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Short Communication

Analysis of flow parameters in blood flow through mild stenosis

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Abstract

A buildup of plaque that contracts arteries and decreases blood flow to the heart causes chest pain, difficulties in breathing, or another coronary artery disease, medically called stenosis puts our lives at risk. We have used Navier-Stokes equations in a cylindrical polar coordinate system to study this problem by considering the flow is steady, axially symmetrical, fully developed, and laminar. Flow parameters like velocity profile, pressure drop, shear stress, and volumetric flow rate in the stenosed regions are analyzed after getting analytical solutions. Results revealed that the pressure drop decreases with the increased thickness of stenosis. The volumetric flux decreases highly as the viscosity increases. The velocity of blood flow decreases exponentially for a small increment of viscosity. Shear stress increases with indices of the power law of shear rate.

Keywords: Arterial stenosis; Pressure drop; Viscosity of blood; Velocity profile; Volumetric flow rate

1 | Introduction

The human circulatory system is a closed cardiovascular network of arteries, veins, capillaries and the blood. The flow behavior of blood in arteries and veins is complicated since it comprises deformable solid particles including red blood cells, white blood cells, platelets, and plasma, a viscous fluid (Kafle et al. 2022; Pokharel et al. 2020). Arteries are the vascular vessels that transport oxygen-rich blood to every cell of the body, hence cardiovascular network plays a vital role in sustaining life (Charon and Kurland 1965; Kafle et al. 2022; Pokharel et al. 2020). Deformability of erythrocytes, percentage of hematocrit, wall shear stress, and viscosity of the blood are key variables affecting the blood flow in arteries (Jain et al. 2010). Through experimental research, it has been established that blood is non-Newtonian fluid under low shear rates and arteries with small radii (Mandal & Chakravarty 2007; Young 1968).

Deposition of cholesterol, fatty particles, and other foreign particles form atherosclerotic plaque called stenosis, and abnormal tissue development is also a cause of stenosis (Nosovitsky et al. 1997). There are various types of stenosis such as symmetrical, triangular, trapezoidal, bell-shaped and composite (Ponalagusamy & Manchi 2020). The role of each of this stenosis is to obstruct the blood flow and reduce the blood supply bringing problems to the blood flow system (Young 1968). The activation of platelets by extremely high shear stress close to the stenosis neck might completely obstruct the passage of blood to the heart, such type of atherosclerosis breaks the cardiovascular system (Singh 2012). To know the blood flow system, we should investigate the blood flow through different forms of stenosis (Chakravarty 1987). Fluid dynamical quantities play a crucial role when the stenosis continues to worsen, and blood flow is dramatically altered, eventually leading to the development of cardiovascular diseases (Charon & Kurland 1965). The major effects of stenosis are ischemia, atherosclerosis, angina pectoris, heart attacks, strokes, and cerebral strokes (Forrester & Young 1970).

Erythrocytes deposits in the area of low shear stress in the form of rouleaux which narrows the lumen and it causes blood to have a non-zero yield stress (Chaturani & Ponalagusamy 1985).

The effect of constriction on blood flow properties like velocity, viscosity, and resistance has been studied both theoretically and empirically by Chaturani and Ponalagusamy (1985). Singh (2012) assumed a mild stenosis is radially non-symmetrical. When they used single loop of stenosis with maximal depression at various places, they ran a graphical analysis and show that the flow resistance decreased as the shape parameter increased (Srivastava 2002). An approximate equation for projecting the pressure drop in region of stenosis is established and empirically verified by Young and Tasi (1973). Seeley and Young (1976) have studied pressure drop in the region of multiple stenosis by taking two blunt plugs.

2 | Materials and methods

Taking into consideration, steady flow of blood in an axially symmetrical stenosed artery, let *Ro* and *r* denotes the radius of artery without stenosis and with stenosis, respectively. The artery is considered an inelastic circular tube. The radial blood flow is neglected in light of stenosis. The blood is considered to flow along axial direction only.

