*Nepal Journal of Mathematical Sciences (NJMS)*
*ISSN*: **2738-9928 (*online*), 2738-9812 (*print*)**
*Vol. 5, No. 2, 2024 (August): 13-20*
*DOI:* https://doi.org/10.3126/njmathsci.v5i2.76445


*Research Article*
*Received Date: December 10, 2024*
*Accepted Date: February 21, 2025*
*Published Date: March 16, 2025*

# On Difference Sequence Space $\ell_M(X, \alpha, P)$ Defined by Orlicz function

**Prem Prakash Kaphle[1], Gyan Prasad Paudel[2*] & Purushottam Parajuli[3]**

[1]*Department of Mathematics, Tribhuvan University, Tribhuvan Multiple Campus, Palpa, Nepal*
[1]*Central Campus of Science and Technology, Mid-Western University, Surkhet, Nepal*
[3]*Department of Mathematics, Tribhuvan University, Prithvi Narayan Campus, Pokhara, Nepal*

Corresponding Author: *gyan.math725114@gmail.com

**Abstract:** *The idea of difference sequence space is introduced by Kizmaz. Lindenstrauss and Tzafriri used the concept of Orlicz function M to construct the sequence space $\ell_M$. In this paper, we introduce the class $\ell_M(X, \alpha, P)$ of the generalized form of difference sequence space and study some inclusion and linear properties.*

**Keywords:** Sequence space, Difference sequence space, Orlicz function, Banach spaces.

## 1. Introduction:

Sequence spaces have a major role in mathematics, especially in functional analysis and mathematical analysis. They give researchers a framework for examining infinite numerical sequences and provide information on their convergence, summability, and other characteristics. Mathematicians have extensively researched sequence spaces, which are vector spaces of sequences with elements in real or complex numbers ($\mathbb{R}$ or $\mathbb{C}$). Many mathematicians in classical analysis have investigated these spaces, which include convergence, bounded, null, and $l_p$ spaces.

If ω denotes the set of all functions from the set of positive integers $\mathbb{N}$ to the field $\mathbb{K}$ of real or complex numbers then it becomes a vector space. Any Sequence space is defined as a set of all sequences $x = (x_n)$ linear subspace of ω over the field $\mathbb{C}$ with the usual operations defined as

$$(x_n) + (y_n) = (x_n + y_n) \text{ and } \lambda(x_n) = (\lambda x_n).$$

Several researchers, including Kamthan and Gupta [4] (1980), Maddox [7] (1981), Ruckle [15 ] (1981), and Malkowski and Rakocevic[8] (2004), have made substantial contributions to the theory of vector and scalar valued sequence spaces using Banach sequences. Similarly, Orlicz used the idea of Orlicz function to construct the space ( $L^M$). Lindentrauss and Tzafriri [6] investigated Orlicz sequence spaces in more detail, and proved that every Orlicz sequence space $\ell_M$ contains a subspace isomorphic to $\ell_p (1 \leq p < \infty)$.

Subsequently different classes of sequence spaces defined by Murasaleen et al.[9], Parashar and Choudhary [10], Subramanian [17 ] , Tripathy et al. [18] , and many others are studied.

A function $M: [0, \infty\ ) \longrightarrow [0, \infty\ )$ is called Orlicz function if it is continuous, non-decreasing, and convex with

$$M(0) = 0, M(t) > 0 \text{ for } t > 0 \text{ and } M(t) \rightarrow \infty \ as\ t\ \rightarrow \infty.$$

If the convexity of Orlicz function $M$ is replaced by $M(t+u) \leq M(t) + M(u)$ then this function is called modulus function.

