The Nepali Mathematical Sciences Report, Vol. 39, No. 2, 2022: 54-66
DOI: https://doi.org/10.3126/nmsr.v39i2.51694

INFLUENCE OF FRICTION ANGLES ON EARTH PRESSURES IN DRY GRANULAR FLOW DYNAMICS AND SOIL MECHANICS

JEEVAN KAFLE [1], RAMUNA PANDEY [1], BEKHA RATNA DANGOL [1,2], CETH NATH TIWARI[1,3] AND PARAMESWARI KATTEL[3,*]

[1] *Central Department of Mathematics, Tribhuvan University, Kirtipur, Kathmandu, Nepal*
[2] *Department of Mathematics, Patan Multiple Campus, Tribhuvan University, Kathmandu, Nepal*
[3] *Department of Mathematics, Tri-Chandra Multiple Campus, Tribhuvan University, Kathmandu, Nepal*

**Correspondence to: Parameshwari Kattel, Email: parameshwari.kattel@trc.tu.edu.np*

Abstract: The earth pressure coefficient (K) determines the nature of (tendency of) deformation of the granular mass during flow or deposition. When flow velocity is increasing, K takes its active state K_{act} and the flow is divergent. When the flow velocity is decreasing, K takes its passive state K_{pas} and the flow is convergent. The mathematical relations presented here and their 2-D and 3-D plots highlight that the passive and active earth coefficients strongly depend on the internal angle (δ) and basal angle (ϕ) of frictions. The mathematical relation for dry granular mass flow is extended to find these coefficients in soil mechanics. Results further show that active earth pressure drops as the internal angle of friction increases, but passive earth pressure rises. The earth pressure is at rest if the wall is in its natural position.

Key Words: Earth Pressure Coefficient, Active Earth Pressure, Passive Earth Pressure, Internal friction angle, Basal friction angle.

AMS (MOS) Subject Classification: 74G15; 74H15; 74L10

1. Introduction

Landslides, debris flow, snow and rock avalanches are extremely destructive natural hazards caused by massive mass movements (earth mass failure) of the mixture of soil, ice, rock and fluid down mountain slopes [4, 5, 6, 10, 11, 12, 19, 21]. Such events occur when the shearing resistance (resisting force) fails to bear the destabilizing force (normal stress). A massive landslide had struck Lidi village, Sindhupalchok on August 14, destroying 17 houses and damaging 37 others. Nearly 39 villagers died and 135 households were displaced in this disaster [8]. For these hazard mitigation perspective, it is crucial to understand the dynamics of how massive amounts of gravel, snow, or soil originate and move rapidly. The dynamics of these mass flows depend on initial volume of mass, the material properties and

Received August 10, 2022 Accepted/Published Online: December, 2022.

channel geometries with initial and boundary conditions [7, 8, 9, 19]. A variety of models have been put out for the analysis of granular mass flowing rapidly in recent decades. In the Savage and Hutter model [21], the earth pressure coefficient dominates the force balances in the flow dynamics for the flow of a finite mass of granular material down an inclined plane. It is the ratio of longitudinal pressure p_L to the hydrostatic pressure p of the release mass:

$$K = \frac{p_L}{p}$$

When $p_L = p$, the pressure distribution is that of the liquid and $K = 1$ (Isotropic pressure). In this case, the model becomes a hydraulic model also known as Saint-Venant or Boussinesq equations [19, 21]. If $p_L \neq p$, then $K \neq 1$, then the pressure distribution will be non-isotropic. In such circumstances, two different values namely active and passive pressure coefficients are suggested in soil mechanics [19]. The dimensionless quantity is called active pressure coefficient if velocity in the flow is increasing and it corresponds to the dilatational mode of deformation. On contrary, passive earth pressure coefficient refers to the compressive deformation posing the decreasing flow-velocity. Accordingly [19],

$$K = K_{\text{act/pas}} = \begin{cases} K_{\text{act}} & \text{for} \quad \partial u/\partial x > 0, \\ K_{\text{pas}} & \text{for} \quad \partial u/\partial x < 0, \end{cases}$$

It is also worth pointing out that fluid and earth materials have significant variations. The lateral pressure coefficient K is always equal to 1 in a fluid. The earth coefficient varies between the active and passive if the flowing material is a dry granular substance with friction. Earth pressure plays an essential role in the stability of the granular mass, and also in the flow dynamics, runout and depositional behaviour of the material [18]. This coefficient substantially depends on the internal friction of the granular mass and the roughness of the bed in the channel geometry or topography. Passive earth pressure directly varies to the internal angle of friction whereas the active one varies inversely, but natures of the coefficients are opposite in relation to the basal angle of friction.

In the mountainous countries, disasters protective engineering structures are abundant for mountain roads, landslide controlling, drainage, river embankments, hydroelectric power generation and town planning. Earth pressure concerns arise in such engineering procedures when evaluating internal stresses impacting on soil masses or stresses between soils and surrounding structures [1]. The pressure coefficient determines the retention or instability of the structures against vertical and lateral stresses. In the geotechnical stability analysis, the engineering structures like retaining wall depend on the formation of soil that determines the internal friction angles and topography. The ability of a unit of rock or soil to resist a shear force is determined by the internal angle of friction. It is commonly estimated between the normal force and the resulting force when failure occurs in response to a shearing stress [13]. When grains and the basal surfaces on which they flow are smooth, friction angles are low; when grains and the basal surface are coarse, friction angles are high. Higher values are referred with coarse materials such as gravel, whereas lower values are for ultra smooth particles such as glass beads. In such soil mechanics, bed