2.1 | Model equation

Let $u\ (x, y, z, t)$, $v\ (x, y, z, t)$, $w\ (x, y, z, t)$ and $p\ (x, y, z, t)$ be the three components of velocity and pressure at (x, y, z) and at time t respectively. The continuity equation in a steady-state form is (Gaire 2021; Kapur 1985)

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \quad (1)$$

Here $\rho\ (x, y, z, t)$ is the density of a fluid. The density ρ of an incompressible viscous fluid is constant. The N-S equations for Newtonian, viscous and incompressible fluid are (Gaire 20021; Kapur 1985).

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=\rho g_x-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right) \quad (2)$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=\rho g_y-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right) \quad (3)$$

$$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=\rho g_z-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right) \quad (4)$$

Here t is the time, $f = \rho(g_x, g_y, g_z)$ are the external body forces, mainly gravity. The terms $\partial p/\partial x$, $\partial p/\partial y$, $\partial p/\partial z$ are forces due to differences in pressure and the last term on the right are viscous forces with constant viscosity coefficient μ in x, y and z- direction respectively. The system (1)-(4) is closed for four unknown functions u, v, w and p. When there are no external body forces, i.e., $g_x = 0$, $g_y = 0$ and $g_z = 0$ and the motion is steady, i.e., $\partial u/\partial t = 0$, $\partial v/\partial t = 0$, $\partial w/\partial t = 0$, and $\partial p/\partial t = 0$, and the motion is in two dimension.

By using Navier-Stokes equations for fluid flow inside a cylinder, blood flow in arteries can be modeled. Let r and p denote the radius and pressure drop of the blood flow in the artery. Component of the velocities along radial, angular and axial directions are v^r, v^θ and v^z respectively. The system (1)-(4) can be expressed in cylindrical form as (Gaire 2021; Kapur 1985)

$$\frac{1}{r}\frac{\partial}{\partial r}(rv^r)+\frac{\partial}{\partial z}(v^z)=0 \quad (5)$$

$$\rho\left(\frac{\partial v^r}{\partial t}+v^r\frac{\partial v^r}{\partial r}+v^z\frac{\partial v^r}{\partial z}\right)=-\frac{\partial p}{\partial r}+\mu\left(\frac{\partial^2 v^r}{\partial r^2}+\frac{\partial^2 v^r}{\partial z^2}+\frac{1}{r}\frac{\partial v^r}{\partial r}-\frac{v^r}{r^2}\right) \quad (6)$$

$$\rho\left(\frac{\partial v^z}{\partial t}+v^r\frac{\partial v^z}{\partial r}+v^z\frac{\partial v^z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 v^z}{\partial r^2}+\frac{\partial^2 v^z}{\partial z^2}+\frac{1}{r}\frac{\partial v^z}{\partial r}\right) \quad (7)$$

In axisymmetric flow, we assume $v^\theta = 0$, and v^r, v^z, and p are independent of θ, viscosity μ and density ρ are assumed constant for the steady flow of blood. Velocity component parallel to z- axis is v. We have considered axially symmetric flow along z-axis only, so $v^r = 0$, $v^\theta = 0$, and $v^z = v$, then equations (6) - (7) become

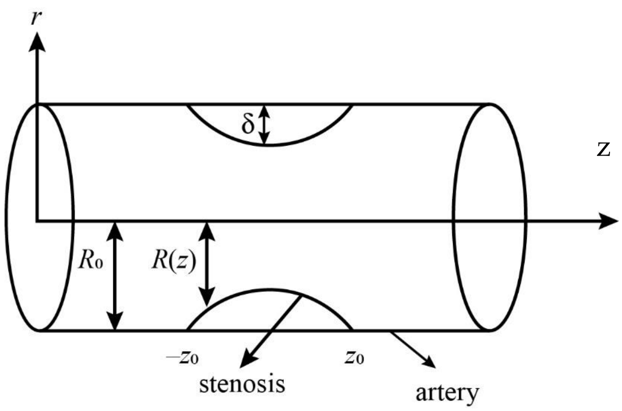

Figure 1. Representative sketch for artery and stenosis.

