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PRIORITIZED MAX-FLOWLOC FOR EVACUATION PLANNING

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Abstract: From different disasters such as earthquakes, flooding, landslides, hurricanes, blizzards, etc., thousands of people have lost their lives and millions of people have to be rescued each year. To reduce the losses from different disasters, appropriate emergency plannings are essential. The allocation of facility in an appropriate location of prioritized network are evenly used in such plannings. The prioritized maximum flow problem is to obtain the maximum flow in a network by taking a predetermined leaving pattern at the sources and/or reaching pattern at the sinks. But placing the facility on an arc, which is a very important task to supply the basic needs of the evacuees, reduces the arc capacity so that the maximum flow value in the network may be reduced. Here, we introduce the prioritized maximum static and dynamic flow location (FlowLoc) problems in a single source and multi-sink network. The mathematical models are given for both single and multi facility problems. The polynomial time algorithms are presented in single facility cases. After proving the $\mathcal{NP}$-completeness, the polynomial-time heuristics are presented for optimal or near optimal solutions in case of multi-facility cases.

Key Words: Evacuation planning, multi-terminal network, FlowLoc, prioritized flow
2010 Mathematics Subject Classification. *Primary: 90B10, 90C27, 68Q25; Secondary: 90B06, 90B20.*

1. Introduction

Motivation. A disaster is a sudden and severe event or series of events that result in significant damage, destruction, loss of life, and disruption of normal functioning within a community or society. Due to different disasters, thousands of people have lost their lives, and millions of people have to be rescued every year. Evacuation planning involves developing strategies and procedures to safely and efficiently move of people from potentially dangerous or hazardous regions to a safer place during a disaster. It is a proactive measure aimed at protecting lives and minimizing injuries. In evacuation network, the disastrous places and safe places are considered as sources and sinks, respectively. Several mathematical models are commonly utilized in evacuation plans, among them:

(1) Maximum flow problem: The maximum flow model aims to determine the optimal flow of evacuees through the transportation network from the disaster affected areas to the safe destinations. This

We are very shocked to share untimely demise of Prof. Dr. Urmila Pyakurel who was passed away at the early age of 42 on April 12, 2023. She was a very energetic young professor and role model Nepalese women with outstanding career in mathematics. This untimely demise of Urmila is really a huge loss to the scientific community. On this deep sadness, we express heartily tribute to her.

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model helps identify the most efficient allocation of resources, such as transportation vehicles and routes, to maximize the evacuees towards the safe place.

(2) Prioritized maximum flow problem: The model of the prioritized maximum flow problem aims to maximize the flow in a network while considering a predetermined leaving pattern at the sources and/or reaching pattern at the sinks. By assigning priorities to the arcs, conflicts in the flow distribution can be minimized, resulting in a smoother flow throughout the network. This approach ensures that the flow is optimized based on the predefined patterns, enhancing the efficiency of evacuation planning.

(3) Maximum flow location (FlowLoc) problems: The model of the FlowLoc problems focus on identifying the optimal location of resources, such as food stall, security camp, medical stores,etc., within the transportation network. This problem addresses the trade-off between facility placement and the minimum cut capacity of the network, aiming to achieve an optimal flow while ensuring necessary facilities are strategically positioned.

Now, we show the significance of prioritized maximum flow and FlowLoc problems. For this suppose that an emergency network with disastrous place s and two safe places d_1 and d_2 are shown in Figure 1, where the number allocating on each arc be the capacity of the arc. Let the priority order of the safe places be $d_1 \succ d_2$ (i.e. d_1 has more priority than d_2). Due to the priority ordering on the sinks, we first maximize the flow towards d_1 and then towards d_2. Thus first of all, we push 3 units of flow towards d_1 and then 2 units of flow towards d_2. The prioritized solution is given in Figure 2.

Again, assume a facility of size 2 has to be fixed on any one of the locations $\{(s, v), (v, d_2)\}$. After fixing the facility on the different locations, the maximum flow value is obtained as:

$$\text{Prioritized max FlowLoc} = \begin{cases} 3 \text{ (3 towards } d_1 \text{ \& 0 towards } d_2) & \text{if we fix the facility on } (s, v) \\ 4 \text{ (3 towards } d_1 \text{ \& 1 towards } d_2) & \text{if we fix the facility on } (v, d_2) \end{cases}$$

From the above result, if we fix the facility on (v, d_2), the flow value is maximum. So the location (v, d_2) is appropriate. The FlowLoc solution is given in Figure 3. Therefore, an integrated strategy that addresses the prioritized maximum flow and FlowLoc in a multi-terminal network is essential to study.

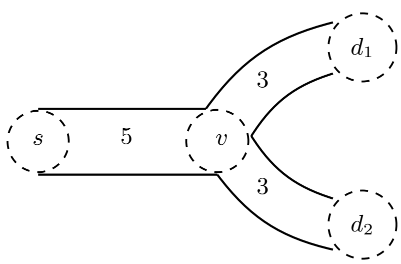


FIGURE 1. Given network with priority order $d_1 \succ d_2$: d_1 before d_2

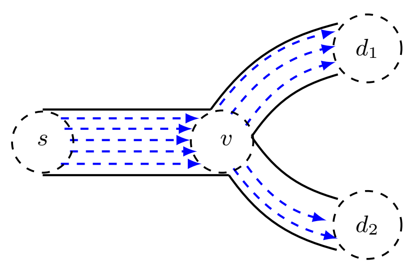


FIGURE 2. Prioritized solution

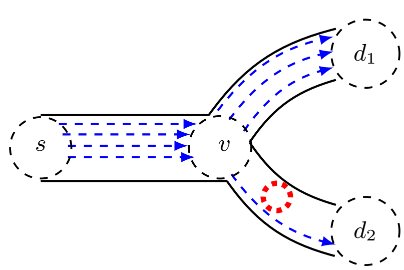


FIGURE 3. FlowLoc solution

Literature Review. The maximum flow problem is to determine the largest possible flow that can be pushed from the source to the sink within a given time period, Ford and Fulkerson [8]. The maximum flow in a network is equivalent to the capacity of the minimum cut. The earliest arrival flow (EAF) problem maximizes the flow value out from the source in each time steps, Gale [9]. The EAF problem in a series parallel network was solved polynomially by Ruzika et al. [23]. The contraflow problem deals with the maximization of flow from the source to the sink by reversing the direction of arcs towards the sink. The earliest arrival contraflow problem in a two terminal series parallel graph was introduced and solved in [20]. The abstract contraflow problem was introduced and solved polynomially by Dhamala et al. [4].