FIGURE 1. **Left:** Concrete retaining wall turned outward by slope failure above it during the Chi-Chi earthquake in Taiwan (1999). **Right:** Retaining walls constructed in the BP highway in the eastern Nepal where various slope stabilization techniques are adopted (Photo: N. Sitar).

friction (degree of smoothness or roughness of the basal surface) has almost no impact on the failure or retention of the mass. The pressure applied by the material while the wall is at rest and the material is in its natural form is known as earth pressure at rest. As the wall travels away from the backfill, the pressure on the wall lowers. This reduction lasts until a minimum value is reached, at which point there is no more pressure reduction and the value becomes constant. Such least stable value is referred as active earth pressure. In contrast, the pressure on the wall increases as the wall moves into the back fill and this increase persists until a maximum value is measured, after which there is no further increase in pressure and the value becomes constant. The greatest stable value is known as passive earth pressure [14].

Mohr's circle [19] is the geometric representation of the transformation of the stresses and principal stresses to estimate the maximum shear stress and stresses on the inclined plane. In particular, it is used to calculate the relationship between normal and shear stresses. Savage and Hutter [21] proposed Mohr-Coulomb plastic yield criterion for rapid

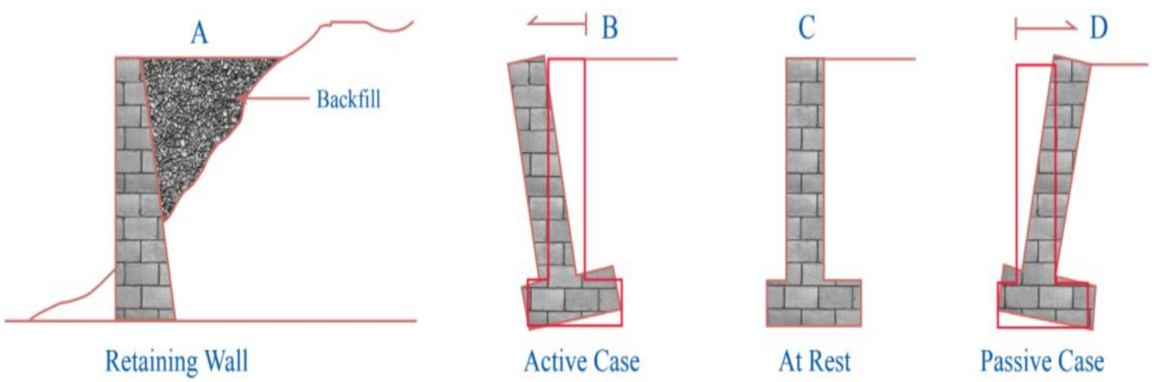

FIGURE 2. **A**: Retaining wall supporting the backfill, **B**: Active case where wall moves away from the backfill, **C**: The wall is at rest and the back fill in the natural state, **D**: Passive case where wall moves towards the backfill.

granular flows. This model has been extensively applied to study the deformation of the effectively single-phase materials, such as avalanches and granular flows. It supposes that the material fails plastically after the Mohr's circle of two-dimensional state of stress (in $p-\tau$ plane) meets the Coulomb failure envelope. Mohr's circle is often used in geotechnical engineering to determine the failure surface and in measuring the mechanical properties of many deformable solids of earth material like soil, sand, clay, rock, and other granular materials or powders [15, 16].

Understanding the influence of earth pressure in the deformation of the granular material and behaviour of retaining walls subjected to earth pressures are essential but complex phenomena. Coulomb made the first significant contribution to the study of earth pressures when he analyzed a cohesionless rigid mass of soil sliding across a shear surface [3]. In contrast to Coulomb's solution, which regarded a soil mass restricted by a single failure surface, Rankine advanced earth pressure theory by finding a solution for a complete soil mass in a condition of failure [20]. Mohr circles have also been effectively employed to construct solutions for the Rankine analysis. Rankine's concept was initially limited to cohesionless soils. Bell further expanded on this theory to include the case of soils that have both cohesion and friction [2]. The active and passive lateral earth pressures are often calculated in the engineering profession using conventional methods based on the Rankine theory and Coulomb theory. Nevertheless, both of these theories imply that the backfills can stretch far enough to allow the failure plane to fully emerge. As a result, they are unable to account for the impact of the backfill width behind the wall. In reality, narrow backfill widths have been considered in many cases. According to the agreement, in mountainous areas where rock formations are close to the wall, mechanically stabilized earth walls are built in front of already stabilized walls to expand existing routes [23]. With the influence of backfill width, internal soil friction angle, and wall-soil friction angle, Yang and Deng [24] presented an analytical technique to compute active earth pressure coefficients and their classes on the rigid retaining wall.

Yan et al. [22] determined the distribution of earth pressure by differentiating the influences of the total active force's starting phase, amplification factor, and soil friction angle on the distribution. Levesque et al. [14] demonstrated that the accuracy of the Marston approach for estimating horizontal earth pressure would be improved by using K-values that are acquired as a function of back fill geotechnical parameters and also excavation geometry. Both components differ necessarily from one back filled site to the next. A method is developed to calibrate the K-value depending on these parameters for the prediction with improved accuracy within the Marston analytical method of earth pressure. Mayne et al. [17] discussed about earth pressure coefficient at rest.

