The Nepali Mathematical Sciences Report
Regd No.: 53/033-034, ISSN: 2392–411X
http://www.journal.cdmathtu.edu.np
Year: 2024, Volume: 41, No. 1
Pages: 43–53
https://doi.org/10.3126/nmsr.v41i1.67458
Research Article

On The Determination of a Convex Sequence of Signals by Absolute Sum of Factors of Trigonometric Series

Suresh Kumar Sahani[*1], S.K.Tiwari[2], B. Sonat[3], and Madhav Prasad Poudel[4]

[*1]Department of Science and Technology, Rajarshi Janak University, Janakpurdham, Nepal
sureshsahani@rju.edu.np
[2]Department of Mathematics, Dr. C.V.Raman University, C.G., India
sk10tiwari@gmail.com
[3]Department of Mathematics, Dr. C.V.Raman University, C.G., India
bindeshwarikurrey04@gmail.com
[4]School of Engineering, Pokhara University, Pokhara-30, Kaski, Nepal
pdmadav@gmail.com

Received: 22 February, 2024 Accepted: 10 May, 2024 Published Online: 30 June, 2024

Abstract

Five theorems on the identification of a convex sequence of signals via the absolute sum of elements of trigonometric series are established in this study. Several well-known results are specific cases of these theorems. When the function has bounded variation, it also addresses several special circumstances of fuzzy numbers.

Keywords: Absolute sum, Convex sequence, Trigonometric series, Bounded variation.
AMS(MOS) Subject Classification: 40D15, 40F05,40G99,42A24.

1 Introduction

Let $\xi(z)$ be a function that is Lebesgue integrable over $(-\pi, \pi)$, and has a period of 2π, then

$$\xi(z) = \frac{\alpha_0}{2} + \sum_{g=1}^{\infty}(\alpha_g cosgz + \beta_g singz)$$
$$= \sum_{g=0}^{\infty} V_g(z) \tag{1.1}$$

*Corresponding author

Defination:

We consider an infinite series $\sum_{g=0}^{\infty}\alpha_g u^g$ which is convergent in $(0 \leq u < 1)$, where

$$l(u) = \sum_{g=0}^{\infty}\alpha_g u^g \tag{1.2}$$

If the Abel limit $\lim_{x\to 1-0} l(u)$ exists finitely, then the infinite series $\sum_{g=0}^{\infty}\alpha_g$ is known as summable by Abel method.

Example 1.1. The divergent series $\sum_{g=1}^{\infty}(-1)^{g-1}g$ has the Abel sum $\frac{1}{4}$.

Proof. Since

$$\begin{aligned}\xi(z) &= \sum_{g=1}^{\infty}(-1)^{g-1}.gz^g\\ &= z\sum_{g=1}^{\infty}(-1)^{g}.g.z^{g-1}\\ &= \frac{d}{dz}\left(\frac{z}{1+z}\right) = \frac{z^2}{(1+z)^2}\end{aligned}$$

put $z = 1$ then we obtain the sum equals $\frac{1}{4}$. □

Example 1.2. The Abel's sum $\sum_{g=0}^{\infty}(-1)^g = 1 - 1 + 1 - 1 + ... = \frac{1}{2}$.

Proof.

$$\sum_{g=1}^{\infty}\frac{(-1)^{g-1}}{g} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + ...$$

Since it is alternating series. So it is convergent.
We can apply Abel's theorem to the function

$$\begin{aligned}\xi(z) &= \sum_{g=1}^{\infty}\frac{(-1)^{g-1}.z^g}{g}\\ \implies \xi'(z) &= \sum_{g=0}^{\infty}(-1)^g.z^g\\ \implies \xi'(z) &= \frac{1}{z+1}\\ \therefore \xi(z) &= log(1+z)\end{aligned}$$

Put $z = 0$, then $\xi(0) = log(1 + 0) = 0$ and $\xi(1) = log2$ □

Defination: If equation (1.2) is of bounded variation in $(0, 1)$ then the Abel limit will necessarily exists then the infinite series $\sum_{g=0}^{\infty}\alpha_g$ is known as absolutely summable (A) and is denoted by $\mid A \mid$.

Known Results:

Numerous studies have been written about the absolute summability factors of infinite series and Fourier series (refer to [1]–[5], [8–10], [12–14], [16–21]).

