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An Actuarial Comparison of General Business Insurance Loss Reserving: Evidence from Classical Chain Ladder and Cape-Cod Technical Provisions Numerical Techniques

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Abstract: *The assets and liabilities of a general insurance underwriter constitutes two main variables usually adopted when evaluating the solvency requirements of a general insurance firm under which technical provisions form an unprecedented part of insurance liabilities. The technical provision of a general insurance business comprises provisions for unearned premiums and provisions for claims. The technical provisions specified under solvency II requirements asserts that the classical actuarial techniques for evaluating the best estimate for provisions in general business insurance obligations contains the run off triangles. The objective of this study are to* **(i)** *estimate the chain ladder reserve* **(ii)** *estimate the cape-cod reserve and* **(iii)** *compare the chain ladder with the cape-cod mathematical techniques with the disposition of estimating losses and technical provisions. These techniques evaluated through some run-off triangles can be adopted to estimate technical provisions for the outstanding claims. Computational evidence from our results over the periods considered revealed that despite the fact that the cape-cod technique is a mathematical variant of the chain ladder technique which seems less dependent on the variations of a single observation, the chain ladder reserve is numerically less than the* corresponding cape-cod reserve and hence $CL_{RESERVE} = 18534.42 < CC_{RESERVE} = 19123.84$

Key words: *Technical provisions, Chain ladder, Cape Cod, Solvency, Run-off triangles*

1. Introduction to Actuarial Loss Reserving

Let $f : \mathbf{R}^+ \to \mathbf{R}^+; y \mapsto f(y)$ such that $f(y)$ defines a continuous claim density function representing the value of a claim momentarily at time y. The value $V_L(s,t)$ defining the total loss incurred within some time interval $s \le y \le t$ is numerically modelled as $V_L(s,t) = \int_s^t f(y)dy$

However in actuarial practice, the functional form $f(y)$ seems quite difficult to model since all observable technical provisions represent observations on differing aggregate amounts and moreover these observable claim values could fall within intervals short of ultimate developments. Development in loss reserving usually describes the numerical difference between observable values of a defined actuarial variables at consecutive valuation dates which could be applied to loss reserve computations hence the modeling of actuarial claim function is required. In order to model the claim function, it becomes pertinent to introduce a continuous development function $g(\xi):\mathbf{R}^{+} \rightarrow \mathbf{R}^{+}$ with the following conditions.

$$\begin{cases} g(\xi)=1 \quad for \quad \xi > S \\ g(\xi)=0 \quad for \quad \xi < 0 \\ \end{cases}$$

Where the claim development is assumed to continue over the period S

Consequently, the aggregate claim within the time interval developed through the time α is obtained as $V_L(s,t)=\int_s^t g(\alpha-y)f(y)dy$ and assuming that $f(y)$ is a real constant β, then $V_L(s,t)=\int_s^t \beta\times g(\alpha-y)dy$ where the aggregate loss model assumes a requisite functional form such that the functions $g(y)$ and $f(y)$ are well defined and well behaved to fit the observable calendar period aggregate claim data. In (Weindorfer, 2012; Adams, 2018; Suwardi & Purwono, 2020), an underwriting firm is expected to put in place a sufficient buffer funds to cover expected future costs of claims on the insured schemes that are still in force. Following (Ramos de Carvalho & de Franca Carvalho, 2019), this buffer fund defines the claims reserve called the technical provisions which is actuarially estimated as a sum of the best estimate reserve and safety loading. In (Rahmawati, Darti & Marjono, 2019; Karmila, Nurrohmah & Sari, 2020), we observe that the technical provisions is essentially required to enable an underwriter satisfy her contractual obligations to the insured so as to create a requisite cover against emergence of sudden losses and to generate an unconstrained evolution of profit dynamics. In (Wuthrich & Merz, 2015; Dina, 2019), the best estimate reserve defines the expected discounted value of the outstanding claims payments while another actuarial variable alternatively referred to as the risk margin is a quantity that protects the underwriter against implicit uncertainties.

Following (Wuthrich & Merz, 2015; Sakthivel, 2016; Dina, 2019), the EU regulatory framework on solvency II defines this risk margin as a value such that the amount of the technical provisions is the amount another entity would require in order to analyse the run-off of the liabilities of the insurer. Dina (2019) compared the classical chain ladder with the Munich chain ladder and discovered that claim the reserve projections under Munich chain ladder based on paid methodology underestimates the outstanding loss liability while the classical chain ladder based on incurred method overestimates the claim reserves.