The objective of prioritized maximum flow (PMF) problem is to obtain the maximum flow value by assuming a predetermined pushing pattern at the source and/or the sending pattern to the sinks. Mineka[16] solved the prioritized maximum static flow (PMSF) problem in polynomial time. By using the time expanded graph, the prioritized maximum dynamic flow (PMDF) problem was solved in [15, 16]. Using prioritized minimum cost flows, the building evacuation problem was solved in [11]. By using the chain decomposale flows in the original graph, Hoppe and Tardos [12, 13] solved the PMDF problem. The different network flow models with their application in evacuation planning are found in the survey papers [1, 3]. Takizawa et al. [24] introduced an emergency evacuation model, by using the concept of prioritized quickest flow. Hrydziushka et al. [14] introduced an extended prioritized dynamic flow model for the multi-commodity aid distribution problem and tested for the Hagibis typhoon disaster in Japan (2019). The quickest multi-commodity partial contraflow problem was introduced and solved by presenting a fully polynomial-time approximation scheme in [2]. The maximum static and dynamic multi-commodity flow and earliest arrival multi-commodity flow problems were introduced and solved in [21].

Using the prioritized concept of [12, 13, 16], Pyakurel and Dempe [18, 19], introduced maximum static and dynamic flow problems with excess flow storage and presented polynomial time algorithms to solve them. In an abstract network, the prioritized maximum static and dynamic flow problems with excess flow storage are solved in polynomial time by Pyakurel et al. [22].

The flow location (FlowLoc) problem aims to determine the precise positions of facilities and optimizing the flow value. When a facility is located on a location, it reduces the capacity of the corresponding arcs in the network. Consequently, the maximum flow value that can be achieved, may be decreased. Weber [25] presented a model to determine the optimal location and minimal cost for the manufacturing plants of industries. Hamacher et al.[10] combined the network flows and locational analysis and introduced the FlowLoc problems for single as well as multiple facility cases. They solved the single flow location (1-FlowLoc) problem in polynomial time and presented polynomial time heuristics for the multi flow location (q-FlowLoc) problem. Nath et al. [17] introduced the quickest FlowLoc problem, which focuses on minimizing the increase in the quickest transshipment time caused by reducing arc capacities due to the placement of a facility on a location. They developed a strongly polynomial time algorithm for the single facility case and heuristic algorithm for multi facility case. The maximum ContraFlowLoc problem maximizes the flow towards the sink from the source by reversing the arcs. Dhungana and Dhamala [6] introduced maximum static and dynamic Contra1-FlowLoc problems and solved polynimially. Recently, Dhamala et al. [5] have introduced the maximum static and dynamic FlowLoc problem with excess storage that maximizes the flow form the source to the sink plus the excess flow towards the prioritized intermediate nodes by locating the given facilities at appropriate locations. They present polynomial time algorithms for single facility case and polynomial time heuristics for multi facility case.

Research Gap. To maximize the flow out from the disastrous place (source) during the evacuation time, the mathematical models for the maximum FlowLoc and the prioritized maximum flow problems can be found in literature. However, there is a significant research gap in combining these two problems into a unified approach. To fill this gap, we propose four new problems: prioritized maximum static 1-FlowLoc (PMS1FL), prioritized maximum dynamic 1-FlowLoc (PMD1FL), prioritized maximum static q-FlowLoc (MSqFL) and prioritized maximum dynamic q-FlowLoc (MDqFL).

Our Contribution. The prioritized maximum static and dynamic FLowLoc problems for the single as well as multiple facility cases are introduced for the first time in this paper. We find the appropriate locations for the given facilities, fix them there and compute the prioritized maximum flow in the reduced network. The polynomial time algorithms are presented for the single facility cases. For multi-facility cases, we present the polynomial time heuristics to approximate the solution.

Organization of the Paper. The basic terminologies used throughout the paper are provided in Section 2. In Section 3, we introduce the prioritized maximum static 1-FlowLoc (PMS1FL) problem, provide its mathematical formulation and solve it in polynomial time. The results of Section 3 are extended for dynamic FlowLoc problem in Section 4. In Section 5, we provide a mathematical formulation of the prioritized maximum static q-FlowLoc (PMSqFL) problem. Realizing its $\mathcal{NP}$-completeness, we present a polynomial time heuristic for the solution. In Section 6, the prioritized maximum dynamic q-FlowLoc (MDqFL) problem and its mathematical formulation is introduced and presented polynomial time heuristic to find the optimal or near-optimal solution. The paper is concluded in Section 7.

2. Preliminaries

Assume that, $G = (V, A)$ be a network, where V and A represent the sets of nodes with $|V| = n$ and arcs with $|A| = m$, respectively. Let $L \subseteq A$ be the set of all feasible locations for given facilities, s be the source, D be the set of sinks and $b_a : A \to \mathbb{Z}^+$ be the arc capacity function. In the case of dynamic flow, the transit time function $\tau : A \to \mathbb{Z}^+$ measures the time to transship the flow from $tail(a)$ to $head(a)$ for $a \in A$. For any node $v \in V$, $B(v)$ and $A(v)$ be the sets of incoming and outgoing arcs, respectively, i.e., $B(v) = \{(u, v) : u \in V\}$ and $A(v) = \{(v, u) : u \in V\}$. Now, the network is denoted as $G = (V, A, L, b_a, \tau, s, D, \mathbf{T})$, where the predefined time horizon $\mathbf{T}$ be the permissible time window by which the flow has to be completed. In discrete time steps, $\mathbf{T} = \{0, 1, \ldots, T\}$. If we discard the the time parameter, the network becomes static. So, $G = (V, A, L, b_a, s, D)$ be a static network.

Suppose that P be the set of all given facilities, the functions $b : P \to \mathbb{N}$ and $n_p : L \to \mathbb{N}$ measure size of facility and the number of facilities that can be placed on each $a \in L$, respectively. The FlowLoc problem concerns with the allocation $Lc : P \to L$ of the given facilities on arcs such that the source-sink flow value is maximized in the reduced network $G^R = (V, A, b_a^R, \tau, s, D, \mathbf{T})$, where $b_a^R = b_a - \max\{b_p : Lc(p) = a\}$. If we place more than one facility on a location $a \in L$, then only the arc capacity of a is reduced by the size of the largest facility.

3. Static 1-FlowLoc

In this section, we introduced the prioritized maximum static 1-FlowLoc (PMS1FL) Problem and present a polynomial time algorithms for the solution procedure. In two terminal network Hamacher et al. [10] introduced MS1FL problem and solved polynomially.

