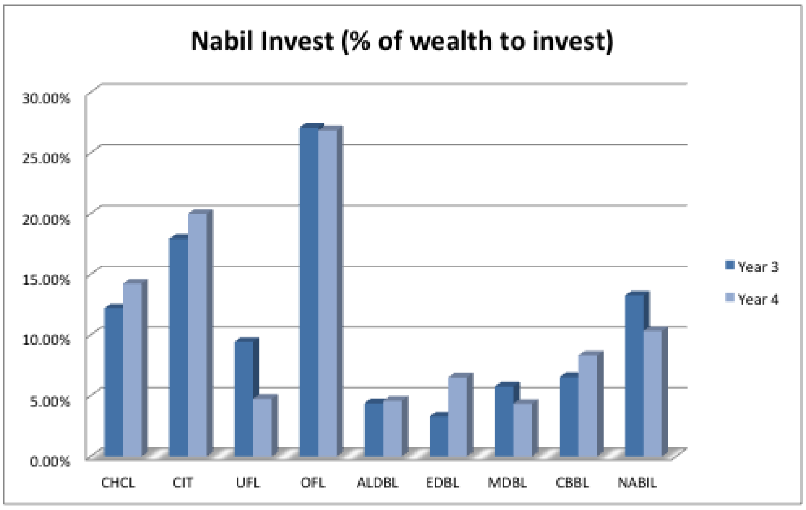
The figure 5 shows the plot of projected return and variation associated with the two mutual fund companies for the given years. These lie quite close to the minimum variance portfolio thus the investment needed to produce the stated level of return can be considered relatively less risky. This indicates the risk averse nature of mutual fund companies in the country. The other portfolios, preferably higher up the Markowitz bullet or the tangency portfolio, provide them a better return for the risk they take.

Figure 5: Plot of Returns for Mutual Fund Schemes



Nabil Invest

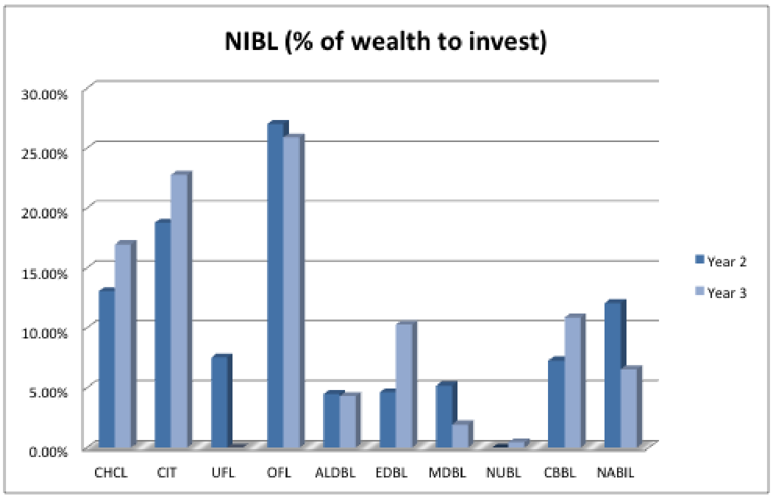
Figure 6: Nabil Invest Portfolio Weights



A large part of the return for Nabil Invest in both the third and the fourth year comes from OFL and CIT stocks. It is worth noting that the low level of risk associated with both these stocks provides stability to the return earned by the investment of Mutual Fund Company. This is partly due to the closeness of the returns provided by these mutual fund schemes to the minimum variance portfolio, and the risk reducing characteristics of these stocks. The weights for 11 other sample stocks were zero. There is need to change the weightage during the end of the year, in order to achieve the desired returns. The UFL and EDBL stocks, in particular, need quite a bit reallocation if the mutual fund is to provide the stated level of returns to the unit holders.