We consider a function $\chi(z)$ which is defined as following way.

$$\chi(z) = \frac{\xi(v+z)\xi(v-z) - 1 - \xi(v)}{2} \tag{1.3}$$

Among them authors [6], [15], [11] and [7] respectively proved the following theorems.

Theorem 1.3. *If*

$$\int_0^z \mid \chi(\beta) \mid d\beta = o(z) \tag{1.4}$$

as $z \to 0$ *then the infinite series* $\sum_{g=1}^{\infty} \frac{U(v)}{logg}$ *is convergent.*

Theorem 1.4. *If*

$$\int_0^z \mid \chi(\beta) \mid d\beta = o(z) \tag{1.5}$$

as $z \to 0$ *holds betterly, then* $\int_z^{\pi} \frac{\chi(\beta)|}{\beta} d\beta = o(log\frac{1}{z})$*, as* $z \to 0$*.*

Theorem 1.5. *If* $\int_z^{\pi} \frac{|\chi(\beta)|}{\beta} d\beta = o(log\frac{1}{z})$ *holds betterly, then*

$$\int_0^z \mid \chi(\beta) \mid d\beta = o(zlog\frac{1}{z}) = o(-zlogz) \tag{1.6}$$

as $z \to 0$.

Theorems 1.4 and 1.5 are proved by author [15].

Theorem 1.6. *If If* $\int_z^{\pi} \frac{|\chi(\beta)|}{\beta} d\beta = o(-logz)$ *as* $z \to 0$*, holds, then the infinite series* $\sum_{g=1}^{\infty} \frac{U_g(v)}{logz}$ *is convergent.*

Theorem 1.7. *If If* γ_g *is one of the following sequences*

$$\frac{1}{(logg)^{1+m}}, \frac{1}{(logg)(log_2 g)^{1+m}}, \frac{1}{(logg)(log_2 g)(log_3 g)^{1+m}}, \cdots, (m > 0) \tag{1.7}$$

and if $\int_0^z \mid \chi(\beta)\beta = o(z)$ *as* $z \to 0$*, then the infinite series* $\sum \gamma_g U(v)$ *is summable* $\mid A \mid$

In this research note, we generalize the results of [6], [15], [11] and [7] by using (or proving) the following theorem.

2 Main Results

Theorem 2.1. *If If γ_g is one of the following sequences*

$$\frac{1}{(logg)^{1+m}}, \frac{1}{(logg)(log_2 g)^{1+m}}, \frac{1}{(logg)(log_2 g)(log_3 g)^{1+m}}, \cdots, (m > 0)$$

where m is a positive number and if

$$\int_z^{\pi} \frac{\mid \chi(\beta) \mid}{\beta} d\beta = o(-logz) \tag{2.1}$$

as $z \to 0$,then the series $\sum \gamma_g U(v)$ is absolutely summable.

Our theorem requires several lemmas for proof.

Lemma 2.2. *If $\epsilon \in (0,1)$ and $x = sin^{-1}\left[\frac{1-\epsilon}{2(1+\epsilon^2)}\right]$ and $N(u) = \frac{1-u^2}{1+u^2-2u(cos2z)}$, then*

$$\int_0^{\epsilon} \mid N'(u) \mid du = f(x) = \begin{cases} o(x^{-1}), & z \in [0,x] \\ o(\frac{1}{2}), & z \in [x, \frac{\pi}{4}] \\ o(1), & z \in [\frac{\pi}{4}, \pi] \end{cases}$$

Lemma 2.3. *If $L(U) = \frac{\alpha_0}{2} + \sum_{g=1}^{\infty} U_g(v)u^g$ and $\int_z^{\pi} \frac{|\chi(\beta)|}{\beta} d\beta = o(-logz)$ as $z \to 0$, satisties then*

$$\int_0^{\epsilon} \mid L'(u) \mid du = O\left(log\frac{1}{1-\epsilon}\right) \tag{2.2}$$

where $\epsilon \in (0,1)$.