Problem 1. *Let G be a given static network with a given facility p of size b_p. The PMS1FL problem maximizes the amount of flow from the source s to the prioritized sinks D by locating p optimally.*

Mathematical formulation. Suppose that $G^R = (V, A, L, b_a^R, s, D)$ be a reduced network which is obtained from the given network G by fixing the given facility p on an arc with static flow $x : A \to \mathbb{R}^+$. Then the mathematical formulation of the PMS1FL problem is given by

$$\max \sum_{a \in A(s)} x_a = \sum_{a \in B(D)} x_a \tag{3.1a}$$

$$\text{subject to} \sum_{a \in B(v)} x_a - \sum_{a \in A(v)} x_a = 0, \ \forall v \in V \setminus \{s, D\} \tag{3.1b}$$

$$0 \leq x_a \leq b_a^R, \ \forall a \in A. \tag{3.1c}$$

Objective (3.1a) maximizes the amount of flow. Equations (3.1b) are the flow conservation constraints. Constraints (3.1c) bound the flow value on each arc by b_a^R, where $b_a^R = \begin{cases} b_a - b_p & \text{if we fix the facility } p \text{ on } a \\ b_a & \text{otherwise.} \end{cases}$

To solve Problem 1, we first obtain the shortest distance with arc counts of each sink node from s. We give the first priority to the farthest sink from s and so on because in the disastrous period, the farther sink is comparatively safer than the nearer sink from the source. Suppose that the priority order of sinks be $d_1 \succ d_2 \succ \cdots \succ d_\alpha$. Now, we place the facility p on one of the given locations $a_1 \in L$, then the network is reduced to $G^R(a_1) = (V, A, b_a^R, s, D)$. Let $D_1 = \{d_1\}$, $D_2 = \{d_1, d_2\}$, $\ldots$, $D_\alpha = \{d_k, k = 1, 2, \ldots, \alpha\} = D$, then there exists an order $D_1 \subset D_2 \subset \cdots \subset D_\alpha = D$. By Minieka [16], if the maximum flow value entering the the sinks in D is $x_F(D)$, then $x_F(D_i)$ represents the maximum flow value to every $D_i, i = 1, 2, \ldots, \alpha$, which is a prioritized maximum static flow on sinks. The facility fixed on another $a_2 \in L$ by removing it from a_1 and computed the PMSF in the reduced network $G^R(a_2) = (V, A, b_a^R, s, D)$. The iteration is continued. Finally, we take the maximum flow values from the over all flow values. Here we present Algorithm 1.

Algorithm 1: Prioritized MS1FL

Input : Given network $G = (V, A, L, b_a, s, D)$ with location L, facility p with size b_p.
Output: Prioritized MS1FL.

(1) Compute the shortest distance to each $d \in D$, by using Dijkstra's algorithm [7].
(2) Give first priority to the sink $d \in D$ with the longest distance from s and so on.
(3) Fix p on one of the locations $a_1 \in L$, the reduced network is $G^R = (V, A, b_a^R, s, D)$.
(4) Compute the prioritized maximum static flow in G^R
(5) Repeat Steps (3-4) with location p on another $a_2 \in L$, and so on.
(6) Pickup the maximum flow value among these maximum flow values.

Algorithm 1 iterates each $a \in L$ in order to obtain the maximum flow with the optimal location. To improve the running time, Algorithm 2 is presented. At first, after prioritizing the sinks, the PMSF is computed in G without fixing the facility on any location $a \in L$. We obtain the residual capacity of every $a \in L$ and check whether there exists a location $a \in L$ with the residual capacity greater or equal the size of p. If we find such a location then, p is placed there and obtain the PMSF which is already obtained. Otherwise, Algorithm 1 is applied. Although, the worst case time complexity of the both algorithm are same, but in general, for large sized network, there may be fewer such arcs whose residual capacity is enough to host the given facility. As a result, the number of calculations to obtain the maximum flow is reduced.

Algorithm 2: Improved prioritized MS1FL

Input : Given network $G = (V, A, L, b_a, s, D)$ with location L, facility p with size b_p.
Output: Prioritized MS1FL.

(1) Compute Steps (1)-(2) of Algorithm 1 to prioritize the sinks.
(2) Obtain the prioritized maximum static flow in the network G.
(3) Obtain residual arc capacity, $b_a^r = b_a - x_a$, $\forall a \in L$.
(4) If $\exists\ a \in L$, $b_a^r \geq b_p$, place the facility p on the location a, the reduced network is $G^R = (V, A, b_a^R, s, D)$.
(5) Compute the PMSF in G^R which is already obtained in Step 2.
(6) If $b_a^r < b_p$, $\forall a \in L$, apply Algorithm 1.
(7) Neglect $a \in L$ if $k_a < r_p$.

Theorem 3.1. *The PMS1FL problem can be solved optimally in a polynomial time.*

Proof. To prove this theorem, at first, we prove the feasibility of Algorithms 1 and 2. We know, the allocation of the given facility, construction of the reduced network and the PMSF in the reduced network are feasible.

Now, we prove optimality. Using Algorithm 2, we compute the PMSF in the original network and calculate the residual capacity of each $a \in L$. If we find a location $a \in L$ with $b^r_a \geq b_p$, then the facility p is fixed there and the PMSF is computed in the reduced network G^R. If such location does not exists, then p is fixed on one of the locations $a_1 \in L$ and the PMSF is computed in the reduced network. This process is continued to each remaining $a_i \in L$. We find an optimal solution in each time. Finally, the maximum flow value among these solutions is selected which is the PMS1FL solution.