$$\frac{\partial v}{\partial z}=0,\ 0=-\frac{\partial p}{\partial r}\ ,\quad 0=-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 v}{\partial r^2}+\frac{\partial^2 v}{\partial z^2}+\frac{1}{r}\frac{\partial v}{\partial r}\right) \quad (8)$$

Suppose pressure term as $P = -\frac{\partial p}{\partial z}$, equation (8) reduces to

$$-P\frac{r}{\mu} = \frac{\partial}{\partial r}\left(r\frac{\partial v}{\partial r}\right) \quad (9)$$

2.3 | Geometry of stenosis

The geometry of the stenosis developed is considered as a plaque or abnormal tissue buildup in the inner wall of a cylindrical shaped artery is depicted in Fig. 1 where, the ratio of the radii with and without stenosis is modeled as

$$\frac{R}{R_0} = 1 - \frac{\delta}{2R_0}\left(1 + \cos\frac{\pi z}{z_0}\right) \quad (10)$$

where R is artery radius with stenosis, R_O is radius without stenosis and δ is height of the stenosis.

The boundary condition according to Kapur (1985) is

$$v = \begin{cases} 0 & at \quad r = R\ , \quad -z_0 \le z \le z_0 \\ 0 & at \quad r = R_0\ , \quad |z| > z_0 \end{cases}$$

Integrating (9) with respect to r taking z constant $r\frac{\partial v}{\partial r} = -P\frac{r^2}{2\mu} + C(z)$. (11)

Using $\partial v/\partial r = 0$ at $r = 0$, gives $C(z) = 0$. Integrating (11) and using the boundary condition we get

$$v(r) = -\frac{Pr^2}{4\mu} + D(z)$$

Using $v = 0$ at $r = R$, gives $D(z) = PR^2/4\mu$, then the velocity is $v = \frac{P}{4\mu}(R^2 - r^2)$

The volumetric flow rate through the cylindrical tube can be obtained by (Kapur 1985)

$$Q = \int_0^R 2\pi r v dr = \frac{2P\pi}{4\mu}\int_0^R (rR^2 - r^3)dr = \frac{P\pi}{2\mu}\left[\frac{r^2R^2}{2} - \frac{r^4}{4}\right]_0^R = \frac{\pi P}{8\mu}R^4$$

2.3 | Pressure drop

The volumetric flow rate Q (Kapur 1985)

$$Q = \int_0^R 2\pi r . v dr = \frac{n\pi}{3n+1}\left(\frac{P}{2\mu}\right)^{1/n} R^{\frac{1}{n}+3} \quad (12)$$

From (12), $P = 2\mu Q^n\left(\frac{3n+1}{n\pi}\right)^n \frac{1}{R^{3n+1}}$ (13)

The pressure drop in the stenosed part is obtained after integrating (13) as

$$\Delta P = \int_{-z_0}^{z_0} P dz = \int_{-z_0}^{z_0} 2\mu Q^n\left(\frac{3n+1}{n\pi}\right)^n \frac{1}{R^{3n+1}} dz$$

For mild stenosis, we have used the radial surface from (10), and the pressure drop across the length (Kapur 1985), then

$$\Delta P = \int_{-z_0}^{z_0}\left(\frac{3n+1}{n\pi}\right)^n \frac{2\mu Q^n dz}{{R_0}^{3n+1}\left(a - b\cos\frac{\pi z}{z_0}\right)^{3n+1}} \quad (14)$$

where $a = 1 - \frac{\delta}{2R_0}$, $b = \frac{\delta}{2R_0}$. Putting $\frac{\pi z}{z_0} = u$, so that $\pi\, dz = z_0\, du$. When $z = -z_0$, $u = -\pi$, when $z = z_0$, $u = \pi$. The equation (18) reduces to

$$\Delta P = \frac{4\mu z_0 Q^n}{\pi {R_0}^{3n+1}}\left(\frac{3n+1}{n\pi}\right)^n \int_0^\pi \frac{du}{(a - b\cos u)^{3n+1}}$$