Proof. By hypothesis

$$\begin{aligned}
\int_0^{\epsilon} \mid L'(u) \mid du &\leq \frac{2}{\pi}\int_0^{\pi} \mid \chi(z) \mid \int_0^{\epsilon} \mid N'(u) \mid dudz \\
&\leq \frac{2}{\pi}\int_0^{x} \mid \chi(z) \mid \int_0^{\epsilon} \mid N'(u) \mid dudz + \int_x^{\frac{\pi}{4}} \mid \chi(z) \mid \int_0^{\epsilon} \mid N'(u) \mid dudz \\
&+ \int_{\frac{\pi}{4}}^{\pi} \mid \chi(z) \mid \int_0^{\epsilon} \mid N'(u) \mid dudz \\
&= O\left(\frac{1}{x}\right)\int_0^{x} \mid \chi(z) \mid dz + O(1)\int_x^{\frac{\pi}{4}} \frac{\mid \chi(z) \mid}{z} dz + O(1)\int_{\frac{\pi}{4}}^{\pi} \mid \chi(z) \mid dz \\
&= O\left(\frac{1}{x}\right)\left[z.log\frac{1}{z}\right]_0^x + O\left(\frac{1}{x}\right)\left[log\frac{1}{z}\right]_x^{\frac{\pi}{4}} + O(1)\left[log\frac{1}{z}\right]_{\frac{\pi}{4}}^{\pi} \\
&= O(log\frac{1}{1-\epsilon})
\end{aligned}$$

□

Confirmation of the theorem: Let $\gamma_g = \frac{1}{(logg)^{1+m}}$, m is a positive number.
Let

$$R(u) = \sum_{g=2}^{\infty} U_g(v)u^g \tag{2.3}$$

By using lemma 2.2,

$$\int_0^{\epsilon} \mid R'(u) \mid du = O\left(log\frac{1}{1-\epsilon}\right) \tag{2.4}$$

where $\epsilon \in (0,1)$. Then as $u \to 0$, we have

$$\int_0^{\epsilon} \mid R'(u) \mid du = O(\epsilon^3) \tag{2.5}$$

and subsequently, we obtain

$$\int_0^{\epsilon} \mid R'(u) \mid du = O(1)\left(log\frac{1}{1-\epsilon} - \epsilon - \epsilon^2\right) \tag{2.6}$$

For taking $m > 0$, then

$$\begin{aligned} R_\delta(u) &= \sum_{g=2}^{\infty} \frac{1}{(logg)^{m+1}} U_g(v)u^g \\ &= \frac{1}{\Gamma(1+m)} \int_0^{\infty} \frac{y^m}{\Gamma(y)} \int_0^1 R(u\beta)(log\frac{1}{\beta})^{y-1}\frac{1}{\beta}d\beta dy \end{aligned}$$

Then the total variation of $R_m(u)$ in (0,1) is

$$\begin{aligned} \int_0^{\epsilon} \mid R_\delta'(u) \mid du &= \int_0^{\epsilon} \mid \frac{1}{\Gamma(1+m)} \int_0^{\infty} \frac{y^m}{\Gamma(y)} \int_0^1 R'(u\beta)(log\beta)^{y-1} d\beta \mid du \\ &= \frac{1}{\Gamma(1+m)} \int_0^{\infty} \frac{y^m}{\Gamma(y)} dy \int_0^1 (-log\beta)^{y-1}\frac{d\beta}{\beta}\left(\int_0^{\epsilon} \beta \mid R'(u\beta) \mid du\right) \end{aligned} \tag{2.7}$$

Using (2.6), then we have

$$\begin{aligned} \int_0^1 (-log\beta)^{y-1}\frac{d\beta}{\beta}\int_0^{\epsilon} \mid R'(u\beta) \mid .\beta du &= \int_0^1 (-log\beta)^{y-1}\frac{d\beta}{\beta}\int_0^{\epsilon\beta} \mid R'(u\beta) \mid .\beta du \\ &= O(1)\int_0^1 (-log\beta)^{y-1}\frac{1}{\beta}\left(log\frac{1}{1-\epsilon} - \epsilon\beta - \frac{\epsilon^2\beta^2}{2}\right)d\beta \\ &= O(1)\int_0^1 \sum_{g=2}^{\infty}\frac{1}{g}\epsilon^g\beta^{g-1}(-log\beta)^{y-1}d\beta \\ &= O(1)\sum_{g=2}^{\infty}\frac{1}{g}\epsilon^g\Gamma(y)g^{-y} \end{aligned}$$