Here, the time complexity to compute PMSF in G is $O(\Omega mn)$, where Ω be the number of terminals. As the reduced network can be constructed in linear time, so the PMS1FL problem can be solved in $O(|L|\Omega mn)$ time. □

Example 3.2. Let $G = (V, A, L)$ be a given network with source s, set of sinks $D = \{d_1, d_2, d_3\}$ and $L = \{(s, y_1), (s, y_3), (y_1, y_2), (y_2, d_3)\}$ as in Figure 4(a). Each arc has two attributes, capacity and cost (distance). Applying shortest path algorithm of [7], d_1 is the farthest from s, so it gets the first priority. Arbitrarily we take $d_2 \succ d_3$ as d_2 and d_3 are at equal distance from s. We plan the facility p of size $b_p = 3$ to be placed in G so that the optimal PMS1FL can be computed. For this, at first we compute PMSF in G without fixing p on any $a \in L$. Using paths $\mathcal{P}_1 := s - y_1 - d_1$, $\mathcal{P}_2 := s - y_3 - d_1$, $\mathcal{P}_3 := s - y_1 - d_2$, $\mathcal{P}_4 := s - y_1 - y_2 - d_2$, $\mathcal{P}_5 := s - y_2 - d_2$, $\mathcal{P}_6 := s - y_2 - d_3$ and $\mathcal{P}_7 := s - y_3 - d_3$, the prioritized maximum flow values 6, 3, 5, 2, 3, 5 and 3 units, respectively, can be pushed towards the prioritized sinks. The residual capacity of each location is:

$$b^r_{(s,y_1)} = b_{(s,y_1)} - x_{(s,y_1)} = 13 - 13 = 0, \qquad b^r_{(s,y_3)} = b_{(s,y_3)} - x_{(s,y_3)} = 8 - 6 = 2$$

$$b^r_{(y_1,y_2)} = b_{(y_1,y_2)} - x_{(y_1,y_2)} = 3 - 2 = 1, \qquad b^r_{(y_2,d_3)} = b_{(y_2,d_3)} - x_{y_2,d_3} = 8 - 5 = 3$$

Since $b^r_{(y_2,d_3)} = b_p$, we fix p on (y_2, d_3). The reduced network is given in Figure 4(b), where the prioritized maximum static flow value does not decrease. Hence, the PMS1FL is 27 among them 9, 10 and 8 units of flow values are reached at d_1, d_2 and d_3, respectively, and the optimal location is (y_2, d_3).

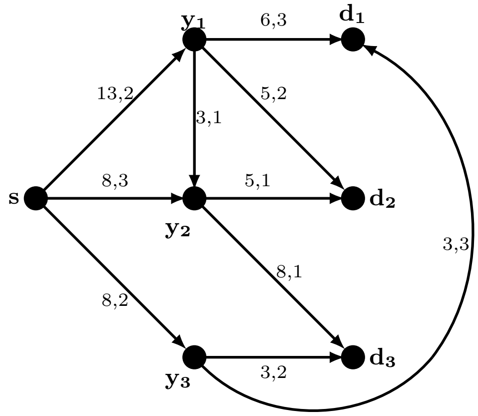
Example 3.3. Suppose that $b_p = 4$ in Example 3.2, then $b^r_a < b_p$, $\forall a \in L$. As $b_p > b_{(y_1,y_2)}$, we neglect (y_1, y_2) from L. If we fix p on (s, y_1), the arc capacity is reduced to 9. In this case, we can push at most 23 units of flow from s towards the sinks (9, 8 and 6 units of flow towards d_1, d_2 and d_3, respectively). If p is placed on (s, y_2), we can push at most 23 units of flow from s towards the sinks (9, 10 and 4 units of flow towards d_1, d_2 and d_3, respectively). If p is placed on (s, y_3), we can push at most 25 units of flow from s towards the sinks (9, 10 and 6 units of flow towards d_1, d_2 and d_3, respectively). If p is fixed on (y_2, d_3), we can push at most 26 units of flow from s towards the sinks (9, 10 and 7 units of flow towards d_1, d_2 and d_3, respectively). Thus, the location (y_2, d_3) is optimal and the PMS1FL solution is 26 (c.f. Figure 5(b)).

4. Dynamic 1-FlowLoc

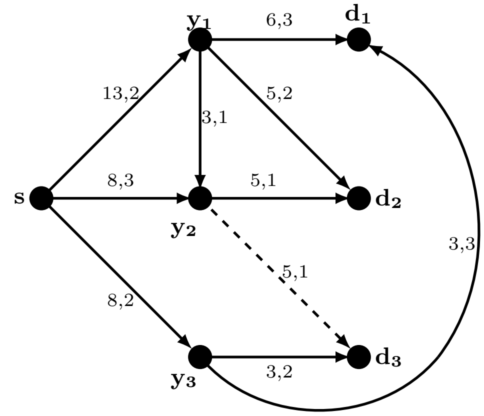
In this section, the prioritized maximum dynamic 1-FlowLoc (PMD1FL) problem is introduced. The mathematical formulation and the polynomial time algorithms are presented for the problem. The MD1FL problem in two terminal network is solved polynomially in ([10]).

Problem 2. *Let G be a given dynamic network with facility p of size b_p. The PMD1FL problem is to maximize the amount of flow from the source s to the prioritized sinks D within the given time T by locating p optimally.*

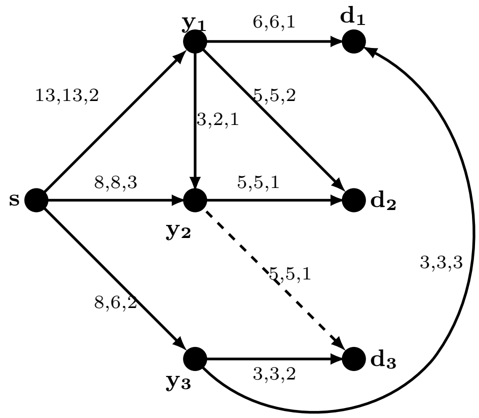
Mathematical formulation. Let the given network G be reduced to $G^R = (V, A, b^R_a, \tau, s, D, \mathbf{T})$ with dynamic flow $\phi(\theta) : A \times \mathbf{T} \to \mathbb{R}^+$, by fixing p on a location $a \in L$. Then the mathematical formulation of



a. arc: capacity,cost

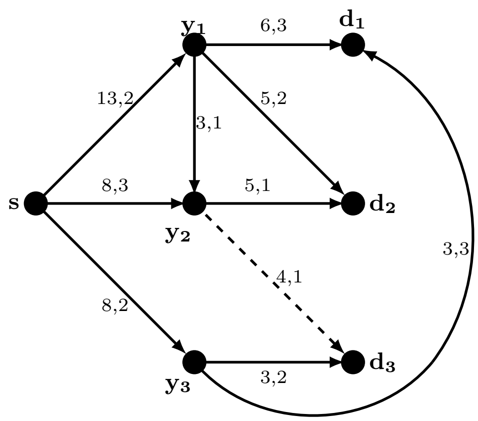


b. optimal location

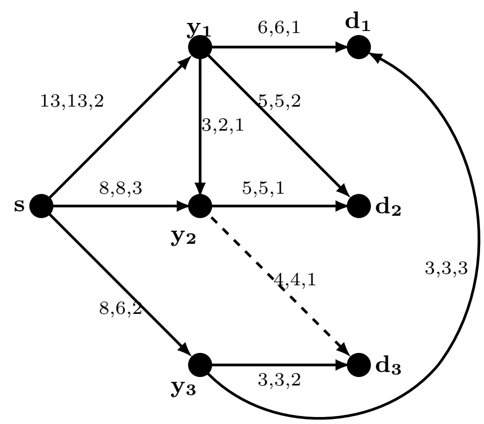


c. PMS1FL

FIGURE 4. a. Given network, b. Reduced network after placing the facility on the optimal location (y_2, t_3) c. PMS1FL solution