Based on laboratory study or analytical and numerical models, different researchers proposed the various values for the earth pressure coefficient (K). For the agreement, Marston [16] identified K as the active earth pressure established by Rankine ($K_a = 0.33$) however Li et al. [15] recommended it as 0.5 to to analyze mass deformation. However, various analytical and computational studies indicate that the granular material properties and the roughness of the basal surface determine these coefficients. The present paper demonstrates

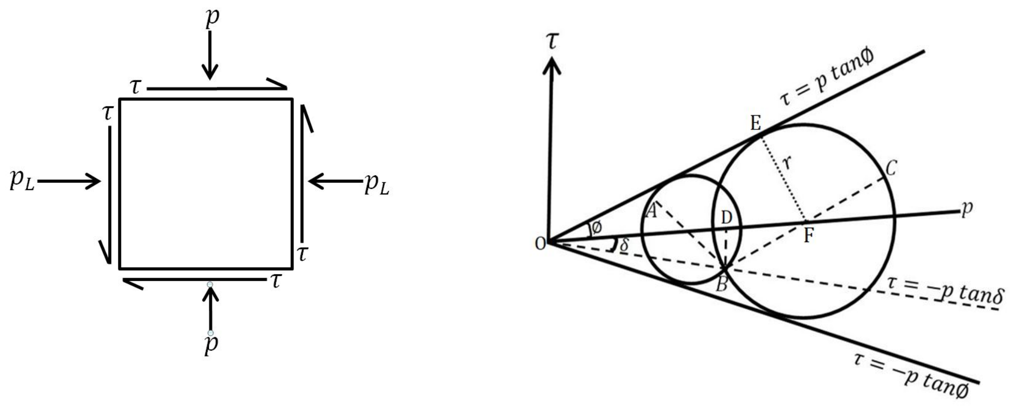

FIGURE 3. **Left:** Material plane element at the basal plane with normal stresses p and p_L and shearing stress τ. **Right:** Mohr circle diagram representing the stress states. The respective states of stresses at points A, B, C on the figure are (p_L, τ), $(p, -\tau)$ and (p_L, τ) [19].

the characteristics of earth pressure coefficient in active and passive conditions in different ranges of parameters and also varying the ratio of parameters used in the formulation of earth pressure coefficient in Savage-Hutter model. Firstly, the mathematical formulations of the active or passive earth pressure coefficient in Savage-Hutter Model for flow of granular mass in one dimension are discussed. Secondly, the reduction of the pressure coefficients in the soil mechanics is exhibited by neglecting the basal angle of friction. Finally, some plots are carried out to study the variations of active and passive earth pressure coefficients due to the changes in internal and basal friction angles and their ratios.

2. Mathematical Model

According to the Savage and Hutter theory [21], an avalanche experiences a very simple condition of stress, as shown in Fig. (3). A material plane element is considered at the base with normal stresses p (vertical) and p_L (longitudinal), and shearing stress τ. Thus, two Mohr stress circles that simultaneously obey the basal sliding rule and the internal yield criterion can be built for a given stress state (p_L, τ) at the base. The yield criterion corresponds to the two straight lines at angles $\pm\phi$ to the horizontal, whereas the line at an angle $-\delta$ to the horizontal indicates the Coulomb basal dry friction. The smaller circle represents active stress state which is tangent to both yield criterion curves and passes through the state stress $A(p_L, \tau)$ and $B(p, -\tau)$. Correspondingly, the passive basal stress state is indicated by the bigger circle with center at $\left(\frac{p+p_L}{2}, 0\right)$ and radius $r = \sqrt{\tau^2 + \frac{1}{4}(p_L - p)^2}$ (Fig. 3) that also satisfies the yield conditions [19, 21].

From the right angled triangle OBD,

$$\tau = -p \tan\delta.$$

Similarly, in triangle OEF,

$$\sin\phi = \frac{r}{\frac{1}{2}\left(p_L + p\right)}. \tag{2.1}$$

Using the values of τ and r, relation (2.1) becomes

$$\sin\phi = \frac{\sqrt{p^2\tan^2\delta + \frac{1}{4}\left(p_L - p\right)^2}}{\frac{1}{2}\left(p_L + p\right)}.$$

Since $K_{act/pas} = p_L/p$, on squaring both sides of the above relation gives

$$\begin{aligned}
&\frac{1}{4}\sin^2\phi\left(K_{act/pas}+1\right)^2 = \tan^2\delta + \frac{1}{4}\left(K_{act/pas}-1\right)^2\\
&\text{or, } 4\tan^2\delta + \left(K^2_{act/pas} - 2K_{act/pas} + 1\right) - \sin^2\phi\left(K^2_{act/pas} + 2K_{act/pas} + 1\right) = 0\\
&\text{or, } 4\tan^2\delta + K^2_{act/pas}\cos^2\phi - 2K_{act/pas}\left(1 + \sin^2\phi + 1 - 1\right) + \cos^2\phi = 0\\
&\text{or, } K^2_{act/pas} - 2\left(\frac{2}{\cos^2\phi} - 1\right)K_{act/pas} + 1 + \frac{4\tan^2\delta}{\cos^2\phi} = 0\\
&\text{or, } K^2_{act/pas} - 2\left(\frac{2}{\cos^2\phi} - 1\right)K_{act/pas} + 1 + \frac{4\tan^2\delta}{\cos^2\phi} = 0.
\end{aligned}$$

This is quadratic in $K_{act/pas}$. So, its solutions are [19, 21]

$$\begin{aligned}
K_{act/pas} &= \left(\frac{2}{\cos^2\phi} - 1\right) \mp \sqrt{\left(\frac{2}{\cos^2\phi} - 1\right)^2 - \left(\frac{4\tan^2\delta}{\cos^2\phi} + 1\right).1}\\
\text{or, } K_{act/pas} &= \left(\frac{2}{\cos^2\phi} - 1\right) \mp \sqrt{\frac{4}{\cos^4\phi} - \frac{4}{\cos^2\phi}\left(1 + \tan^2\delta\right)}\\
\text{or, } K_{act/pas} &= \frac{2}{\cos^2\phi}\left(1 \mp \sqrt{1 - \frac{\cos^2\phi}{\cos^2\delta}}\right) - 1. \qquad (2.2)
\end{aligned}$$

This shows that the two phenomenal parameters, the internal angle ϕ and bed angle δ of frictions describe the material response of the grains in the flow dynamics. The minus sign is used for active pressure which characterizes that the deformation of the granular material is extensive. Similarly, the plus sign is for passive pressure and suggests us that the mode of deformation is compressive.