again using (2.7), we may obtain

$$\begin{aligned}\int_0^{\epsilon} \mid R'_m(U) \mid du &= O(1)\frac{1}{\Gamma(1+m)}\int_0^{\infty}\frac{y^m}{\Gamma(y)}\sum_{g=2}^{\infty}\frac{1}{g^{1+y}}\epsilon^g\Gamma(y)dy \\ &= O(1)\frac{1}{\Gamma(1+m)}\sum_{g=2}^{\infty}\frac{1}{g}\epsilon^g\int_0^{\infty}y^m u^{-ylogg}dy \\ &= O(1)\frac{1}{\Gamma(1+m)}\sum_{g=2}^{\infty}\frac{1}{g}\epsilon^g\frac{\Gamma(1+m)}{(logg)^{1+m}} \\ &= O(1)\sum_{g=2}^{\infty}\frac{1}{g(logg)^{1+m}} \\ &= O(1) \qquad (2.8)\end{aligned}$$

where m is a positive number. Hence $\sum_{g=2}^{\infty}U_g(v)\frac{1}{(logg)^{m+1}}$ is absolutely summable. The proof runs parallel if we consider any value. The sequences γ_g given in known theorem (1.7), with the same line of derivation. Then for $m > 0$,

$$\sum_{g=2}^{\infty}U_g(v)u^g\frac{1}{(logg)(logg)(log_2g)(log_2g)(log_vg)^{m+1}}$$

$$= \frac{1}{\Gamma(1+m)}\int_0^{\infty}du_v\frac{U_v{}^m}{\Gamma(u_v+1})\int_0^{\infty}du_{v-1}\frac{(U_{v-1})^{u_v}}{\Gamma(u_{v-1}+1)}\int_0^{\infty}...\int_0^{\infty}du_1\frac{U_1{}^{u_2}}{\Gamma(y)}\int_0^{\infty}R_q(u\beta)(-log\beta)^{y-1}\frac{1}{\beta}d\beta$$

This concludes the demonstration of the validity of the theorem.

An applications:

The coefficients of the Fourier transform are defined as follows:

$$\alpha_0 = \frac{1}{\pi}\int_{-\pi}^{\pi}\xi(z)dz,$$
$$\alpha_g = \frac{1}{\pi}\int_{-\pi}^{\pi}\xi(z)cosgzdz$$

and

$$\beta_g = \frac{1}{\pi}\int_{-\pi}^{\pi}\xi(z)sin\alpha zdz$$

Let a_g denotes the g^{th} partial sum of infinite series $\sum a_g$, ${A_g}^P$ and ${b_g}^P$ denote the g^P Cesàro mean of order q where q is non-negative number of the sequences $< a >_g$ and $< ga >_g$ respectively. The series $\sum a_g$ is absolute Cesàro summable if

$$\sum_{g=2}^{\infty} \mid {A_g}^q - {A_{g-1}}^q \mid < \infty \qquad (2.9)$$

Put

$$b_g{}^q = \frac{1}{U_g{}^q}\sum_{c=0}^{q} U_{g-(c+1)}{}^{q-1}(c+1)\alpha_{1+c} \tag{2.10}$$

$$b_g{}^q = g(A_g{}^q - A_{g-1}{}^q) \tag{2.11}$$

where

$$U_g{}^q = \frac{\Gamma(g+q+1)}{\Gamma(g+1)\Gamma(q+1)} \sim \frac{g^q}{\Gamma(q+1)} = O(g^q) \tag{2.12}$$

$$\Delta^0 C_n = C_n, \Delta C_n = \Delta' C_n = C_n - C_{n+1},$$

$$\Delta^r C_n = \sum_{d=0}^{\infty} U_d{}^{-r-1} C_{d+g}$$

provided this series is convergent.

Theorem 2.4. *If If $\xi(\beta) \in BV(0, \pi)$, and using the equations (1.3), (1.4), (2.9), (2.10), (2.11) and (2.12) then the infinite series*

$$\sum_{g=0}^{\infty} \frac{U_{g+2}(z)}{[log(2+g)]^{1+\delta}}, (\delta > 0) \tag{2.13}$$

is summable $| C, q |, (q > 1)$. (see [1.7]).