When there is no stenosis, $\delta = 0$, but $f(\delta/R_0) = 1$, then the pressure drop in the stenosed part becomes

$$(\Delta P)_P = \frac{4\mu z_0 Q^n}{{R_0}^{3n+1}}\left(\frac{3n+1}{n\pi}\right)^n.$$

2.4 Pressure drop ratio

The ratio of pressure with and without stenosis is (Kapur 1985)

$$\frac{\Delta P}{(\Delta P)_P} = \frac{1}{\pi}\int_0^\pi \frac{du}{(a - b\ \cos\ u)^{3n+1}} \quad (15)$$

For Newtonian fluid (n = 1), using Leibnitz's rule equation (15) reduced to

(16)

2.5 | Shear stress

$$\frac{\Delta P}{(\Delta P)_P} = \frac{1}{\pi}\int_0^\pi \frac{du}{(a - b\ \cos\ u)^4}$$
$$= \left(1 - \frac{\delta}{2R_0}\right)\left(1 - \frac{\delta}{R_0} + \frac{5}{8}\frac{\delta^2}{{R_0}^2}\right)\left(1 - \frac{\delta}{R_0}\right)^{-7/2}$$

The shear stress on the stenosis surface is obtained from (Kapur 1985)

$$\tau = \frac{1}{2}PR = \frac{1}{2}\ 2\mu Q^n\left(\frac{3n+1}{n\pi}\right)^n \frac{R}{R^{3n+1}}$$
$$= \mu Q^n\left(\frac{3n+1}{n\pi}\right)^n \frac{1}{R^{3n}}$$

When $\delta = 0$, there is no stenosis, so that $f\left(\frac{\delta}{R_0}\right) = 1$. The shear stress in absence of stenosis is

$$\tau_P = \mu Q^n\left(\frac{3n+1}{n\pi}\right)^n \frac{1}{{R_0}^{3n}\left(1 - \frac{\delta}{2R_0} - \frac{\delta}{2R_0}\cos\ \frac{\pi z}{z_0}\right)^{3n}}$$
$$= \mu Q^n\left(\frac{3n+1}{n\pi}\right)^n \frac{1}{{R_0}^{3n}}$$

2.6 | Shear stress ratio

The ratio of shear stress with and without stenosis is (Kapur 1985)

$$\frac{\tau}{\tau_P} = \left(\frac{R_0}{R(z)}\right)^{3n} = \frac{1}{\left(1 - \frac{\delta}{2R_0} - \frac{\delta}{2R_0}\ \cos\ \frac{\pi z}{z_0}\right)^{3n}}$$

Ratio of maximum and minimum stress is

$$\frac{\tau_{max}}{\tau_{min}} = \frac{1}{\left(1 - \frac{\delta}{2R_0} - \frac{\delta}{2R_0}\cos 0\right)^{3n}} = \left(1 - \frac{\delta}{R_0}\right)^{-3n} \quad (17)$$

For Newtonian fluid (n = 1) and $\frac{\delta}{R_0} \ll 1$, by using Maclaurin expansion (17)

$$\frac{\tau_{max}}{\tau_{min}} = \left(1 - \frac{\delta}{R_0}\right)^{-3} = 1 + \frac{3\delta}{R_0} \quad (18)$$

3 | Results and discussion

The present model has been developed to analyze blood flow parameters in an artery with mild stenosis along axial velocity flow. Analytical solutions of the velocity profile, volumetric flow rate, pressure drop, shear stress and effective viscosity of the blood flowing in an artery with mild stenosis are obtained and analyzed. The following simulated result for different blood flow parameter are obtained by using computational software.