In flows of granular materials, the bed friction angle has an influence in the longitudinal and overburden pressures. Nonetheless, in the study of soil mechanics for geotechnical stability, there in no rapid flows of soil [16]. Consequently, the effect of the bed friction is neglected in the relation (2.2)

$$K_{act/pas} = \frac{2}{\cos^2\phi}\left(1 \mp \sin\phi\right) - 1.$$

Considering the lower sign,

$$K_{pas} = \frac{2\left(1+\sin\phi\right) - \left(1 - \sin^2\phi\right)}{1 - \sin^2\phi} = \frac{\left(1+\sin\phi\right)^2}{1 - sin^2\phi} = \frac{1+\sin\phi}{1-\sin\phi}. \tag{2.3}$$

The relation $\dfrac{1+\sin\phi}{1-\sin\phi} = \left(\dfrac{\cos\frac{\phi}{2}+\sin\frac{\phi}{2}}{\cos\frac{\phi}{2}-\sin\frac{\phi}{2}}\right)^2 = \left(\dfrac{1+\tan\frac{\phi}{2}}{1-\tan\frac{\phi}{2}}\right)^2$ gives

$$K_{pas} = \tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right). \tag{2.4}$$

Likewise, taking the upper sign in the relation (2.2),

$$K_{act} = \frac{1-\sin\phi}{1+\sin\phi} = \left(\frac{1-\tan\frac{\phi}{2}}{1+\tan\frac{\phi}{2}}\right)^2 = \tan^2\left(\frac{\pi}{4}-\frac{\phi}{2}\right). \tag{2.5}$$

Relations (2.3), (2.4), and (2.5) are frequently used in the soil mechanics when stability of the retaining wall against the backfill is to be analyzed [16].

3. Results and Discussion

The two phenomenological parameters, the internal friction angle ϕ and the bed friction angle δ play major roles in the determination of the earth pressure coefficients. Based on the aforementioned mathematical relations, various two- and three-dimensional plots for the active and passive earth pressure coefficients are demonstrated with regards to the change in basal angle and/or internal angle of friction and their ratios. Also, the work explained how these physical parameters describe the material responses in the granular flows in an inclined channel as well as in the soil mechanics.

3.1. **Effect of the variation of separate friction angles on pressure coefficients.** With constant bed friction angle $\delta = 30°$, Fig. 4(A) exhibits the graph of active and passive pressure coefficients as functions of internal angles of friction. The coefficients become complex valued when $\delta > \phi$ and the theory fails. In fact, granular mass does not flow along the incline in this situation. This natural phenomenon is revealed in Fig. 4(A), where the active and passive coefficients have no values for $\phi < \delta = 30°$. Initially, when $\phi = \delta = 30°$, both active and passive coefficients are 1.7. When the internal friction angle is progressively

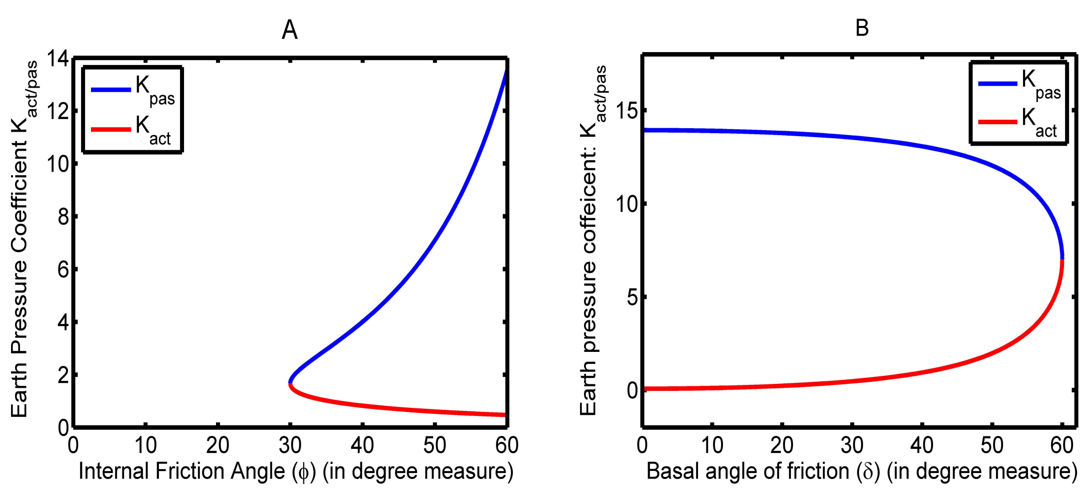

Figure 4. Active and passive earth pressure coefficients due to **A:** variation in internal friction angle with constant basal angle of friction as 30° and **B:** change in bed friction angle with constant angle of internal friction as 60°.

increased to 60°, however, active earth pressure coefficient gradually decreases from 1.7 to 0.4, while passive earth pressure coefficient increases sharply from 1.7 to 13.5. Also,it can be seen that passive earth pressure increases more sharply as ϕ varies from 45° to 60° than when it varies from 30° to 45°. This demonstrates that the internal friction angle has a major influence on the passive earth coefficient. This is due to the fact that as the internal friction angle decreases, the lateral shearing of the material also decreases, making the granular mass more compressive. When both friction angles are equal or nearer to each other, the values of the earth pressure coefficients in the granular mass are equal, or vary within a smaller range. Instead, the earth pressure has large variation when internal and bed frictions substantially deviate.