Theorem 2.5. *If If $\xi(\beta) \in BV(0, \pi)$ and using the equations (1.3), (1.4), (2.9), (2.10), (2.11) and (2.12) we get an equation (1.5).*

Theorem 2.6. *If If $\xi(\beta) \in BV(0, \pi)$ and using the equations (1.3), (1.4), (2.9), (2.10), (2.11), (2.12) and $\int_z^\pi \frac{|\xi(\beta)|}{\beta} d\beta = o(-logz)$ we get an equation (1.6).*

Theorem 2.7. *If If $\xi(\beta) \in BV(0, \pi)$, and using the equations (1.3), (1.4), (2.9), (2.10), (2.11) and (2.12) then the infinite series*

$$\sum_{g=0}^{\infty} \frac{U_{g+2}(z)}{[log(2+g)]^{1+\delta}}, \tag{2.14}$$

is summable $| C_1, q |, (q > 1)$. (see [2.13]).

3 Conclusion

Summability theory, which began in 19^{th} century is a part of analysts[the branch of mathematics dealing with limits and related theories]. It generalises the concept of convergence ones. It attempts to create an algorithm that analyses a limit to non convergent sequences, the theory makes a non convergent series, in a general sense. whereas a sequence of positive linear operators does not ordinary convergent(see [32-43]).

Funding

No funding is obtained for this research paper

Authors' contributions

All authors have equally contributed to this work.

Conflict of interest

The authors declare that they have no conflicts of interest regarding the publication of this paper.

References

[1] H. Bor; Absolute summability factors for infinite series, Indian J. Pure Appl. Math., 19 (7), 1988, 664-671.

[2] H. Bor; Some new results on absolute Riesz summability of infinite series and Fourier series, Positivity, 20 (3), 2016, 599-605.

[3] H. Bor; A new note on factored infinite series and trigonometric Fourier series, C. R. Math. Acad. Sci. Paris, 359 (2021).

[4] H. Bor; Factored infinite series and Fourier series involving almost increasing sequences, Bull. Sci. Math., 169 (2021), Paper no. 102990, 8 pp.

[5] K. K. Chen; Functions of bounded variation and Cesàro mean of a Fourier series, Acad. Sinica Science Record, 1, 1945, 283-289.

[6] G. H. Hardy; Divergent series, Oxford University Press, Oxford, 1949.

[7] B.N. Prasad; On the summability of Fourier series and bounded variation of Power series, Proc. London Math. Soc., 2, 35, 1933, 407-424.

[8] A. Karakaş; On absolute matrix summability factors of infinite series, J. Class. Anal., 13 (2), 2018, 133-139.

[9] B. Kartal; New results for almost increasing sequences, Ann. Univ. Paedagod Crac. Stud. Math., 18, 2019, 85-91

[10] B. Kartal; Generalized absolute Riesz summability of infinite series and Fourier series, Inter. J. Anal, Appl. 18 (6), 2020, 957-964.

[11] R. Mahapatra; A note on Nörlund summability factors; J. India Math. Soc., 31, 1967, 213-224.

[12] H. S. Özarslan; On almost increasing sequences and its applications, Int. Math. Math. Soc., 25 (5), 2001, 293-298.

[13] H. S. Özarslan; On the local properties of factored Fourier series, Proc. Jangieon Math. Soc., 9 (2), 2006, 103-108.

[14] H. S. Özarslan; Local properties of factored Fourier series, Int. J. Comp. Appl. Math., 1 (1), 2006, 93-96.

[15] M. L. Mishra; On the determination Jump of a function by its Fourier coefficients, Quart. J. Math. Oxford, Ser. 18, 1947, 147-156.

[16] H. S. Özarslan A. Karakas; A new result on the almost increasing sequences, J. Comput. Anal. Appl., 22 (6), 2017, 989-998.

[17] S. M. Mazhar; A note on absolute summability factors, Bull. Inst. Math. Acad. Sinica, 25, 1997, 233-242.

[18] W. T. Sulaiman; Inclusion theorems for absolute matrix summability methods of an infinite series (IV), Indian J. Pure Appl. Math. 34 (11), 2003, 1547-1557,.

[19] H. S. Özarslan H.N. Öğduk; Generalizations of two theorems on absolute summability methods, Aust. J. Math. Anal, Appl. 1, 2004, Article 13, 7 pages.

[20] H. S. Özarslan A. Karakaș; On Generalized Absolute Matrix Summability of Infinite Series, Communications in Mathematics and Applications; 10, No. 3, 2019, 439-446.

[21] Ș. Yildiz; A new extension on absolute matrix summability factors of infinite series, AIP conference Proceedings 2183, 050016, 2019.