NIBL Capital

Figure 7: NIBL Capital Portfolio Weights



A large part of return for NIBL comes from OFL, CIT and CHCL stocks. The weights of 10 other stocks are zero for both the years. The UFL stock should be held only for the year 2 but not for the following year. A fraction of NUBL stock needs to be newly purchased in the year 3 in order to achieve the return on investment ambitions. The relative consistency in both the years for OFL stock can be attributed to its less volatility. The standard deviation of OFL is 40.62%, which is among the lowest for the gives sample of stocks. The relative consistent level of OFL stock in the portfolio adds stability to the returns.

From the unit holder's perspective, the analysis of minimum variance portfolio and the returns provided by the mutual funds together provide a point for consideration. The investment in mutual fund scheme does not seem to be more profitable than looking for an optimal portfolio to invest on one's own. This presents the value of investment related knowledge to the perspective investors. It is very important for the investors to consider the stock investment opportunities present in the market and analyze the risk-return tradeoff, than locking into a modest return proved by a mutual fund.

4.5 Tangency Portfolio

Figure 8: Tangency Portfolio at r_f =0.0061

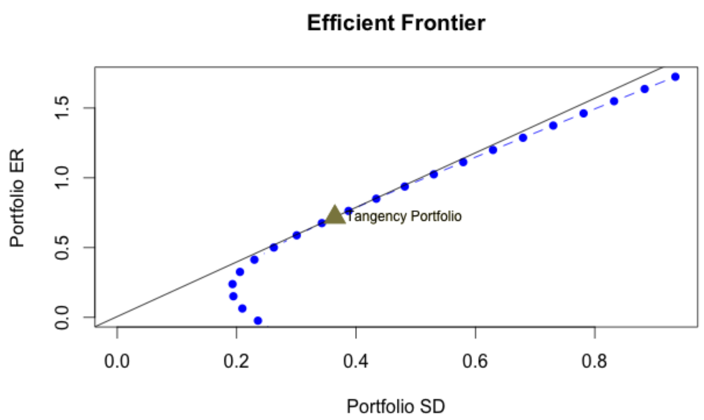


Table 3: Tangency Portfolio at Risk Free Rate 0.61%

Portfolio Exp. Ret.	0.7185
Portfolio Standard Dev.	0.3645

The investors have an option to invest in large universe of risky assets bounded by the Markowitz bullet. In an economy with lending and borrowing possible at the risk free rate, an investor can choose to combine the risk free asset with any portfolio on the frontier. The best portfolio to hold in combination with the risk free rate is the portfolio yielding highest expected return per unit increase in risk i.e. the one with highest Sharpe slope, the tangency portfolio. This is the portfolio any investment analyst would suggest to the client. The choice of allocating the wealth would depend on the risk preference of the client. Risk averse investors tend to place themselves within the origin and the tangency portfolio point thus, lending at risk free rate. One the other hand, risk seeking investors aim to reach a point further away in the line than the tangency portfolio, borrowing at the risk free rate and investing the proceeds at the tangency portfolio (Cochrane, 1999; Mayanja, 2011).

The 364-day Treasury bill rate as of August 2015 is 0.61%. The institutional investors can use this rate as the risk free rate of investment for them. The weights to be allocated for the tangency portfolio obtained from the model are given in table 4.

Table 4: Tangency Portfolio Weights

CHCL	NLIC	PLIC	CIT	KAFIL	UFL	OFL	ACEDBL	ALDBL	EDBL
0.495	0.348	-0.068	0.329	-0.071	-0.672	0.215	-0.010	0.221	0.350
MDBL	**NLBBL**	**NUBL**	**CBBL**	**SUBBL**	**MDB**	**EBL**	**CZBIL**	**NABIL**	**ADBL**
-0.267	-0.223	0.057	0.288	-0.184	0.259	-0.073	-0.075	-0.145	0.227

A risk averse investor would lie exactly on the y-intercept in figure 8, thus yielding a total portfolio return equal to the return provided by the treasury bill. If the investor holds equal of both the treasury bills and the tangency portfolio, a return of 36.23% would be obtained with a reduction in the portfolio volatility. A risky investor would borrow money at the rate of Treasury bill and invest proceeds in the tangency portfolio, thus earning return higher than 71.85%. But, there is a sharp increase in the risk in such scenario.