Fig. 4(B) represents the graph of the active and passive pressure coefficients depending on the bed friction angles while internal friction among the grains is assumed to be constant, $\phi = 60°$. Initially, when basal friction angle $\delta = 0°$, the active and passive earth coefficients are 0.07 and 13.9 respectively. As δ increases, K_{pas} decreases from 13.9 to 7 while K_{act} increases from 0.07 to 7. The evolution of the pressure coefficients is somehow opposite to that in Fig. 4(A). The coefficients deviate remarkably when there is large difference in the friction angles and $K_{act} = K_{pas}$ when $\delta = \phi$. Also, it can be seen that when ϕ varies from 0° to 45°, K_{pas} slowly decreases whereas K_{act} slowly increases. But K_{pas} rapidly decreases and K_{act} rapidly increases when ϕ varies from 45° to 60°.This is so because, when ϕ is increased, compressive mode of deformation increases whereas extensive mode of deformation decreases. The large changes in the values can be observed when δ exceeds 45°. Similar development can be seen in Fig. 4(A) when $\phi > 45°$. From the results, it can be concluded that the value of K is consistent if the values of δ and ϕ are close to each other. The passive earth pressure coefficient is strongly controlled by the internal angle of friction while the active pressure coefficient is by the basal friction angle. The further values of K can not be obtained when δ exceeds 60°.

In Fig. 4(A) and Fig. 4(B), it is observed that $K_{pas} \geq K_{act}$. It is in line with the natural phenomenon that the compressive mode of deformation (K_{pas}) is always greater than or equal to extensive mode of deformation (K_{act}). It is noteworthy to observe that K_{pas} is increased by increasing ϕ, whereas it is decreased by increasing δ. On contrary, K_{act} shows exactly the opposite behavior.

3.2. **Earth pressure coefficients against internal friction angle for different ratios of basal to internal friction angles.** Internal friction and bed friction angles have major influence in the flow dynamics. Also, their difference and closeness have the significant impact in the dynamics. To observe this, we plot the pressure coefficients depending on the different ratios of the friction angles.

Figure 5(A) shows the graph of active pressure coefficients as a function of internal friction angle for three different ratios of δ and ϕ. As $\frac{\delta}{\phi} = 1$, K_{act} increases from 1 to 7, whereas it decreases from 1 to 0.47, and from 1 to 0.07 when $\frac{\delta}{\phi} = \frac{1}{2}$ and $\frac{\delta}{\phi} = 0$, respectively. It means that if the velocity gradient is positive, the flow is highly extensive whenever ϕ and δ are higher and equal. Instead, the value of the coefficient is the least if internal angle of friction is much lager than that of bed. For $\frac{\delta}{\phi} = 1$, value of K_{act} increases more rapidly for

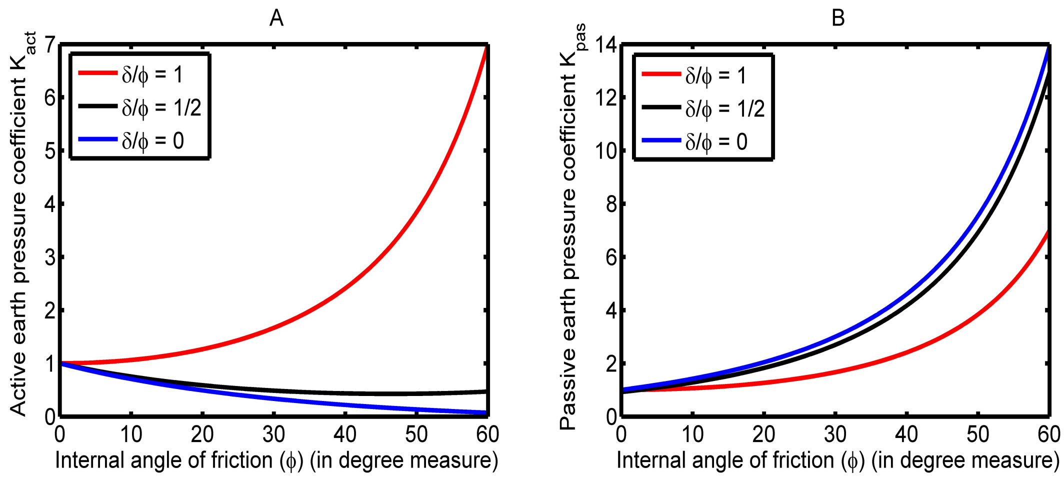

FIGURE 5. **A:** Active earth pressure coefficients and **B:** Passive earth pressure coefficients, depending on internal angle of friction that constantly varies to basal angle.

values of ϕ higher than 45°. On contrary, it decreases significantly when $\phi < 45°$, in case if $\frac{\delta}{\phi} = \frac{1}{2}$ and $\frac{\delta}{\phi} = 0$. So, $\phi = 45°$ is somehow behaves as a critical friction angle. There is not much larger difference in the values of K_{act} when the ratio $\frac{\delta}{\phi} \leq \frac{1}{2}$. It is equally interesting to observe that K_{act} is increasing when $\frac{\delta}{\phi} = 1$, i.e., for $\delta = \phi$ but slowly decreases when $\frac{\delta}{\phi} = \frac{1}{2}$, i.e., $\delta = \frac{1}{2}\phi$ and rapidly decreases for $\frac{\delta}{\phi} = 0$, i.e., $\delta = 0$.