Following (Gisler & Wuthrich, 2008; Adams, 2018), the chain ladder method represents a numerical technique in estimating provisions for the outstanding claim payments. Actuaries usually adopt this technique to extrapolate the expected future claims from the claims already reported or paid. In chain ladder technique, it is presumed that the time series of claims is stable in time hence a run-off triangle is needed for the input data. We observe in (Ting, 2016; Teja, Barua, Mudigonda & Kandala, 2018; Balona & Richman, 2020; Georgieva, 2021; Raeva & Pavlov, 2021), that the run off triangle collects cumulative data on the incurred claims in respect of accident year and development year as a result of possible delay between claim occurrence and claim settlement. The chain-ladder predictor of the ultimate claim is computed by multiplying the current claims value of an accident year by a product of development factors. The chain-ladder predictors of the ultimate claims are directly proportional to the current claims value and hence if the current claims amount tends zero or increases without bounds then, the chain-ladder prediction leads to inadequate results. This is typically the case for long-tailed lines of business in current accident years. Furthermore, the chain-ladder technique is quite responsive to variations in individual claim numbers. The chain-ladder technique is wholly dependent on the claims data though it ignores the information on earned premiums.

The development pattern for the Bornhuetter Ferguson (BF) technique is usually obtained through the development pattern resulting from the chain-ladder method. The (BF) presumes that unreported claims would develop by reason of expected claims as a combination of chain ladder and the expected loss ratio techniques. However (Mack, Quarg & Braun, 2006; Merz & Wuthrich, 2008) evaluate the application of the chain-ladder development pattern and concluded that the chain-ladder technique assumes a multiplicative relationship between the past and future loss value. Nonetheless (Merz & Wuthrich, 2008) concluded that BF method is an additive relationship. In the cape-cod method, the reserve of an accident year defines the product of an estimate of the expected ultimate claim and the estimated still-to-come factor of the corresponding accident year. For the estimation of the still-to-come factor, it is presumed that there is a development pattern which is the same for all accident years. The development pattern is a fraction of claims that develops up to a particular development year in relation to the ultimate claim value. The cape-cod method was modeled so as to address some of the deficiencies of the chain ladder technique. The cape-cod technique seems more impervious to any form of inadequacies irrespective of unexpected claim condition and hence it is not really influenced by errors in the assumptions about the distribution of claim sample errors.

2. Materials and Methods

Chain Ladder Estimation

Let $\left\{S_{i,r}\right\}_{i,r\in\{0,1,2,3,\ldots m\}}$ be a collection of random variables. In (Schmidt & Wunsche, 1998), the random variables $S_{i,r} > 0$ which is assumed to be strictly positive represents the aggregate claim size over all claims occurring in occurrence year i and which are expected to be settled prior to the end of calendar year $i+r$. Furthermore in (Schmidt, 2006), the enumeration of the development years describes the delays in respect of the occurrence years. Following (Kaas, Goovaerts, Dhaene & Denuit, 2001), the number of diagonals with $i-1+r=a$ confirm the payments progression which have been made in occurrence year a. However, it is assumed that claims are settled prior to the end of development year m. The random variables $S_{i,m}$ therefore defines the ultimate aggregate claims. The ultimate aggregate claims $S_{i,m}$ conforms to the aggregate claims of occurrence year i.

In (Schmidt & Wunsche, 1998), the observed aggregate claims could be represented by the run-off triangle. The information on the area of the triangle below the main run off triangle is undeterminable because it deals with future development parameters about different group of claims.

Table** 1: **Run off Triangle

Occurrence year	Development year 0	1	…	r	.	$m-i$	.	$m-1$	m
0	$S_{0,0}$	$S_{0,1}$	…	$S_{0,r}$	.	$S_{0,m-i}$	.	$S_{0,m-1}$	$S_{0,m}$
1	$S_{1,0}$	$S_{1,1}$	…	$S_{1,r}$	.	$S_{1,m-i}$	.	$S_{1,m-1}$	
.									
i	$S_{i,0}$	$S_{i,1}$	…	$S_{i,r}$	…	$S_{i,m-i}$			
.									
$m-r$	$S_{m-r,0}$	$S_{m-r,1}$		$S_{m-r,r}$					
.									
$m-1$	$S_{m-1,0}$	$S_{m-1,1}$							
m	$S_{m,0}$								

A cumulative claim $S_{i,r}$ is observed whenever $i+r \le m$ but non-observable $i+r > m$. Furthermore $S_{i,r}$ is present when $i+r=m$ and $S_{i,r}$ will be ultimate where $r=m$. The objective of loss reserving is to estimate

(i) the future cumulative losses $S_{i,r}$

(ii) the future losses $\zeta_{i,r} = S_{i,r} - S_{i,r-1}$

(iii) the calendar year reserves $\sum_{j=q-m}^{m} \zeta_{j,q-j}$

(iv) the total reserve $\sum_{j=1}^{m}\left(\sum_{l=m-j+1}^{m} \zeta_{j,l}\right)$ where $i+r \ge m+1$ and $q=m+1,...2m$

The chain ladder technique heavily depends on the observed cumulative losses in the run off triangle and involving no prior estimators. However, it fundamentally assumes that each accident year has the same pattern of claim development and that there is a development pattern for factors.