4.6 The Applicability Test of Model

The model uses historical performance to obtain estimates of the future characteristics of the stocks. Fabozzi et al (2002) stated that the performance of developing markets could be different from the expected performance thus, raising question over the applicability of the mean variance theory in developing markets like Nepal. Thus, a test check was necessary to validate the model in case of Nepali stock market.

In order to check the applicability of the model in NEPSE, a test was carried out at the expected return of 30%. The portfolio weights obtained at the 30% level of returns are given in table 5.

Table 5: Test Check Weights

CHCL	NLIC	PLIC	CIT	KAFIL	UFL	OFL	ACEDBL	ALDBL	EDBL
0.169	0.021	0.032	0.262	0.000	0.000	0.202	0.000	0.016	0.010
MDBL	**NLBBL**	**NUBL**	**CBBL**	**SUBBL**	**MDB**	**EBL**	**CZBIL**	**NABIL**	**ADBL**
0.00	0.000	0.016	0.143	0.000	0.016	0.000	0.000	0.0171	0.000

A hypothetical wealth of Rs. 100,000 was used to purchase these stocks on December, 2015 market price as per the weights assigned by the model. The weights were divided by the then share price to come up with the number of each shares to be held. This number of shares was then multiplied with the market price of the share on April 23, 2014 in order to come up with the portfolio value.

The portfolio value on April 23, 2014 was Rs. 106,070.68, which amounted to a 110-day return of 6.06%. This figure was then annualized using,

$$R_A = (1 + 0.0606)^{3.3181} - 1$$

Giving, the annualized return of 21.56%.

The test shows that the mean-variance model was quite close in yielding the desired level of return for the investor in NEPSE.

The closeness of suggested weight allocation can be calculated using,

$$100\% - ((30 - 21.56) / 30 * 100\%)$$

giving, the closeness figure of 71.86%.

Thus, the model provided 71.86% of the return desired by the investor. This can be further increased with the increase in accuracy of the parameters being estimated for the model. Inclusion of real market forces in the model would help the model provide a more accurate result, as sought by the investor.

V. CONCULSION

This study found that the Markowitz mean-variance method of portfolio analysis is useful in construction of optimal portfolio from stocks traded at NEPSE. The model provided 71.86% of the desired rate of return by the investors, and has potential to be further increased with the use of better estimation of return by considering more of the real world factors. Furthermore, the inclusion of CIT stock in majority of the peculiar-risk-return portfolios illustrates the importance of active portfolio construction compared to a mere collection of individual assets. The CIT stock despite having asset specific risk on the higher side was able to reduce the risk associated with the portfolio due to its negative correlation coefficient with most of the other stocks present in the portfolio.

The research found a significant lacking of such negatively (or low) correlation stock in the Nepal stock exchange. Majority of banking stocks, understandably move together with each other. But the data analysis showed that insurance and hydropower stocks move in the same trend as the banking stocks do. This has relative disadvantage for the investors and calls for expansion of the Nepali stock market to encourage the privately owned companies to enter the market.

The bringing of privately owned real sector companies into the public trading realm should create a win-win situation for all the stakeholders. The government's current treating of the companies who have gone public on par with the companies who have not has discouraged the latter to issue public shares, and forced them to remain content with their current business volume. These privately owned companies should be provided some tangible benefits in return for their public disclosure of financial position thus, encouraging their converting to a public company. The existing mechanism of companies having to float Initial Public Offering only after three years of registering profit and at a face value of Rs. 100 discourages a profit-making firm. A flexible provision on allowing the companies to offer their share to public at a premium rate, after an independent valuation, will definitely encourage their entering the public realm. A mandatory requirement can be made for companies above a scientifically determined threshold level of capital to list a certain fraction of their shares for public trading.