In 1971, Lindenstrauss and Tzafriri[ 6] used the Orlicz function to construct the following class

$$\ell_M = \{\bar{x} = (x_k) \in \omega\colon \textstyle\sum_{k=1}^{\infty} M(\frac{|x_k|}{\rho}) < \infty \text{ for some } \rho > 0\}.$$

This class together with the norm defined by

$$\|\bar{x}\| = \inf\{\rho > 0 : \sum_{k=1}^{\infty} M(\frac{|x_k|}{\rho}) \leq 1\}$$

forms a Banach space called an Orlicz sequence space and is related to the sequence space $\ell_p$ with

$$M(t) = t^p, 1 \leq p < \infty.$$

The concept of difference sequence spaces was first introduced by Kizmaz[5] in 1981 and defined as

i. $c_0(\Delta(\mathrm{X})) = \{\ \bar{x} = (x_k)\ \in X : \ \|\Delta x_k\|\ \rightarrow 0\ as\ k\ \rightarrow \infty\}$

ii. $c(\Delta(\mathrm{X})) = \{\ \bar{x} = (x_k)\ \in X : \ \exists\ l \in X\ s.t.\ \|\Delta x_k - l\|\ \rightarrow 0\ as\ k\ \rightarrow \infty\}$

iii. $\ell_\infty(\Delta(\mathrm{X})) = \{\ \bar{x} = (x_k)\ \in X : \ sup_k\|\Delta x_k\| < \infty\}$

iv. $\ell_p(\Delta(\mathrm{X})) = \{\ \bar{x} = (x_k)\ \in X : \ \sum_{k=1}^{\infty}\|\Delta x_k\|^p < \infty, 0 < p < \ \infty\}$

where $\Delta x_k = x_k - \ x_{k-1}$ .

These spaces are Banach spaces with norm $\|x\|_\Delta\ =\ |x_1| +\ \|\Delta x\|_\infty$ .

Et[2], in 1993, generalized the concept of Kizmaz to study the $\Delta^2$ sequence spaces of Banach space $X$-valued sequences defined as follows:

i. $c_0(\Delta^2(X)) = \{\ \bar{x} = (x_k)\ \in X : \ \|\Delta^2 x_k\|\ \rightarrow 0\ as\ k\ \rightarrow \infty\}$

ii. $c(\Delta^2(\mathrm{X})) = \{\ \bar{x} = (x_k)\ \in X : \ \exists\ l \in X\ s.t.\ \|\Delta^2 x_k - l\|\ \rightarrow 0\ as\ k\ \rightarrow \infty\}$

iii. $\ell_\infty(\Delta^2(\mathrm{X})) = \{\ \bar{x} = (x_k)\ \in X : \ sup_k\|\Delta^2\, x_k\| < \infty\}$

where

$$\Delta^2\, x_k\ = \Delta x_k \text{ - } \Delta x_{k-1}\ = x_k -\ x_{k-1} - x_{k-1} -\ x_{k-2}\ =\ x_k -\ 2x_{k-1} + x_{k-2}.$$

These space are Banach spaces with the norm defined by

$$\|x\|_\Delta\ =\ |x_1| +\ |x_2| + \|\Delta^2 x\|_\infty .$$

Similarly, in 1995, Et and Colak [3] defined the following classes

i. $c_0(\Delta^m(X)) = \{\ \bar{x} = (x_k)\ \in X : \ \|\Delta^m x_k\|\ \rightarrow 0\ as\ k\ \rightarrow \infty\}$

ii. $c(\Delta^m(\mathrm{X})) = \{\, \bar{x} = (x_k) \in X : \exists\, l \in X\; s.t.\; \|\Delta^m x_k - l\| \to 0 \; as\; k \to \infty\}$

iii. $\ell_\infty(\Delta^m(\mathrm{X})) = \{\, \bar{x} = (x_k) \in X : \; sup_k \|\Delta^m x_k\| < \infty\}$

where $m \in \mathbb{N}, \Delta^0 x = (x_k), \Delta x_k = x_k - x_{k-1}$,

and $\Delta^m x_k = \Delta^{m-1} x_k - \Delta^{m-1} x_{k-1} \;=\; \sum_{r=1}^{\infty}(-1)^r \binom{m}{r} x_{k+r}$ .