a. optimal location



b. PMS1FL

FIGURE 5. a. Optimal reduced network b. PMS1FL solution

the PMD1FL problem is

$$\text{(4.1a)} \qquad \max \sum_{a\in A(s)} \sum_{\sigma=0}^{T} \phi_a(\sigma) = \sum_{a\in B(D)} \sum_{\sigma=\tau(a)}^{T} \phi_a(\sigma - \tau(a))$$

$$\text{(4.1b)} \qquad \text{subject to} \sum_{a\in B(v)} \sum_{\sigma=\tau(a)}^{\theta} \phi_a(\sigma - \tau(a)) - \sum_{a\in A(v)} \sum_{\sigma=0}^{\theta} \phi_a(\sigma) \geq 0,\ \forall v \in V \setminus \{s, D\},\ \theta \in \mathbf{T}$$

$$\text{(4.1c)} \qquad \sum_{a\in B(v)} \sum_{\sigma=\tau(a)}^{T} \phi_a(\sigma - \tau(a)) - \sum_{a\in A(v)} \sum_{\sigma=0}^{T} \phi_a(\sigma) = 0,\ \forall v \in V \setminus \{s, D\}$$

$$\text{(4.1d)} \qquad 0 \leq \phi_a(\theta) \leq b_a^R,\ \forall a \in A,\ \theta \in \mathbf{T}.$$

Objective (4.1a) maximizes the amount of flow to be sent towards the prioritized sinks within time T. The flow conservation constraints are represented by (4.1c). Constraints (4.1b) show the holding over of the flow at the intermediate vertices. Constraints (4.1d) bound the flow value on each arc.

To solve Problem 2, at first, the sinks are prioritized as in static case with $c_e = \tau_e$. Now, we choose a location $a_1 \in L$ and fix the given facility p there. The network is then reduced to $G^R(a_1) = (V, A, b_a^R, \tau, s, D, \mathbf{T})$. We compute the PMDF on $G^R(a_1)$ by using the solution techniques of [12, 13]. For this, a super terminal node v^* is constructed and the source terminal s is joined to v^* with $b_{(v^*,s)} = \infty$ and $\tau(v^*, s) = 0$. Let us denote this network by $G^R_{\alpha+2}$ with zero flow $\phi^{\alpha+2}$. For every iteration $i = \alpha + 1, \alpha, \ldots, 1$, we take a terminal node s_i. If s_i is a source, we remove the arc (v^*, s_i) from G^R_{i+1} to obtain G^R_i and compute the min-cost max-flow from v^* to s_i by taking time as a cost. On the other hand, if s_i is a sink, we construct an arc (s_i, v^*) with $b^R_{(s_i,v^*)} = \infty$ and $\tau(s_i, v^*) = -(T + 1)$ to get G^R_i and obtain min-cost circulation f^i by taking the time as a cost. Update $\phi^i = \phi^{i+1} + f^i$. If γ_i be the standard chain decomposition of f^i, then the accumulated chain flow is given by $\Gamma_i = \Gamma_{i+1} + \gamma_i$. If $\Gamma = \Gamma_1$, the PMDF is obtained.

After this, p is removed from a_1, placed it on the another location $a_2 \in L$ and PMDF is computed in $G^R(a_2)$. This process is continued to each remaining $a_i \in L$ and we get a sequence of the maximum flow values. Finally, the reduced network G^R with overall maximum PMDF is selected (c.f. Algorithm 3).

Algorithm 3: Prioritized MD1FL

Input : Given dynamic network G, location L, facility p with size b_p.
Output: Prioritized MD1FL.

(1) Compute the shortest distance to each $d \in D$, by using Dijkstra's algorithm [7].
(2) Give first priority to the sink $d \in D$ with the longest distance from s, and continue for other nodes similarly.
(3) Fix p on one of the locations $a_1 \in L$, the reduced network is $G^R = (V, A, b_a^R, \tau, s, D, \mathbf{T})$.
(4) Compute the prioritized maximum dynamic flow in G^R
(5) Remove p from a_1, fix it to $a_2 \in L$, goto Step (4) and repeat likewise for other locations.
(6) Pickup the maximum flow value among these maximum flow values.

To improve the running time of Algorithm 3, we present Algorithm 4. After prioritizing the sinks, the PMSF is computed in G without fixing p on any $a \in L$ and calculated the residual capacity of each $a \in L$. We check, whether there exist a location $a \in L$ so that its residual capacity is enough to host p. If we find such a location, then we fix p there and compute the PMDF which is already obtained. Otherwise Algorithm 3 is applied.

Theorem 4.1. *The PMS1FL problem can be solved optimally in a polynomial time.*

Proof. As in Theorem 3.1, Algorithms 3 and 4 are feasible. To obtain the PMD1FL in G, the PMSF is computed in G without placing the facility p and the residual capacity of each $a \in L$ are calculated. If we find a location $a \in L$ with $b_a^r \geq b_p$, then p is placed there and the PMDF is computed in the reduced network G^R. If we cannot find such a location, then p is placed on one of the locations $a_1 \in L$ and the

Algorithm 4: Improved prioritized MD1FL

Input : Given network G, location L, facility p with size b_p.

Output: Prioritized MD1FL.

(1) Compute Steps (1)-(2 using Algorithm 3 to prioritize the sinks.
(2) Obtain the PMSF in G.
(3) Obtain residual arc capacity $b_a^r = b_a - \phi_a$, $\forall a \in L$.
(4) If $\exists\ a \in L$, $b_a^r \geq b_p$, place the facility p on the location a, the reduced network is $G^R = (V, A, b_a^R, \tau, s, D, \mathbf{T})$.
(5) Obtain the PMDF in G^R.
(6) If $b_a^r < b_p$, $\forall a \in L$, apply Algorithm 3.
(7) Neglect $a \in L$ if $k_a < r_p$.

PMDF is computed. This process is to be continued to each remaining $a_i \in L$. Each time we obtain the PMDF. Finally the maximum flow value among these solutions is selected, which is the optimal solution.