The minimum variance portfolio provides return close to 20% and has a similar risk as well. This sums up the current investment scenario of the country. The high political and economic risk surrounding the country has translated into significant high minimum level of risk for the investments. But, the once willing to take on the risk get the reward through a relatively high level of return. The return provided by the mutual fund is comparable with the minimum variance portfolio return. This largely means that a knowledgeable investor able to analyze the market situation should explore the opportunities present in the stock market than locking the investment in mutual fund schemes. These schemes seem well suited for elderly and retired people with less ability or willingness to regularly analyze the investment situations in the market. Tangency portfolio is the ideal portfolio to consider for every investor and is the one recommended by portfolio manager to all the investors. There is an immense return of 71.85% for holding the tangency portfolio. Though the risk associated is quite high, it is largely compensated by the high return potential, thus the portfolio with highest Sharpe slope.

This research can be a base for additional studies to come. The ability of the model to come up with asset weights giving the stated rate of return as expected by the investor seemingly decays over time as the input data and the expected return period get further and further apart, the characteristic of autoregressive process. This relation could be a subject of future study through the use of Autoregressive model. The study of portfolio construction could be widened by consideration of all the scrips available for trading in NEPSE. With the development of commodities market in Nepal, various commodities, and financial and non-financial derivatives could be added into the consideration for construction of an optimal portfolio.

Investors in developing markets like that of Nepal should actively seek more information rather than following the herd. Investing without an education and research leads to lamentable investment decision. Research is much more than just listening to the herd. Phillip Fisher once rightly stated, “The stock market is filled with individuals who know the price of everything, but the value of nothing.” We would want to do better.

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Appendices

A. Stocks Used in the Sample & their Abbreviations

Commercial Banks (4)
Agriculture Development Bank (ADBL)
Everest Bank Limited (EBL)
Citizens Bank Limited (CZBIL)
Nabil Bank (NABIL)

Finance Companies (4)
Kaski Finance Company (KAFIL)
United Finance Company (UFL)
Om Finance Limited (OFL)
Citizen Investment Trust (CIT)

Insurance Companies (2)
Prime Life Insurance Company (PLIC)
Nepal Life Insurance Company (NLIC)

Development Banks (9)
Ace Development Bank (ACEDBL)
Alpine Development Bank (ALDBL)
Excel Development Bank (EDBL)
Malika Bikas Bank (MDBL)
Nerude Laghubitta Bikas Bank (NLBBL)
Nirdhan Utthan Bank (NUBL)
Subechha Bikas Bank (SUBBL)
Miteri Development Bank (MDB)
Chhimek Laghu Bitta Bikas Bank (CBBL)

Others (1)
Chilime Hydropower Company (CHCL)

B. Portfolio Characteristics

Table 6: Expected Annual Return

CHCL	NLIC	PLIC	CIT	KAFIL	UFL	OFL	ACEDBL	ALDBL	EDBL
0.227	0.440	0.417	0.442	0.101	-0.040	0.060	0.116	0.132	0.293
MDBL	**NLBBL**	**NUBL**	**CBBL**	**SUBBL**	**MDB**	**EBL**	**CZBIL**	**NABIL**	**ADBL**
0.041	0.304	0.389	0.457	-0.002	0.272	0.182	0.172	0.094	0.286