For different ratios of δ and ϕ, Fig. 5(B) represents the graph of passive earth pressure coefficients against internal friction angles. As $\frac{\delta}{\phi} = 1$, K_{pas} increases from 1 to 7 which is the same graph of K_{act} as in Fig. 5(A). It means that active and passive pressure coefficients are both equal when $\phi = \delta$. When $\frac{\delta}{\phi} = \frac{1}{2}$, K_{pas} increases from 1 to 13.5. In similar manner, it increases from 1 to 13.9 when $\frac{\delta}{\phi} = 0$. Like the active pressure, K_{pas} also has smaller deviation when $\frac{\delta}{\phi} \leq \frac{1}{2}$. As the ratio of basal angle and internal angle of friction decreases, the active pressure decreases but the passive pressure increases. When ϕ exceeds 40°, the pressure coefficients change significantly.

3.3. **Changes in coefficients against basal friction angle for different ratios of basal and internal friction angles.** Earth pressure coefficients not only depend on the internal angle of friction but also on the basal angle of friction. With regards to the different ratios of friction angles, Fig. 6 demonstrates the active and passive earth pressure coefficients as functions of bed friction angles. Figure 6(A) is the evaluation of K_{act} for the different values of basal friction angles. As $\delta = \phi$, K_{act} increases from 1 to 1.7, and in contrast, it decreases from 1 to 0.73 and 1 to 0.48 when $\frac{\delta}{\phi} = \frac{3}{4}$ and $\frac{\delta}{\phi} = \frac{1}{2}$, respectively.

We clearly observe that K_{act} in Fig. 5(A) is much higher than Fig. 6(A). So, influence of internal friction angle (ϕ) is higher than the basal friction angle (δ) for the variation of active earth pressure K_{act}. K_{act} decreases as internal friction angle increases relative to the basal friction angle. But, the passive earth pressure shows the completely opposite behaviour that can be seen in Fig. 6(B). As the ratio of basal and internal friction angles decreases, K_{pas} increases for increasing δ. When $\frac{\delta}{\phi} = 1$ and $\frac{\delta}{\phi} = \frac{3}{4}$, K_{pas} increases from 1 to

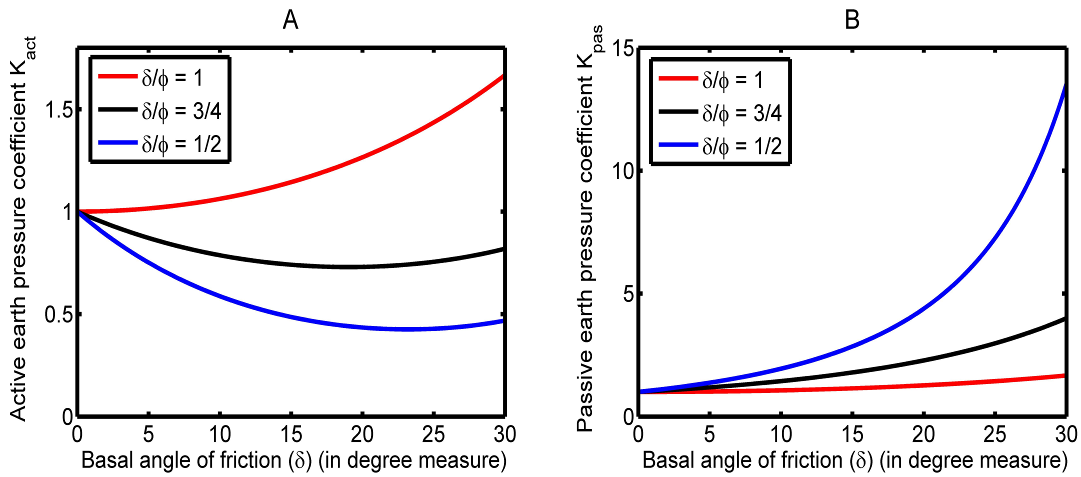

FIGURE 6. **A:** Active earth pressure and **B:** passive earth pressure coefficients, depending on basal angle that constantly varies to internal angle of friction.

1.7 and 1 to 3.99, respectively. But it rapidly increases from 1 to 13.5 when $\frac{\delta}{\phi} = \frac{1}{2}$. As the ratio $\frac{\delta}{\phi}$ increases, the active pressure also increases and passive pressure decreases for the increasing basal friction angles. When internal and basal friction angles are equal, active and passive pressure coefficients are the same.

3.4. **Earth pressure coefficients as functions of both basal angle and internal angle of friction.** Figure 7 is the 3-D representation of the values of earth pressure coefficients as functions of both basal angle and internal angles of friction. Two dimensional graph revealed that K_{act} is maximum when $\delta = \phi$, and this result can also be seen in Fig. 7A. Within the entire domain $0 \leq \phi \leq 60°$, $0 \leq \delta \leq 60°$, maximum of K_{act} is 7 when $\delta = \phi = 60°$. Similarly, the maximum value of K_{pas} is 14 at $\delta = 0°$, $\phi = 60°$ and minimum value is 1 at $\delta = \phi = 0°$ which can also be in observed in Fig. 5(B). When the internal