They showed that these classes are Banach spaces with norm defined as

$$\|x\|_\Delta = \sum_{r=1}^{m} |x_r| + \|\Delta^m x\|_\infty .$$

In 2006, Tripathy and Esi [18] studied a new type of difference sequence spaces $c(\Delta_m), c_0(\Delta_m), c(\Delta_m)$ where $m \in \mathbb{N}$ as defined by

$$Z(\Delta_m) = \{\, \bar{x} = (x_k) \in \omega : \; \Delta_m x \in Z\}, \quad \text{for } Z = \ell_\infty, c \text{ and } c_0$$

where, $\Delta_m x = (\Delta_m x_k) = (x_{k+m} - x_{k,})$for all $k \in \mathbb{N}$.

For $m$ =1, $\ell_\infty(\Delta_m) = \ell_\infty(\Delta)$ , $c(\Delta_m) = c(\Delta)$, $c_0(\Delta_m) = c_0(\Delta)$.

He proved that these spaces are Banach spaces with norm defined by

$$\|x\|_\Delta = \sum_{r=1}^{m} |x_r| + \sup_k |\Delta_m x_k|.$$

and the inclusion relations $c_0(\Delta_m) \subset c(\Delta_m) \subset \ell_\infty(\Delta_m)$

and $Z(\Delta) \subset Z(\Delta_m)$ for, $Z = \ell_\infty, c, c$ and $c_0$.

In 2012, Srivastava and Pahari [16] introduced the following class

$$\ell_M(X, \alpha, P) = \left\{ x = (x_k) \in \omega(X) : \sum_{k=1}^{\infty} M\left(\frac{\|\alpha_k x_k\|^{p_k}}{\rho}\right) < \infty \text{ for some } \rho > 0 \right\},$$

and

$$\ell_M(X, \alpha, P, L) = \left\{ x = (x_k) \in \omega(X) : \sum_{k=1}^{\infty} M\left(\frac{\|\alpha_k x_k\|^{p_k/L}}{\rho}\right) < \infty \text{ for some } \rho > 0 \right\},$$

where, $sup_k p_k = L < \infty$.

For $\alpha = (\alpha_k)$ for a sequence of nonzero complex numbers and $P = (p_k)$ for any sequence of strictly positive real numbers, they also investigated conditions for containment relation of $\ell_M(X, \alpha, P)$ and explore the linear topological structure of the class $\ell_M(X, \alpha, P, L)$.

Recently, in 2022 and 2023, Paudel, Pahari and et al. [11], [12] , [13] studied the various properties of sequence space using fuzzy concept. In 2023, Pokharel et al. [14] introduced a new class of double sequences in a normed space $X$ to generalized the well-known sequence space $\ell$ by introducing and studying a new class $\ell^2\,((X, \|\,.\,\|), \bar{\gamma}, \bar{w})$ of double sequences with their terms in a normed space $X$ focusing on exploring some of the preliminary results that characterize the linear topological structures.

**Lemma** [1]: Let $(p_k)$ be a bounded sequence of strictly positive real numbers with $0 < p_k \le \sup p_k = L,\ D = \max\{1,\ 2^{L-1}\}$ then

i. $|x + y|^{p_k} \le D\{|x|^{p_k} + |y|^{p_k}\}$;
ii. $|\alpha|^{p_k} \le \max(1,\ [\alpha]^L)$.

Throughout the article, we shall denote for $\alpha = (\alpha_k),\ \beta = (\beta_k)$

$$\gamma_k = \frac{\alpha_k}{\beta_k} \text{ and } \delta_k = \left|\frac{\alpha_k}{\beta_k}\right|^{p_k}.$$

On the basis of the literature mentioned above, here we define the following class

$$\ell_M(X, \alpha, P) = \left\{x = (x_k) \in \omega(X) : \sum_{k=1}^{\infty} M\left(\frac{\|\alpha_k\ \Delta_m\ x_k\|^{p_k}}{\eta}\right) < \infty \text{ for some } \eta > 0\right\}$$

where, $\Delta_m x = (\Delta_m x_k) = (x_{k+m} - x_{k,})$ for all $k \in \mathbb{N}$.

## 2. Main Results

In this section, we shall study some of the containment relations on the class $\ell_M(X, \alpha, P)$ for different values of $\alpha$ and $P$ and examine the linear structures of $\ell_M(X, \alpha, P)$.