3.1 | Velocity profile of blood flow through a stenotic artery

Figure 2 shows that the viscosity affects the velocity of blood flow for the artery having radius 3 mm. Velocity decreases rapidly, when viscosity increases by a small quantity, which can be seen clearly from the figure. For the viscosities (μ) are 0.2 gm mm^{-1}s^{-1}, 0.4 gm mm^{-1}s^{-1}, 0.6 gm mm^{-1}s^{-1}, and 0.8 gm mm^{-1}s^{-1}, the approximate values of velocities are 55 mms^{-1}, 28 mms^{-1}, 18 mms^{-1}, and 12 mms^{-1} respectively. This simulation result shows that the velocity decreases as the viscosity increases and conversely. Near the inner wall of the artery the velocity is approximately zero, which increases gradually when we move towards center. This increment is more rapid when the viscosity changes from 0.2 gram/ (mm s) to 0.4 gram/ (mm s).

3.2 | Volumetric flow rate on stenosis of blood flow through a stenotic artery

Figure 3A shows the effect of viscosity on volumetric flow rate. This simulation result shows that the viscosity and volumetric flow rate hold inverse relation. When viscosity values are (μ) = 0.2 gm mm^{-1}s^{-1}, 0.4 gm mm^{-1}s^{-1}, 0.6 gm mm^{-1}s^{-1}, 0.8 gm mm^{-1}s^{-1}, approximate values of the volumetric flow are 800 mm^3/s, 400 mm^3/s, 270 mm^3/s, 200 mm^3/s, respectively. We can understand

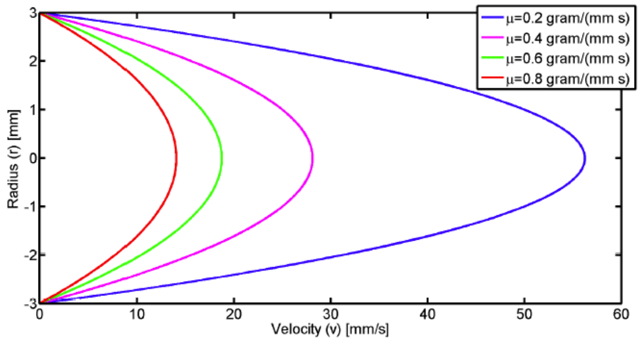

Figure 2. Velocity profile of blood flow through a stenotic artery for different viscosity.

from the figure that volumetric flow becomes half when we double the viscosity. The conclusion is that more viscous fluid flows slowly and less volume passes at the same time comparing with the fluid with less viscosity.

3.3 | Pressure gradient across the stenosis of blood flow through a stenotic artery

Figure 3B shows, the ratio of pressure drop against thickness of the stenosis for different radii. For radii (R_0) = 2.0 mm, 2.5 mm, 3.0 mm, 3.5 mm the pressure drop is 5.5 mm of Hg, 3.3 mm of Hg, 2.5 mm of Hg and 2.1 mm of Hg respectively. As the radius R_0 increases from 2.0 mm to 3.5 mm, the pressure drop increases from 2.1 mm of Hg to 5.5 mm of Hg. From this figure we conclude

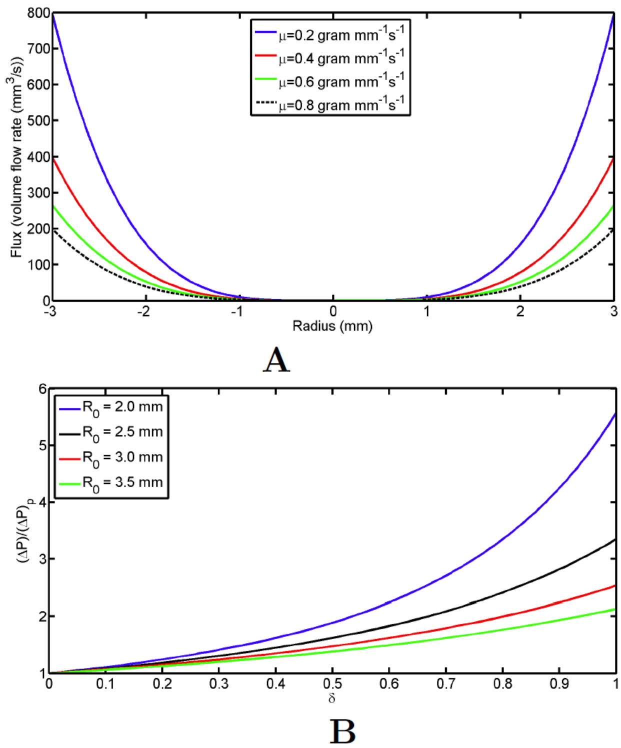

Figure 3. A- Volumetric flow rate for different value of viscosity, B- Pressure drop with the variation of the height of the stenosis.