[22] P. I. Sharma S. C. Kori; Absolute summability factors of Fourier series, Proc. Camb. Phil. Soc., 68 (8), 1970, 61-65.

[23] Ș. Yildiz; On application of matrix summability to Fourier series, Math. Methods Appl. Sci., 41, no. 2, 2018, 664-670.

[24] Ș. Yildiz; On the absolute matrix summability factors of Fourier series, Math. Notes 103, no. 2, 2018, 297-303.

[25] Ș. Yildiz; On the generalization of some factors theorem for infinite series and Fourier series, Filomat, 33, no. 14, 2019, 4343-4351.

[26] Ş. Yildiz; Matrix application of power increasing sequence to infinite series and Fourier series, Ukr, Math. J. 72, no. 5, 2020, 730-740.

[27] Ş. Yildiz; A variation on absolute weighted mean summability factors of Fourier series and its conjugate series, Bol. Soc. Parana. Mat. 38, no. 5, 2020, 105-113.

[28] H. Bor, D. Yu., P. Zhou; On local property of absolute summability of factored Fourier series, Filomat, 28, no. 8, 2014, 1675-1686.

[29] T. M. Flett; On an extension of absolute summability some theorem of Littlewood and Paley, Proc. London Math. Soc., 7, 1957, 113-141.

[30] Y. Okuyama; On the absolute Nörlund summability factors of Fourier series; Bull. Austral. Math. Soc., Vol. 12 (1975), 9-21.

[31] O. P. Varshney; On the absolute harmonic summability of a series related to a Fourier series, Proc. Amer. Math. Soc., 10, 1959, 784-789.

[32] S.K Sahani,et al. Some Problems on Approximations of Functions (signals) in Matrix Summability of Legendre series, Nepal Journal of Mathematical Sciences, Vol. 2(1), 2021, 43-50.

[33] S.K. Sahani, et al. On the Degree of Approximations of a Function by Norund Means of its Fourier Laguerre Series, Nepal Journal of Mathematical Sciences, Vol. 1, 2020, 65-70.

[34] S.K. Sahani L.N. Mishra, Degree of Approximation of Signals by Norlund Summability of Derived Fourier Series, The Nepali Math. Sc. Report, Vol. 38., No. 2, 2021, 13-19.

[35] S.K. Sahani, et al. On a New Application of Positive and Decreasing Sequences to Double Fourier Series Associated with (N, p_m^1, p_n^2), Journal of Nepal Mathematical Society, Vol.5(2), 2022, 58-64.

[36] S.K. Sahani, et al. On Certain Series to Series Transformation and Analytic Continuation by Matrix Method, Nepal Journal of Mathematical Sciences, Vol. 3, 1, 2022, 75-80.

[37] S.K Sahani V.N. Mishra, Degree of Approximation of Function by Norlund Summability of Double Fourier Series, Mathematical Sciences and Applications E-Notes,Vol.11, 2, 2023,80-88.

[38] S.K. Sahani, D. Jha. A Certain Studies on Degree of Approximation of functions by Matrix Transformation, The Mathematics Education, Vol. LV,2, 2021, 21-33.

[39] S.K. Sahani, K.S. Prasad, On a New Application of Almost Non-increasing Sequence to Ultra-spherical Series Associated with (N, p, q_k) Means, Applied Science Periodical Vol.XXIV, 1, 2022, 1-11.

[40] S.K. Sahani, et al. On Norlund Summability of Double Fourier Series, Open Journal of Mathematical Sciences, Vol.6, 1, 2022, 99-107.

[41] J.K. Pokharel, N.P. Pahari, S.K. Sahani, Critical Analyzing on Some Application of Almost Decreasing Sequence to Legendre Series Associated with [B] Sum, Advances in Nonlinear Variational Inequalities, Vol.26, No. 2, 2023, 36-40.

[42] S.K. Sahani, et al. On a New Application of Almost Increasing Sequence to Laguerre Series Associated with Strong Summability of Ultra-spherical Series, Nepal Journal of Mathematical Sciences, Vol. 4, No. 2, 2023, 77-82.

[43] S.K. Sahani, et al. An estimate of Rate of Convergence for the Absolute summability of factors of Infinite series, Power System Technology, Vol. 47, No. 4, December, 2023, 359-370.