The complexity of Algorithm 4 is dominated by Steps 4 and 5 of Algorithm 3, where Step 4 can be computed in $O(\Omega MCF)$ time (c.f. Hoppe and Tardos [12, 13]), where Ω and $O(MCF)$ be the number of terminals and the time complexity to compute single min-cost max-flow, respectively. Thus, the PMD1FL problem can be solved in $O(|L|\Omega MCF)$ time using Step 5. □

Example 4.2. By considering the time as a cost in Figure 4(a), it becomes a single source multi-sink dynamic network with facility p and $b_p = 3$. Here, we have $L = \{(s, y_1), (s, y_3), (y_1, y_2), (y_2, d_3)\}$. As in Example 3.2, after computing prioritized maximum static flow, the residual capacity of each $a \in L$ are calculated. Since, $b^r_{(y_2,d_3)} = b_p$, we place p on (y_2, d_3), the reduced network is given in Figure 4(b). Now the PMDF is computed in the reduced network. The dynamic solution is given in Table 1.

TABLE 1. PMD1FL with excess flow storage and facility locations

S.N.	Path	Time-1	Time-2	Time-3	Time-4	Time-5	Total
1	$s - y_1 - d_1$	..	..	..	..	6	6
2	$s - y_3 - d_1$	..	..	..	..	3	3
3	$s - y_1 - d_2$	..	..	..	5	5	10
4	$s - y_1 - y_2 - d_2$	..	..	..	2	2	4
5	$s - y_2 - d_2$	..	..	..	3	3	6
6	$s - y_2 - d_3$	..	..	..	5	5	10
7	$s - y_3 - d_3$	..	..	..	3	3	6
Total		..	..	..	18	27	45

Example 4.3. Let $b_p = 4$ in Example 4.2, then we have $b_a^r < b_p$, $\forall a \in L$. Since $b_{(y_1,y_2)} < b_p$, neglect (y_1, y_2) from L. If we place p on (s, y_1), we can send at most 37 units of flow to the sinks D within time $T = 5$ (9, 16 and 12 units of flow towards d_1, d_2 and d_3, respectively). If we fix p on (s, y_2), we can send at most 37 units of flow to the sinks D within time $T = 5$ (9, 20 and 8 units of flow towards d_1, d_2 and d_3, respectively). If we fix p on (s, y_3), we can send at most 41 units of flow to the sinks D within time $T = 5$ (9, 20 and 12 units of flow towards d_1, d_2 and d_3, respectively). If we fix p on (y_2, d_3), we can send at most 43 units of flow to the sinks D within time $T = 5$ (9, 20 and 14 units of flow towards d_1, d_2 and d_3, respectively). Now, $\max\{37, 41, 43\} = 43$, so the PMD1FL solution is 43 and the optimal location is (y_2, d_3)

5. Static q-FlowLoc

The prioritized maximum static q-FlowLoc (PMSqFL) problem is introduced. By proving its $\mathcal{NP}$-completeness, we present a mathematical programming model and a polynomial time heuristic to solve it. Hamacher et al. [10] provided a polynomial time heuristic for the MSqFL problem in two terminal network.

Problem 3. *Let G be a given static network. The PMSqFL problem maximizes the amount of feasible flow that is to be sent from the source s which is to be sent towards the prioritized sinks D by locating the q facilities on L and not more than $n_p(a)$ on each $a \in L$.*

Mathematical formulation. Let G be a given network where a set of facilities $\mathbf{P}$ with size $b : \mathbf{P} \to \mathbb{N}$ should be fixed on L so that at most $n_p(a)$ facilities can be placed at $a \in L$. Define the decision variable,

$$\gamma_{ap} = \begin{cases} 1 & \text{if the facility } p \in \mathbf{P} \text{ is placed on } a \in L \\ 0 & \text{otherwise.} \end{cases} \tag{5.1}$$

then the mathematical programming formulation of Problem 3 is to maximize the objective (3.1a) subject to the constraints (5.2a)-(5.2f).

$$\sum_{a \in B(v)} x_a - \sum_{a \in A(v)} x_a = 0, \; \forall v \in V \setminus \{s, D\} \tag{5.2a}$$

$$\sum_{p \in \mathbf{P}} \gamma_{ap} \leq n_p(a), \; \forall a \in L \tag{5.2b}$$

$$\sum_{a \in L} \gamma_{ap} = 1, \; \forall p \in \mathbf{P} \tag{5.2c}$$

$$0 \leq x_a \leq b_a - \max\{b_p : Lc(p) = a\}, \; \forall p \in \mathbf{P}, \; a \in L \tag{5.2d}$$

$$0 \leq x_a \leq b_a, \; \forall a \in A \tag{5.2e}$$

$$\gamma_{ap} \in \{0, 1\}, \; \forall a \in L, \; p \in \mathbf{P}. \tag{5.2f}$$

Objective (3.1a) maximizes the amount of flow reaching to the sinks D which is to be sent from the source s. The flow conservation at each intermediate node is shown by Constraints (5.2a). The number of facilities that can be fixed on each $a \in L$ are bounded by Constraints (5.2b). According to Constraints (5.2c), each facility should be placed on exactly one $a \in L$. Constraints (5.2d) show that the flow value on any arc does not exceed its reduced capacity. Constraints (5.2e) bound the flow on every arcs.

Theorem 5.1. *The PMSqFL problem is $\mathcal{NP}$- complete.*

Proof. At first, the given network is modified as follows. A super terminal node ω with infinite capacity is constructed and each sink node is connected with it. The network becomes two-terminal network. The time complexity of this construction is $O(|D|)$. Now, the problem becomes the maximum static q-FlowLoc problem in a two terminal network. As in Hamacher et al. [10], we can prove that this problem is $\mathcal{NP}$-complete by reducing it into 3-SAT problem. This completes the proof. □

We present a polynomial time heuristic (c.f. Algorithm 5). First of all, sinks are prioritized as in Section 3 and the PMSF is computed in the original network. The locations and the given facilities are sorted in decreasing order according to their residual capacity and size, respectively. Now, we select $a \in L$ with the largest b_a^r value and place $n_p(a)$ facilities there. The process is to be continued until all the facilities are fixed on L. Then, the lexicographic maximum static flow is computed in the reduced network as described in Section 3.