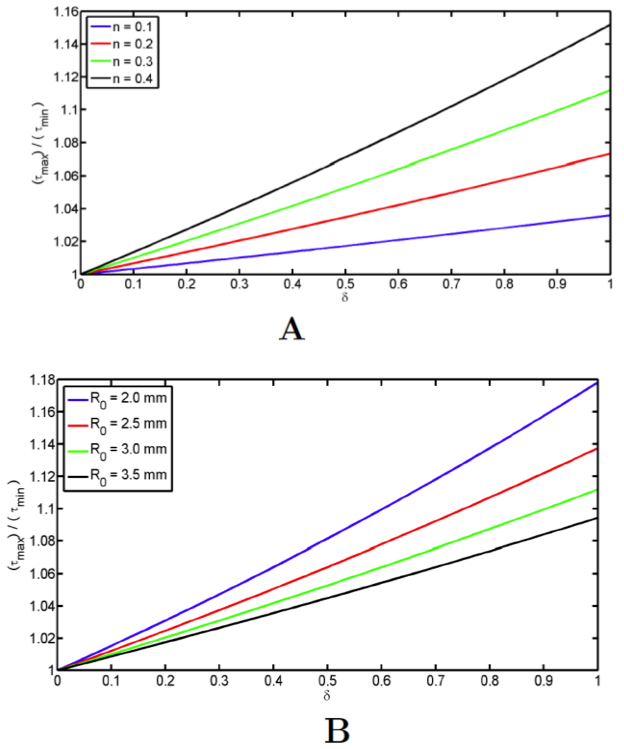

Figure 4. Relation between the shear stress ratio and height of the stenosis for different values of **A**- flow behavior index (n), **B**- radius of the artery (R_0).

that the pressure drop ratio increase more quickly for the narrower artery.

Figure 4A shows, for various values on n (flow behavior index), the ratio of the maximum shear stress to the minimum shear stress. The ratio is directly proportional with the thickness of stenosis. When the thickness of the stenosis increases from 0 to 1mm the ratio increases from 1 to 1.15. As the flow behavior index (n) increases gradually from 0.1 to 0.4, the ratio increases 1.035 to 1.15 approximately.

Figure 4B shows, the relation between the ratios of maximum to minimum shear stresses with the thickness of the stenosis for different values of radii. The ratio of shear stresses increases with the increase of stenosis. The increment of the ratio is linear against the thickness of the stenosis. For the increment of the radii (R_0) = 2.0 mm to 3.5 mm, the ratio of maximum to minimum shear stress increases 1.09 to 1.18.

5 | Conclusions

Stenosis in an artery generally occurs due to the aggregation of cholesterol-laden plaque in its walls resulting in a constricting of the passage of blood as well as a loss of elasticity that leads to stroke and heart attack. The present mathematical analysis brings out many interesting results in the blood flow parameters. We have used the cylindrical polar form of the Navier-Stokes equations to simulate blood flow through the artery. We have analyzed the velocity profile, pressure drop, shear stress, and volumetric flow rate in an artery with mild stenosis. Various heights of stenosis are considered to show its effect on the above-mentioned parameters. The pressure drop decreases with the increased thickness of stenosis. The volumetric flux decreases highly as the viscosity increases. The velocity of blood flow decreases exponentially for a small increment of viscosity. Shear stress increases with indices of the power law of shear rate. Our study can be an applicable better understanding of hemodynamic abnormalities due to mild stenosis and further specialization and upgrading highly-sensitive tools.

Authors' contributions

All authors contributed equally to the manuscript.

Conflicts of interest

Authors declare no conflict of interest.

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