Table 7: Covariance Matrix

	CHCL	NLIC	PLIC	CIT	KAFIL	UFL	OFL	ACEDBL	ALDBL	EDBL	MDBL	NLBBL	NUBL	CBBL	SUBBL	MDB	ADBL	EBL	CZBIL	NABIL
CHCL	0.187																			
NLIC	0.050	0.258																		
PLIC	0.127	0.094	0.232																	
CIT	-0.005	0.076	-0.005	0.243																
KAFIL	0.079	0.094	0.106	-0.035	0.262															
UFL	0.029	0.082	0.020	0.009	0.053	0.178														
OFL	-0.013	0.049	0.024	-0.007	0.063	0.074	0.165													
ACEDBL	0.124	0.051	0.123	-0.023	0.139	0.068	0.007	0.227												
ALDBL	0.042	0.028	0.054	-0.019	0.132	0.091	0.061	0.103	0.172											
EDBL	0.030	0.081	0.062	0.040	0.057	0.097	0.000	0.103	0.064	0.208										
MDBL	0.082	0.066	0.044	-0.016	0.113	0.097	0.040	0.120	0.082	0.087	0.173									
NLBBL	0.099	0.066	0.110	-0.006	0.126	0.038	0.060	0.084	0.092	0.037	0.035	0.304								
NUBL	0.014	0.074	0.073	0.040	0.049	0.094	0.039	0.081	0.097	0.102	0.050	0.067	0.227							
CBBL	0.053	-0.017	0.072	-0.069	0.073	0.023	0.034	0.070	0.086	0.030	0.035	0.166	0.083	0.351						
SUBBL	0.077	0.072	0.042	0.015	0.100	0.105	0.050	0.103	0.097	0.088	0.109	0.050	0.042	0.043	0.277					
MDB	0.034	0.065	0.026	-0.043	0.092	0.156	0.100	0.082	0.118	0.079	0.134	0.091	0.103	0.097	0.124	0.394				
ADBL	0.094	0.095	0.121	0.005	0.099	0.081	0.034	0.081	0.078	0.082	0.100	0.090	0.067	0.042	0.079	0.100	0.194			
EBL	0.132	0.089	0.103	0.010	0.111	0.062	0.003	0.118	0.059	0.065	0.104	0.073	0.048	0.027	0.075	0.073	0.138	0.184		
CZBIL	0.134	0.124	0.111	0.007	0.143	0.110	0.069	0.164	0.097	0.091	0.133	0.101	0.052	0.066	0.127	0.124	0.132	0.166	0.309	
NABIL	0.117	0.078	0.119	0.013	0.113	0.019	-0.011	0.127	0.036	0.062	0.071	0.093	0.020	0.021	0.061	0.010	0.112	0.152	0.154	0.215

Table 7: Correlation Matrix

	CHCL	NLIC	PLIC	CIT	KAFIL	UFL	OFL	ACEDBL	ALDBL	EDBL	MDBL	NLBBL	NUBL	CBBL	SUBBL	MDB	ADBL	EBL	CZBIL	NABIL
CHCL	1.000																			
NLIC	0.225	1.000																		
PLIC	0.609	0.385	1.000																	
CIT	-0.022	0.305	-0.021	1.000																
KAFIL	0.359	0.361	0.429	-0.140	1.000															
UFL	0.158	0.381	0.097	0.042	0.248	1.000														
OFL	-0.077	0.238	0.122	-0.036	0.304	0.430	1.000													
ACEDBL	0.603	0.211	0.538	-0.099	0.571	0.340	0.036	1.000												
ALDBL	0.232	0.131	0.268	-0.094	0.624	0.520	0.359	0.523	1.000											
EDBL	0.150	0.351	0.281	0.177	0.243	0.505	0.001	0.473	0.339	1.000										
MDBL	0.454	0.311	0.218	-0.076	0.529	0.552	0.235	0.604	0.475	0.461	1.000									
NLBBL	0.413	0.235	0.415	-0.022	0.446	0.163	0.267	0.319	0.402	0.146	0.154	1.000								
NUBL	0.070	0.307	0.319	0.170	0.201	0.468	0.203	0.357	0.489	0.471	0.253	0.257	1.000							
CBBL	0.205	-0.056	0.254	-0.237	0.239	0.092	0.142	0.246	0.349	0.110	0.144	0.508	0.293	1.000						
SUBBL	0.337	0.268	0.164	0.058	0.370	0.475	0.235	0.412	0.444	0.365	0.498	0.171	0.166	0.137	1.000					
MDB	0.125	0.203	0.085	-0.140	0.287	0.589	0.394	0.275	0.455	0.276	0.515	0.263	0.346	0.260	0.376	1.000				
ADBL	0.494	0.425	0.570	0.022	0.438	0.434	0.191	0.386	0.425	0.407	0.546	0.369	0.319	0.159	0.339	0.360	1.000			
EBL	0.713	0.408	0.497	0.046	0.504	0.343	0.018	0.575	0.333	0.331	0.582	0.306	0.235	0.105	0.333	0.271	0.728	1.000		
CZBIL	0.558	0.440	0.415	0.026	0.501	0.469	0.304	0.621	0.420	0.358	0.574	0.330	0.198	0.199	0.433	0.356	0.539	0.694	1.000	
NABIL	0.582	0.330	0.532	0.057	0.476	0.096	-0.057	0.573	0.189	0.291	0.368	0.362	0.092	0.076	0.248	0.034	0.545	0.763	0.599	1.000