**Lemma 2.1.** $\ell_M(X, \alpha, P) \subset \ell_M(X, \beta, P)$ if $\inf_k \delta_k > 0$.

Proof:

Let $\inf_k \delta_k > 0$ and let $x = (x_k) \in \ell_M(X, \alpha, P)$ then there exist $m_1 > 0, \eta > 0$ and positive integer K such that

$$M\ |\beta_k|^{p_k} < |\alpha_k|^{p_k}\ \forall\ k \ge K \text{ and } \sum_{k=K}^{\infty} M\left(\frac{\|\alpha_k \Delta_m x_k\|^{p_k}}{\eta}\right) < \infty.$$

Choose $\eta_1 > 0$ such that $\eta < m_1\ \eta_1$. Since $M$ is non-decreasing, we have

$$\begin{aligned}\sum_{k=K}^{\infty} M\left(\frac{\|\beta_k \Delta_m x_k\|^{p_k}}{\eta_1}\right) &= \sum_{k=K}^{\infty} M\left(\frac{|\beta_k|^{p_k}\|\Delta_m x_k\|^{p_k}}{\eta_1}\right) \\ &\le \sum_{k=K}^{\infty} M\left(\frac{|\alpha_k|^{p_k}\|\Delta_m x_k\|^{p_k}}{m_1\ \eta_1}\right) \\ &\le \sum_{k=K}^{\infty} M\left(\frac{\|\alpha_k \Delta_m x_k\|^{p_k}}{\eta}\right) \\ &< \infty\end{aligned}$$

which indicates that $x = (x_k) \in \ell_M(X, \beta, P)$. Thus

$$\ell_M(X, \alpha, P) \subset \ell_M(X, \beta, P).$$

□

**Lemma 2.2.** If $\ell_M(X,\alpha,P) \subset \ell_M(X,\beta,P)$ , then $\inf_k \delta_k > 0$.

Proof:

Assume that $\ell_M(X,\alpha,P) \subset \ell_M(X,\beta,P)$ holds but $inf_k \delta_k = 0$ so that we can find a sequence $(k(n))$ of integers such that $k(n+1) > k(n) \geq 1$ for which

$$n^2 \left|\alpha_{k(n)}\right|^{p_{k(n)}} < \left|\beta_{k(n)}\right|^{p_{k(n)}} \quad \forall n \geq 1.$$

Corresponding to $z \in X$, with $\|z\| = 1$ we can define a sequence $x = (x_k)$ by

$$\Delta_m x_k = \begin{cases} \alpha_{k(n)}^{-1} n^{-2/p_{k(n)}} & \text{for } k = k(n), n \geq 1 \\ 0, & \text{otherwise} \end{cases}$$

By using the convexity of $M$ , we have

$$\sum_{k=1}^{\infty} M\left(\frac{\|\alpha_k \Delta_m x_k\|^{p_k}}{\eta}\right) = \sum_{n=1}^{\infty} M\left(\frac{\left\|n^{-2/p_{k(n)}} z\right\|^{p_{k(n)}}}{\eta}\right) = \sum_{n=1}^{\infty} M\left(\frac{1}{n^2 \eta}\right)$$

$$\leq M\left(\frac{1}{\eta}\right) \sum_{n=1}^{\infty} \frac{1}{n^2} < \infty.$$

$\Longrightarrow x \in \ell_M(X,\alpha,P)$.

But on the other hand for any $\eta > 0$ we have

$$\sum_{k=1}^{\infty} M\left(\frac{\|\beta_k \Delta_m x_k\|^{p_k}}{\eta}\right) = \sum_{n=1}^{\infty} M\left(\frac{\left\|\beta_{k(n)} n^{-2/p_{k(n)}} z\right\|^{p_{k(n)}}}{\alpha_{k(n)} \eta}\right)$$

$$= \sum_{n=1}^{\infty} M\left(\left|\frac{\beta_{k(n)}}{\alpha_{k(n)}}\right|^{p_{k(n)}} \frac{1}{n^2 \eta}\right)$$

$$\geq \sum_{n=1}^{\infty} M\left(\frac{1}{\eta}\right) = \infty,$$

showing that $x \notin \ell_M(X,\beta,P)$ which is a contradiction. So we have $inf_k \delta_k > 0$.