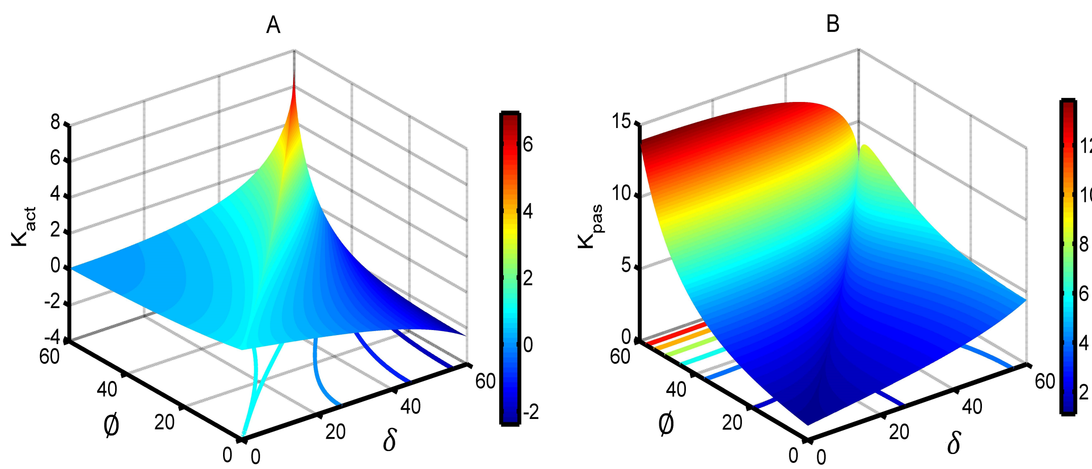

FIGURE 7. Three dimensional plot for **A:** Active earth pressure coefficients, **B:** Passive earth pressure coefficients depending on internal angle of friction and basal angle. The friction angles are measured in degree.

friction angle is near about 60°, active pressure coefficient has the smaller value whereas the passive coefficient has the larger values.

3.5. **Changes in earth coefficients in soil mechanics.** In soil mechanics, the earth pressure coefficient is the ratio of horizontal stress to the vertical stress. These stresses strongly depend on the strength of the soil which is determined by the angle of internal resistance (strength of the soil). When the retaining wall moves away from the soil, vertical stress dominates the horizontal stress and K_{act} becomes less than 1 that can be seen in Fig. 8(A). As ϕ increases, K_{act} decreases and K_{pas} increases because higher internal friction angle makes the soil stronger and more compressive.

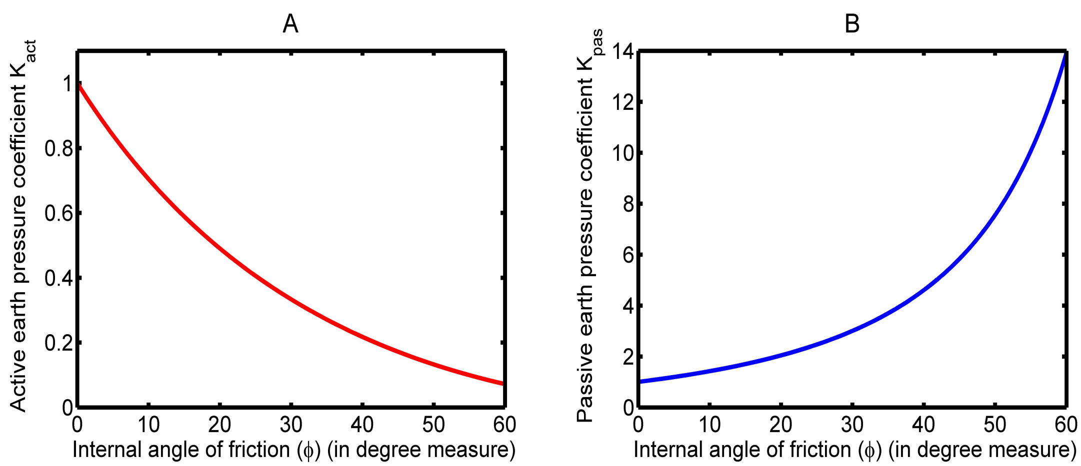

FIGURE 8. **A:** Active earth pressure coefficients **B** passive earth pressure coefficients in soil mechanics depending on internal friction angle.

4. CONCLUSION

The catastrophic mass flows like snow and rock avalanches, heavy landslides and debris flow show variable flow behaviour. Earth pressure coefficient determines whether pattern of the flow is expansive or compressive. Active earth pressure coefficient refers that the flow is divergent affecting the larger area whereas passive earth pressure coefficient describes that the flow is convergent affecting the smaller area in the sliding zone. The derived mathematical relation shows that these coefficients depend on the material response like friction between the grains and friction with the surface in the granular mass flows. The earth coefficient is not effective if basal friction angle exceeds the internal friction angle. The earth coefficient is an isotrophic pressure coefficient if there is no internal friction or basal friction angles. The evaluation of active and passive earth pressure coefficients for the different combinations of internal and basal frictional angles within the domain $0 \leq \phi \leq 60°$, $0 \leq \delta \leq 60°$, maximum of K_{act} is 7 when $\delta = \phi = 60°$. Similarly, the maximum value of K_{pas} is 14 at $\delta = 0°$, $\phi = 60°$ and minimum value is 1 at $\delta = \phi = 0°$. The results justify the direct variation of K_{pas} with internal angle of friction and inverse variation with basal angle of friction. On contrary, K_{act} varies directly with basal angle of friction but inversely with internal angle of friction. Moreover, $K_{act} = K_{pas}$ when $\delta = \phi$, and the values of the coefficients differ more along with the deviations of δ and ϕ from each other. As in soil mechanics, there is a significant increase or decrease in the coefficients when ϕ exceeds 45^0. The earth pressure coefficients are very important in the soil mechanics for the geotechnical stability in engineering structures. The active earth pressure tends the wall to move away from the backfill and K_{act} decreases as the internal frictions of the soil increases. On contrary, the passive earth pressure tends to move the wall towards the backfill and K_{pas} increases as the internal friction of the soil increases.