Table 8: Global Minimum Variance Portfolio

8.1: With Short selling allowed

Portfolio expected return: 0.203

Portfolio standard deviation: 0.191

Portfolio weights:

CHCL	NLIC	PLIC	CIT	KAFIL	UFL	OFL	ACEDBL	ALDBL	EDBL
0.324	0.055	-0.079	0.182	-0.012	0.043	0.345	0.006	0.134	0.132
MDBL	**NLBBL**	**NUBL**	**CBBL**	**SUBBL**	**MDB**	**EBL**	**CZBIL**	**NABIL**	**ADBL**
0.009	-0.131	-0.038	0.142	-0.060	0.0182	-0.030	-0.245	0.236	-0.022

8.2: Not allowing for short sales

Portfolio expected return: 0.237

Portfolio standard deviation: 0.217

Portfolio weights:

CHCL	NLIC	PLIC	CIT	KAFIL	UFL	OFL	ACEDBL	ALDBL	EDBL
0.165	0.000	0.000	0.219	0.000	0.003	0.266	0.000	0.047	0.095
MDBL	**NLBBL**	**NUBL**	**CBBL**	**SUBBL**	**MDB**	**EBL**	**CZBIL**	**NABIL**	**ADBL**
0.030	0.000	0.000	0.099	0.000	0.000	0.000	0.000	0.076	0.000

Table 9: Portfolio Weights subject to Target Returns

Nabil Invest – Year 3

CHCL	NLIC	PLIC	CIT	KAFIL	UFL	OFL	ACEDBL	ALDBL	EDBL
0.122	0.000	0.000	0.179	0.000	0.094	0.271	0.000	0.044	0.033
MDBL	**NLBBL**	**NUBL**	**CBBL**	**SUBBL**	**MDB**	**EBL**	**CZBIL**	**NABIL**	**ADBL**
0.057	0.000	0.000	0.065	0.000	0.000	0.000	0.000	0.132	0.000

Nabil Invest – Year 4

CHCL	NLIC	PLIC	CIT	KAFIL	UFL	OFL	ACEDBL	ALDBL	EDBL
0.142	0.000	0.000	0.200	0.000	0.047	0.268	0.000	0.046	0.065
MDBL	**NLBBL**	**NUBL**	**CBBL**	**SUBBL**	**MDB**	**EBL**	**CZBIL**	**NABIL**	**ADBL**
0.043	0.000	0.000	0.083	0.000	0.000	0.000	0.000	0.103	0.000

NIBL – Year 2

CHCL	NLIC	PLIC	CIT	KAFIL	UFL	OFL	ACEDBL	ALDBL	EDBL
0.130	0.000	0.000	0.187	0.000	0.075	0.270	0.000	0.044	0.046
MDBL	**NLBBL**	**NUBL**	**CBBL**	**SUBBL**	**MDB**	**EBL**	**CZBIL**	**NABIL**	**ADBL**
0.051	0.000	0.000	0.072	0.000	0.000	0.000	0.000	0.120	0.000

NIBL – Year 3

CHCL	NLIC	PLIC	CIT	KAFIL	UFL	OFL	ACEDBL	ALDBL	EDBL
0.169	0.000	0.000	0.227	0.000	0.075	0.258	0.000	0.042	0.102
MDBL	**NLBBL**	**NUBL**	**CBBL**	**SUBBL**	**MDB**	**EBL**	**CZBIL**	**NABIL**	**ADBL**
0.019	0.000	0.004	0.108	0.000	0.000	0.000	0.000	0.065	0.000