Example 5.2. Let $G = (V, A, L)$ be a given network with source s, set of sinks $D = \{d_1, d_2\}$ and $L = \{(s, u_1), (s, u_2), (u_1, v_1), (u_2, v_1), (u_2, v_3), (u_3, v_3)\}$ as in Figure 6(a). Applying the shortest path algorithm of [7], d_1 is the farthest from s, so it gets first priority. So the priority ordering of the sinks be $d_1 \succ d_2$. Suppose that a set of facilities $\mathbf{P} = \{p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9, p_{10}\}$ with size $b_{p_1} = 1$, $b_{p_2} = 3$, $b_{p_3} = 3$, $b_{p_4} = 4$, $b_{p_5} = 7$, $b_{p_6} = 5$, $b_{p_7} = 6$, $b_{p_8} = 2$, $b_{p_9} = 1$, $b_{p_{10}} = 1$ have to be fixed on the exact locations

Algorithm 5: Prioritized maximum static q-FlowLoc

Input : Given network $G = (V, A, L, b_a, s, D)$, facilities $\mathbf{P}$, size $b : \mathbf{P} \to \mathbb{N}$

Output: The PMSqFL and locations.

(1) Prioritize D by using Steps (1)-(2) as in Algorithm 1.
(2) Obtain PMSF with residual capacities of each $a \in L$ in G.
(3) Sort the facilities with their size: $p_1 \geq p_2 \geq \cdots \geq p_q$.
(4) Sort the locations according to their residual capacities: $a_1 \succ a_2 \succ \cdots \succ a_{|L|}$.
(5) Fix the facilities $p_1, p_2, \ldots, p_{n_p}$ on $a_1 \in L$ and so on. The reduced network is $G^R = (V, A, b_a^R, s, D)$.
(6) Compute the prioritized maximum static flow in G^R.

with $n_p(a) = 3$, $\forall a \in L$ so that the PMSqFL can be optimized in G. For this, we prioritize the facilities according to their size, $p_5 \succ p_7 \succ p_6 \succ p_4 \succ p_2 \succ p_3 \succ p_8 \succ p_1 \succ p_9 \succ p_{10}$. Now we compute PMSF, $x_a : A \to \mathbb{R}^+$. Using paths $\mathcal{P}_1 := s - u_1 - v_1 - d_1$, $\mathcal{P}_2 := s - u_2 - v_1 - d_1$, $\mathcal{P}_3 := s - u_2 - v_2 - d_1$, $\mathcal{P}_4 := s - u_2 - v_2 - d_2$, $\mathcal{P}_5 := s - u_3 - v_3 - d_2$ and $\mathcal{P}_6 := s - u_2 - v_3 - d_2$, the prioritized maximum flow values 6, 4, 1, 1, 4 and 3 units, respectively, can be pushed towards the prioritized sinks. The residual capacity of each location is: $b^r_{(s,u_1)} = 0$, $b^r_{(s,u_2)} = 3$, $b^r_{(u_1,v_1)} = 5$, $b^r_{(s,u_2)} = 1$, $b^r_{(s,u_1)} = 0$ and $b^r_{(u_3,v_3)} = 2$

Using the residual capacities, the locations are prioritized as $(u_1, v_1) \succ (s, v_2) \succ (u_3, v_3) \succ (u_2, v_1) \succ (s, u_1) \succ (u_2, v_3)$. Using Algorithm 5, (u_1, v_1) gets p_5, p_7 and p_6, (s, u_2) gets p_4, p_2 and (p_3), (u_3, v_3) gets p_8, p_1 and p_9 and (u_2, v_1) gets p_{10}. After placing the facilities, the reduced network G^R is given in Figure 6(b). Now we compute PMSF in G^R. Using paths $\mathcal{P}_1$, $\mathcal{P}_2$, $\mathcal{P}_3$, $\mathcal{P}_4$, $\mathcal{P}_5$ and $\mathcal{P}_6$ the flow values 4, 4, 1, 1, 4 and 2, are calculated towards the prioritized sinks, respectively. Therefore the PMSqFL is 16 among them 9 and 7 units of flow values can be sent towards d_1 and d_2, respectively (c.f. Figure 6(c)).

6. Dynamic q-FlowLoc

In this section, the prioritized maximum dynamic q-FlowLoc (PMDqFL) problem and its mathematical programming formulation are introduced. A polynomial time heuristic is presented for the solution procedure.

Problem 4. *Let G be a given dynamic network. The PMDqFL problem maximizes the amount of feasible flow that is to be sent towards the prioritized D, by locating the q facilities on L and not more than $n_p(a)$ facilities on each $a \in L$ within time T.*

Mathematical formulation. Let G be a given network with dynamic flow $\phi(\theta) : A \times \mathbf{T} \to \mathbb{R}^+$. Suppose that a set of facilities $\mathbf{P}$ with size $b : \mathbf{P} \to \mathbb{N}$ should be fixed on the location L where, at most $n_p(a)$ facilities can be placed at $a \in L$. Consider the decision variable γ_{ap} defined as in Equation (5.1). Then the mathematical programming formulation of Problem 4 is to maximize Objective (4.1a) with respect to the constraints (4.1b)-(4.1d), (5.2b)-(5.2c), (5.2f) and