For $i \in \{0,1,2,...,m\}$ and $r \in \{1,2,...,m\}$, the development factor is defined as follows $\theta = \dfrac{S_{i,r}}{S_{i,r-1}}$.

Where $r \in \{1,2,...,m\}$, the chain ladder development factor is estimated as follows $\hat{\theta} = \dfrac{\sum_{i=0}^{m-r} S_{i,r}}{\sum_{i=0}^{m-r} S_{i,r-1}}$ (1)

Following (Schmidt & Schnaus, 1996) for the development year r, the chain ladder factor $\hat{\theta}$ seems to be the most appropriate estimation of the observed development factors when the approximation error is given. The weight occurring in the presentation of the chain ladder factor is a weighted average. The chain ladder factors are weighted average and it is adopted in estimating the development factors. The ultimate aggregate claims is expected to meet the following conditions.

$S_{i,m} = S_{i,m-i} \times \left(\prod_{r=m-i+1}^{m} \theta_{i,r} \right)$ and for $i \in \{1,2,...,m\}$,

the chain ladder estimator becomes

$$\hat{S}_{i,m} = S_{i,m-i} \times \left(\prod_{r=m-i+1}^{m} \hat{\theta}_r \right) \qquad (2)$$

Consequently, the collection $\{\zeta_{i,r}\}_{i,r\in\{0,1,2,3,...m\}}$ of incremental claims is obtained as follows

$$\zeta_{i,r} = \begin{cases} S_{i,0} & if \quad r=0 \\ S_{i,r} - S_{i,r-1} & if \quad r>1 \end{cases} \qquad (3)$$

3. Data Analysis

The data used in our computation is obtained from Al-Atar (2017) and covers the periods 2009 to 2016.

3.1 Credibility Premium

Credibility premium $-\alpha$ **(Individual premium)** $=(1-\alpha)$ **(Collective premium)**

where α is the credibility factor and $0 \le \alpha \le 1$. The three key variables stated in premium equation are

(i) credibility factor α

(ii) individual premium $\bar{x}_j$ and

(iii) collective premium ϕ would be needed to compute credibility premium for 5 different auto insurance policies.

We used the previous information about volume of accidents for 4 past years. For each policy, we first compute individual premium which equivalent to the average claim for 4 years using

$$\bar{x}_j = \frac{1}{4}\sum_{i=1}^{4} x_{ij} \tag{4}$$

where $\bar{x}$ is the individual premium i in the year and j is the policy

Table 2 : Past Claims for five different policies

	Policy j				
Year i	1	2	3	4	5
2013	0	6	6	6	3
2014	0	2	3	4	0
2015	0	2	0	3	0
2016	0	2	3	3	1
$\bar{x}_j = \frac{1}{4}\sum_{i=1}^{4} x_{ij}$	**0**	**3**	**3**	**4**	**1**
$\frac{1}{k-1}\sum_{i=1}^{k}\left(\bar{x}_{ij} - x_{ij}\right)^2$	**0**	**4**	**6.667**	**2.333**	**2**

Source: own computations

The collective premium is computed using the equations:

$$\pi = \left(\frac{1}{K\times L}\right)\times\sum_{j=1}^{L}\sum_{i=1}^{K} x_{ij} = \frac{1}{L}\sum_{j=1}^{L}\left(\frac{1}{K}\sum_{i=1}^{K} x_{ij}\right) = \frac{1}{5\times 4}\left(0+12+12+16+4\right) = \frac{44}{20} = 2.2 \tag{5}$$

where K is the number of years, L is the number of schemes and π is the collective premium

$$\hat{\alpha} = \frac{\phi K}{\left(\phi K + \hat{V}\right)} \tag{6}$$

Following Straub (1997), $\hat{\alpha}$ is the credibility parameter $\hat{V}$ is the variance or noise from year to year while ϕ is the variance from risk policy. The values of the variances should be estimated in order to compute the credibility parameter. It is apparent from the credibility equations for credibility factors and credibility premium that as the variance of the each premium becomes bigger, the denominator of credibility factor also becomes bigger which means smaller credibility factor and therefore lower weights assigned to each premium. Therefore $\hat{V}$ measures the noise in each policy. Nonetheless, the larger the variance among the schemes, the smaller the denominator of credibility factor and the larger the credibility parameter and consequently smaller weight is assigned to collective premium. ϕ defines the variance among policies. First $\hat{V}$ is computed to demonstrate that the estimation is not mathematically complex. Therefore, the average of the variances will be computed to obtain $\hat{V}$ hence

$$\hat{V} = \frac{1}{L}\sum_{i=1}^{L}\frac{1}{K-1}\sum_{j=1}^{K}\left(x_{ij} - \bar{x}_{ij}\right)^2 = \frac{1}{5}\left(0+4+6.667+2.333+2\right) = \frac{1}{5}\times 15 = 3 \tag{7}$$