□

**Theorem 2.3.** $\ell_M(X,\alpha,P) \subset \ell_M(X,\beta,P)$ if and only if $\inf_k \delta_k > 0$.

After combining Lemmas 2.1 and 2.2, the result follows:

**Theorem 2.4**. $\ell_M(X,\beta,P) \subset \ell_M(X,\alpha,P)$ if only if $\lim sup_k \delta_k < \infty$.

Proof:

Let $\lim sup_k \delta_k < \infty$. Then we can find a positive integer L such that $L|\beta_k|^{p_k} > |\alpha_k|^{p_k}$ for sufficiently large $k$. Then by using lemma 1 the result follows.

For the necessity, suppose that $\ell_M(X,\beta,P) \subset \ell_M(X,\alpha,P)$ but $\lim sup_k \delta_k = \infty$. we can find a sequence (k(n)) of integers such that $k(n+1) > k(n) \geq 1$ for which

$$\left|\frac{\alpha_{k(n)}}{\beta_{k(n)}}\right|^{p_{k(n)}} > n^2, \forall n \geq 1.$$

Now corresponding to $z \in X$, with $\|z\| = 1$, we can define a sequence $x = (x_k)$ by

$$\Delta_m x_k = \begin{cases} \beta_{k(n)}^{-1} n^{-2/p_{k(n)}} & \text{for } k = k(n), n \geq 1 \\ 0 & \text{otherwise} \end{cases}$$

By using convexity of $M$ , we have

$$\sum_{k=1}^{\infty} M\left(\frac{\|\beta_k \Delta_m x_k\|^{p_k}}{\eta}\right) = \sum_{n=1}^{\infty} M\left(\frac{\left\|n^{-2/p_{k(n)}} z\right\|^{p_{k(n)}}}{\eta}\right) = \sum_{n=1}^{\infty} M\left(\frac{1}{n^2 \eta}\right)$$

$$\leq M\left(\frac{1}{\eta}\right) \sum_{n=1}^{\infty} \frac{1}{n^2} < \infty$$

which shows that $x \in \ell_M( X, \beta, P)$.
On the other hand for any $\eta > 0$ we have

$$\sum_{k=1}^{\infty} M\left(\frac{\|\alpha_k \Delta_m x_k\|^{p_k}}{\eta}\right) = \sum_{n=1}^{\infty} M\left(\frac{\left\|\alpha_{k(n)} n^{-2/p_{k(n)}} z\right\|^{p_{k(n)}}}{\beta_{k(n)}\ \eta}\right)$$

$$= \sum_{n=1}^{\infty} M\left(\left|\frac{\alpha_{k(n)}}{\beta_{k(n)}}\right|^{p_{k(n)}} \frac{1}{n^2 \eta}\right)$$

$$\geq \sum_{n=1}^{\infty} M\left(\frac{1}{\eta}\right) \quad = \infty,$$

showing that $x \notin \ell_M( X, \alpha, P)$, a contradiction.

This completes the proof.

□

**Lemma 2.5** : $\ell_M( X, \alpha, P)$ forms a linear space over $\mathbb{C}$ if $\lim sup_k p_k < \infty$.

Proof:

Let $sup_k p_k < \infty$. Let $x, y \in \ell_M( X, \alpha, P)$and let $a, b \in \mathbb{C}$ then there exist scalars $\eta_1, \eta_2 > 0$ such that
$\sum_{k=1}^{\infty} \alpha_k\, \Delta^m x_k < \infty \quad \sum_{k=1}^{\infty} \alpha_k\, \Delta^m x_k < \infty$.