References

[1] A.F. Abdul, *A Direct Method for the Determination of Earth Pressures on Retaining Walls*, PhD Dissertation, Oklahoma State University Stillwater, Oklahoma, 1966

[2] A.L. Bell, *The lateral pressure and resistance of clay and the supporting power of clay foundations*, *Minutes Proc. Inst. Civ. Eng.*, Vol. 199, pp 233-272, 1915

[3] C.A. Coulomb, An attempt to apply the rules of maxima and minima to several problems of stability related to architecture, *Mémoires l'Académie*, Royale Des Sciences, pp 343-382, 1776

[4] J. Kafle, *Dynamic Interaction Between a Two-phase Submarine Landslide and a Fluid Reservoir*, MPhil Dissertation, School of Science, Kathmandu University, Kavre, Nepal, 2014

[5] J. Kafle, P.R. Pokhrel, K.B. Khattri, P. Kattel, B.M. Tuladhar, S.P. Pudasaini, Submarine landslide and particle transport in mountain lakes, reservoirs and hydraulic plants, *Annals of Glaciology*, Vol. 57(71), pp 232-244, 2016

[6] J. Kafle, P. Kattel, M. Mergili, J.T. Fischer, S.P. Pudasaini, Dynamic response of submarine obstacles to two-phase landslide and tsunami impact on reservoirs, *Acta Mechanica*, Vol. 230(9), pp 3143-3169, 2019

[7] J. Kafle, P. Kattel, P.R. Pokhrel, K.B. Khattri, Numerical experiments on effect of topographical slope changes in the dynamics of landslide generated water waves and submarine mass flows, *J. Appl. Fluid Mech.*, Vol. 14(3), pp 861-876, 2016

[8] J. Kafle, G. Acharya, P. Kattel, P.R.Pokhrel, Impact of variation of size of the initial release mass in the dynamics of landslide generated tsunami, *International Journal of Modeling, Simulation, and Scientific Computing*, Vol. 13(5), 2022

[9] J. Kafle, B.R. Dangol, C.N. Tiwari, P. Kattel, Dynamics of landslide-generated tsunamis and their dependence on the particle concentration of initial release mass, *European Journal of Mechanics - B/Fluids*, Vol. 97, pp 146-161, 2023

[10] P. Kattel, Dynamics of Quasi-Three-Dimensional and Two-Phase Mass Flows, *MPhil Dissertation*, School of Science, Kathmandu University, Kavre, Nepal, 2014

[11] P. Kattel, K.B. Khattri, P.R. Pokhrel, J. Kafle, B.M. Tuladhar, S.P. Pudasaini, Simulating glacial lake outburst floods with a two-phase mass flow model, *Annals of Glaciology*, Vol. 57(71), pp 349-358, 2016

[12] P. Kattel, J. Kafle, J.T. Fischer, M. Mergili, B.M. Tuladhar, S.P. Pudasaini, Interaction of two-phase debris flow with obstacles, *Engineering Geology*, Vol. 242, pp 197-217, 2018

[13] J.R. Keaton, H. Ponnaboyina, Selection of geotechnical parameters using the statistics of small samples, *Georgia geotechnical special publication*, Vol. 234, pp 1532-1541, 2014

[14] Y. Levesque, A. Saeidi, A. Rouleau, An earth pressure coefficient based on the geomechanical and geometric parameters of backfill in a mine stope, *Geo-Engineering*, Vol. 27(8), 2017

[15] L. Li, M. Aubertin, Numerical investigation of the stress state in inclined backflled stopes, *Int J Geomech*, Vol. 27, pp 52-62, 2009

[16] A. Marston, The theory of external loads on closed conduits in the light of the latest experiments, *Highway research board proceedings*, Vol. 96, 1930

[17] P. Mayne, F. Kulhawy, The coefficient of earth pressure at rest, *Canadian Geotechnical Journal*, Vol. 31, pp 788-790, 2011

[18] M. Pirulli, M.O. Bristeau, A. Mangeney, C. Scavia, The effect of the earth pressure coefficients on the runout of granular material, *Environmental Modelling and Software*, Vol. 22, pp 1437-1454, 2007

[19] S.P. Pudasaini, K. Hutter, *Avalanche Dynamics: Dynamics of Rapid Flows of Dense Granular Avalanches*, Springer, Berlin, New York, 2007

[20] W.J.M. Rankine, On the stability of loose earth, *Philos. Trans. R. Soc. Lond.*, Vol. 147, pp 9-27, 1857

[21] S.B. Savage, K. Hutter, The motion of a finite mass of granular material down a rough incline, *J. Fluid Mech.*, Vol. 199, pp 177-215, 1989

[22] Z. Yan, Y. Deng, J. He, Y. Xuan, W. Wu, A pseudodynamic approach of seismic active pressure on retaining walls based on a curved rupture surface, *Mathematical Problems in Engineering*, Vol. 5, pp 1-8, 2020

[23] X.L.Yang, J.H. Yin, Estimation of seismic passive earth pressures with nonlinear failure criterion, *Engineering Structures*, Vol. 28(3), pp 342-348, 2006

[24] M. Yang, B. Deng, Simplified method for calculating the active earth pressure on retaining walls of narrow backfill width based on DEM analysis, *Advances in Civil Engineering*, Vol. 2, pp 1-12, 2019