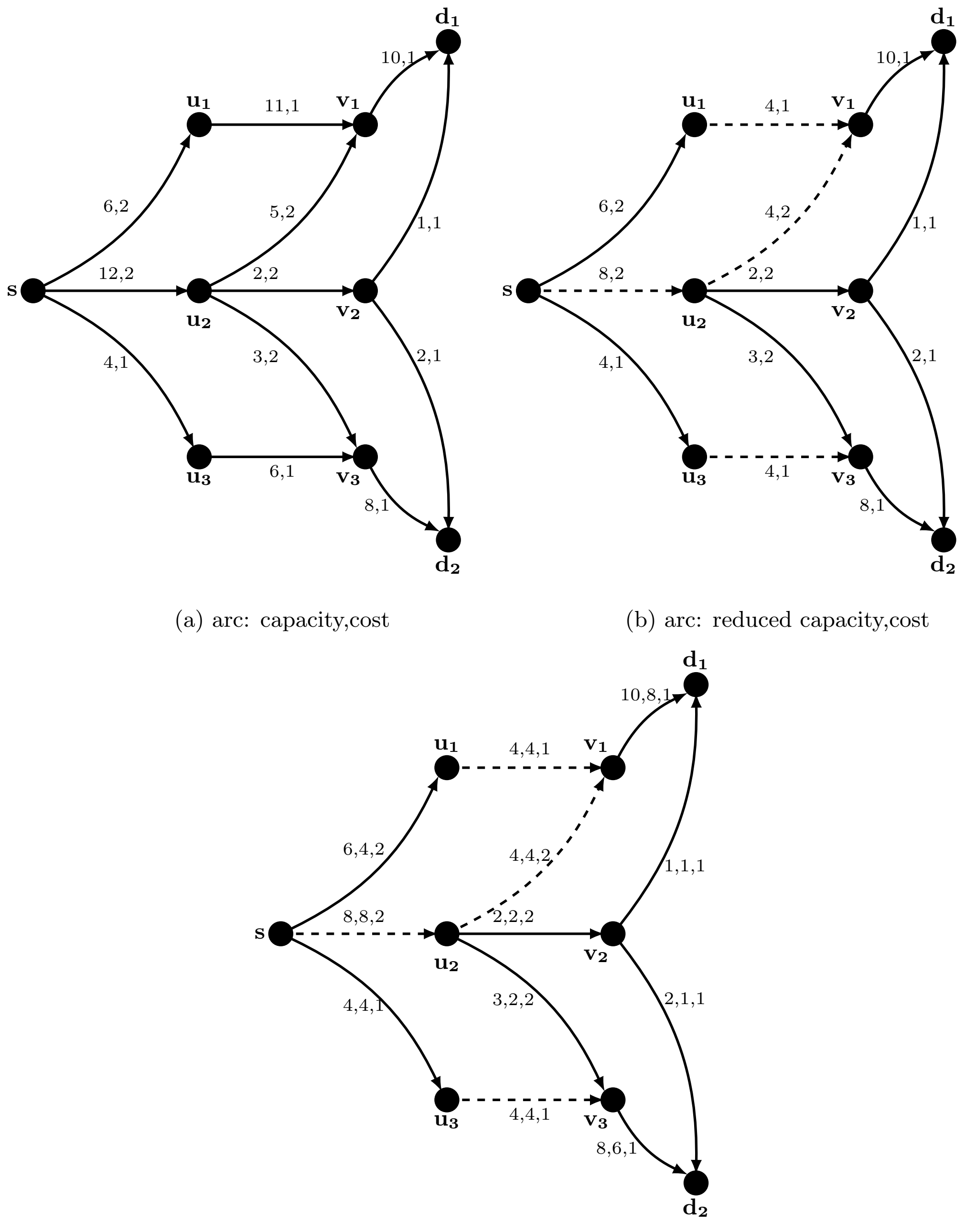
$$0 \leq \phi_a(\theta) \quad \leq \quad b_a(\theta) - \max\{b_p : Lc(p) = a\}, \forall p \in \mathbf{P},\, a \in L,\, \theta \in \mathbf{T} \tag{6.1}$$

As PMSqFL problem is $\mathcal{NP}$-complete, this leads the following theorem:

Theorem 6.1. *The PMDqFL problem is $\mathcal{NP}$-complete.*

Now we present a polynomial time heuristic (c.f. Algorithm 6) to solve Problem 4. First of all, we prioritize the sinks as in the static case and compute the PMSF in the original network. The locations and the given facilities are sorted, and the reduced network is obtained as in Section 5. Then, the PMDF is computed in the reduced network as described in Section 4.

Example 6.2. If we consider time as a cost in Example 5.2, the network is changed into dynamic one. As in Example 5.2, we prioritized the sinks, locations and facilities, and place the given facilities in the appropriate locations. The reduced network G^R is shown in Figure 6(b). Now we compute PMDF as in G^R.



(a) arc: capacity,cost

(b) arc: reduced capacity,cost

(c) arc: capacity, flow, cost

FIGURE 6. (a) Given network (b) Optimal reduced network (c) LMSqFL solution

Algorithm 6: Prioritized maximum dynamic q-FlowLoc

Input : Given dynamic network G, facilities $\mathbf{P}$, size $b : \mathbf{P} \to \mathbb{N}$

Output: The PMDqFL and locations.

(1) Use Steps 1)-(2) of Algorithm 3 to prioritize the sinks.
(2) Compute Steps (2)-(4) (c.f. Algorithm 5) to sort the facilities and the locations.
(3) Fix the facilities $p_1, p_2, \ldots, p_{n_p}$ on $a_1 \in L$ and so on. The reduced network is $G^R = (V, A, b_a^R, s, \tau, D, \mathbf{T})$.
(4) Compute the prioritized maximum dynamic flow in G^R.

The dynamic solution is given in Table 2. From Table 2, the PMDqFL is 28 units among them, 13 and 15 units of flow can be transshipped towards the sinks d_1 and d_2, respectively.

TABLE 2. Prioritized maximum dynamic q-FlowLoc

S.N.	Path	Time-1	Time-2	Time-3	Time-4	Time-5	Total
1	$s - u_1 - v_1 - d_1$	..	..	..	4	4	8
2	$s - u_2 - v_1 - d_1$	..	..	..	..	4	4
3	$s - u_2 - v_2 - d_1$	..	..	..	..	1	1
4	$s - u_2 - v_2 - d_2$	..	..	..	..	1	1
5	$s - u_2 - v_3 - d_2$	..	..	..	..	2	2
6	$s - u_3 - v_3 - d_2$	..	..	4	4	4	12
Total		..	..	4	8	16	28

7. Conclusions

When a facility is fixed in a location, it reduces the capacity of the arc. The decrease in capacity may affect the maximum flow value. Thus in emergency situation, by allocating the facility on an arc may lead to longer evacuation time. However, we need have to provide the emergency facilities like medicine, food, security, etc., to the evacuees, which is very crucial in such periods. On the other hand, the prioritized maximum flow problem involves maximizing the flow value based on a predetermined reaching pattern at the sinks. By doing so, conflicts that arise from sending flow on specific arcs are minimized, resulting in a smoother flow throughout the network. Therefore, the concept of prioritization is crucial during the emergency periods. The FlowLoc models in a two terminal network can be found in literature. However, these models may not resolve problems in the multi-terminal network.

In this paper, the PMS1FL, PMD1FL, PMSqFL and PMDqFL problems are introduced and the FlowLoc models of these problems are investigated. To solve the LMS1FL and LMD1FL problems, polynomial time algorithms are presented. We have shown that the PMSqFL and PMDqFL problems are $\mathcal{NP}$-complete and presented polynomial time heuristics for the approximate solution.

To the best of our knowledge, we have introduced and solved these problems for the first time. These solutions are crucial for disaster management because they will help to maximize the movement of people out from the emergency area by placing the provided facilities in the best possible places. In the context of evacuation planning, we would further like to incorporate these results into the Kathmandu network.

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