Let us choose $\eta_3$ such that

$$2\text{D}\eta_1 \max\{1, |a|\} \leq \eta_3 \text{ and } 2\text{D}\eta_2 \max\{1, |b|\} \leq \eta_3.$$

For such $\leq \eta_3$, by using non-decreasing and convex properties of $M$ ,we have

$$\sum_{k=1}^{\infty} M\left(\frac{\|\alpha_k(a\Delta^m x_k + \text{b}\Delta^m y_k)\|^{p_k}}{\eta_3}\right) \leq \sum_{k=1}^{\infty} M\left(\frac{D\|a\ \alpha_k \Delta^m x_k\|^{p_k}}{\eta_3} + \frac{D\|b\alpha_k \Delta^m y_k\|^{p_k}}{\eta_3}\right)$$

$$= \sum_{k=1}^{\infty} M\left(\frac{D|a|^{p_k}\|\alpha_k \Delta^m x_k\|^{p_k}}{\eta_3} + \frac{D|b|^{p_k}\|\alpha_k \Delta^m y_k\|^{p_k}}{\eta_3}\right)$$

$$\leq \sum_{k=1}^{\infty} M\left(\frac{\|\alpha_k \Delta^m x_k\|^{p_k}}{2\eta_1} + \frac{\|\alpha_k \Delta^m y_k\|^{p_k}}{2\eta_2}\right)$$

$$\leq \frac{1}{2}\sum_{k=1}^{\infty} M\left(\frac{\|\alpha_k \Delta^m x_k\|^{p_k}}{\eta_1}\right) + \frac{1}{2}\sum_{k=1}^{\infty} M\left(\frac{\|\alpha_k \Delta^m y_k\|^{p_k}}{\eta_2}\right) < \infty.$$

This implies that $\ell_M( X, \alpha, P)$ forms a linear space over $\mathbb{C}$. □

**Lemma 2.6 :** If $\ell_M(X, \alpha, P)$ forms a linear space over $\mathbb{C}$, then $\lim sup_k p_k < \infty$.

Proof:

Suppose that $\ell_M(X, \alpha, P)$ is a linear space over $\mathbb{C}$ but $\lim sup_k p_k = \infty$. we can find a sequence $(k(n))$ of integers such that $k(n+1) > k(n) \geq 1, n \geq 1$ for which $p_{k(n)} > n$ for which $n \geq 1$. Now corresponding to $z \in X$, with $\|z\| = 1$ we can define a sequence $x = (x_k)$ by

$$\Delta_m x_k = \begin{cases} \alpha_{k(n)}^{-1} n^{-2/p_{k(n)}} z & \text{for } k = k(n), n \geq 1 \\ 0 & \text{otherwise} \end{cases}$$

Then for $k = k(n)$, we have

$$\sum_{k=1}^{\infty} M\left(\frac{\|\alpha_k a \Delta_m x_k\|^{p_k}}{\eta}\right) = \sum_{k=1}^{\infty} M\left(\frac{\left\| n^{-2/p_{k(n)}} z\right\|^{p_k}}{\eta}\right)$$

$$= \sum_{n=1}^{\infty} M\left(\frac{1}{n^2 \eta}\right) \leq M\left(\frac{1}{\eta}\right) \sum_{n=1}^{\infty} \frac{1}{n^2}$$

$$< \infty.$$

So, $x \in \ell_M(X, \alpha, P)$.

On the other hand since $p_{k(n)} > n$ for any $n \geq 1$ and scalar $a$ = 4 we get

$$\sum_{k=1}^{\infty} M\left(\frac{\|\alpha_k a \Delta_m x_k\|^{p_k}}{\eta}\right) = \sum_{k=1}^{\infty} M\left(\frac{\left\|4\, n^{-2/p_{k(n)}} z\right\|^{p_k}}{\eta}\right)$$

$$\geq \sum_{n=1}^{\infty} M\left(\frac{4^n}{n^2 \eta}\right)$$

$$\geq \sum_{n=1}^{\infty} M\left(\frac{1}{\eta}\right) = \infty \text{ as } \frac{4^n}{n^2} > 1, \text{ for each } n \geq 1.$$

This shows that $\alpha x \notin \ell_M(X, \alpha, P,)$ which gives a contradiction. This completes the proof.

□

On combining Lemmas 2.5 and 2.6, the theorem 2.7 follows.

**Theorem 2.7**: $\ell_M(X, \alpha, P)$ forms a linear space over $\mathbb{C}$ if and only if $\lim sup_k p_k < \infty$.

## 3. Conclusion

Sequence spaces are important in mathematics, particularly in mathematical and functional analysis. Here, using the concept of difference sequence spaces, we have discussed various fundamental topological properties of the generalized form of difference sequence space $\ell_M(X, \alpha, P)$.

## Acknowledgement

This article is the generalization of the works on vector valued paranormed sequence space defined by Orlicz function [16] studied by Srivastava and Pahari. The authors would like to express our deep gratitude and sincerely thank our thesis supervisor, Professor Narayan Prasad Pahari, for his exceptional guidance, valuable advice, and constant support in preparing this article.

## References

[1] Dutta, H., Kilicman, A., & Altun, O. **(2016)**. Topological properties of some sequences defined over 2-normed spaces. *Springer Plus*, *5***(1)**: 1-16.

[2] Et, M.**(1993)**. On some difference sequence spaces. *Tr. Journal of Maths*, **17**: 18-24.

[3] Et, M. & Colok,R.**(1995)**. On generalized difference sequence spaces, *Soochow J. Math*., **21(4)**.

[4] Kamthan, P.K. & Gupta, M.**(1980)**. Sequence and series, *lecture notes*; 65 Marcel Dekker Inc.

[5] Kizmaz,H.(1981). On certain sequence spaces. *Canadian Mathematical Bulletin*, **24(2)**: 169-176.

[6] Lindenstrauss, J. and Tzafriri, L.**(1971)**. On Orlicz sequence spaces. *Israel Jour. Math.*, **10(3)**: 379-390.

[7] Maddox, I.J.**(1980)**. Infinite matrices of operators. *Lecture Notes in Mathematics*, 786, *Springer- Verlag Berlin*, 786.

[8] Malkowsky, E. & Rakocević, V. **(2000)**. An introduction into the theory of sequence spaces and measures of non compactness. *Zbornik radova*, **17**: 143-234.

[9] Mursaleen, M., Khan M.A. & Qamaruddin. Difference sequence spaces defined by Orlicz functions, *Demonstration Math.*, 145-150.

[10] Parashar, S.D. & Choudhary, B. **(1994)**. Sequence spaces defined by Orlicz functions, *Indian J. Pure Appl. Maths.*, **25(4)**: 419-428.

[11] Paudel, G. P., Pahari, N. P. & Kumar, S. **(2022)**. Generalized form of *p*-bounded variation of sequences of fuzzy real numbers. *Pure and Applied Mathematics Journal*, **8(2)**: 47-50.

[12] Paudel, G. P., Pahari, N. P. & Kumar, S. **(2023)**. Topological properties of difference sequence space through Orlicz-paranorm function. *Advances and Applications in Mathematical Sciences*, **22(8)**: 1689-1703.

[13] Paudel, G. P., Sahani, S. K. & Pahari, N. P. **(2022)**. On generalized form of difference sequence space of Fuzzy real numbers defined by Orlicz function. *Nepal Journal of Mathematical Sciences*, **3(2)**: 31-38.

[14] Pokharel, J. K., Pahari, N. P. & Paudel, G. P. **(2024)**. On topological structure of total paranormed double sequence space $(\ell^2((X, \|.\|), \overline{\gamma}, \overline{w}), G)$, *Journal of Nepal Mathematical Society*, **6(2)**: 53-59.

[15] Ruckle, W. H.**(1981)**. Sequence spaces. *Pitman Advanced Publishing Programme.*

[16] Srivastava, J.K. & Pahari, N.P.**(2012)**. On vector valued paranormed sequence space $c_0(X, M, \bar{\lambda}, \bar{p})$ defined by Orlicz function. *J. Rajasthan Acad. of Phy. Sci*.,**11(2)**:11-24.

[17] Subramanian, N.**( 2008)**. The difference sequence space defined on Orlicz function. *Int. Journal of Math. Analysis*, **15(2)**: 721-729.

[18] Tripathy, B. C. & Esi, A.**(2006)**. New type of difference sequence spaces. *International Journal of Science and Technology*, **1(1)**: 11-14.

□□