Now $$\phi = \frac{1}{L-1}\sum_{j=1}^{L}\left(\bar{x}_j - \pi\right)^2 - \frac{\hat{V}}{K} \tag{8}$$

$$\phi = \frac{1}{(5-1)}\left\{(0-2.2)^2 + (3-2.2)^2 + (3-2.2)^2 + (4-2.2)^2 + (1-2.2)^2\right\} - \left(\frac{3}{4}\right)$$

$$\phi = \frac{1}{4}\{4.84 + 0.64 + 0.64 + 3.24 + 1.44\} - (0.75) = 2.7 - 0.75 = 1.95$$

To estimate the credibility factor, we use

$$\hat{\alpha} = \frac{\phi K}{\left(\phi K + \hat{V}\right)} = \frac{1.95 \times 4}{(1.95 \times 4 + 3)} = \frac{7.8}{10.8} = 0.7222 \tag{9}$$

$$\hat{\alpha} = 0.7222 \tag{9a}$$

Therefore credibility factor is obtained as $\hat{\alpha} = 0.7222$ **.** This proves that bigger weight has been assigned to individual experience than to the overall experience. Following (Straub, 1997), the credibility factor is numerically estimated for each policy through the following formula

$$\hat{U}_{ij} = \hat{\alpha} \times \bar{x}_j + \pi \times \left(1 - \hat{\alpha}\right) \tag{10}$$

Table 3 : *Credibility Premium*

Policy j	$\bar{x}_j$	$\hat{U}_{ij}$
scheme 1	0	$\hat{U}_j = \mathbf{0.722} \times 0 + (1 - \mathbf{0.7222}) = 0.6112$
scheme 2	3	$\hat{U}_j = \mathbf{0.722} \times 3 + (1 - \mathbf{0.7222}) = 2.7778$
scheme 3	3	$\hat{U}_j = \mathbf{0.722} \times 3 + (1 - \mathbf{0.7222}) = 2.7778$
scheme 4	4	$\hat{U}_j = 0.\mathbf{722} \times 4 + (1 - \mathbf{0.7222}) = 3.5000$
scheme 5	1	$\hat{U}_j = 0.\mathbf{722} \times 1 + (1 - \mathbf{0.7222}) = 1.3333$

Source: own computations

Chain Ladder Method

The cumulative claims are denoted by $S_{i,j}$ where i is the accident year and j the development year

Table 4 : *Cumulative Run-off Triangle*

Development year j	0	1	2	3	….	y
Accident year i						
2013	$S_{2013,0}$	$S_{2013,1}$	$S_{2013,2}$	$S_{2013,3}$		$S_{2013,y}$
2014	$S_{2014,0}$	$S_{2014,1}$	$S_{2014,2}$	$S_{2014,3}$		
2015	$S_{2015,0}$	$S_{2015,1}$	$S_{2015,2}$			
2016	$S_{2016,0}$	$S_{2016,1}$				
	$S_{y,0}$					

year j defines the year when all claims are paid. Suppose the claims in each year is defined as X_{ij} , then the total claim is

$$X_{2013,0} + X_{2013,1} + X_{2013,2} + X_{2013,3} + \ldots + X_{2013,y}$$

The table 4 above shows the cumulative claims. However, the run-off triangle with incremental claims could also be used. The unknown portion of the triangle is obtained by using development factors.

Development factor is defined as: $$\hat{\theta}_j = \frac{\sum_{i=1}^{y-i-j}\left(S_{ij}+1\right)}{\sum_{i=1}^{y-i-j}\left(S_{ij}\right)} \tag{11}$$

Following (Oliveri & Pitacco, 2010), development factor describes the cumulative aggregate for any accident year i. The increment of the claims are fully covered till year y, $\hat{\theta}_j = 1$ for every development year. Final development factor is estimated as follows:

$$\hat{G}_j = \theta_j \times \theta_{j+1} \times \theta_{j+2} \times \theta_{j+3} \times \ldots \times \theta_{y-1} \tag{12}$$

The final loses are computed as follows $S_{iy} = S_{ij} \times \hat{G}_j = S_{ij}\left(\theta_j \times \theta_{j+1} \times \theta_{j+2} \times \theta_{j+3} \times \ldots \times \theta_{y-1}\right)$ (13)

Table 5 : *Incremental Run-off Triangle*

		Development Year *j*							
		0	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Accident Year i	2009	1232	946	520	722	316	165	48	14
	2010	1469	1201	708	845	461	235	56	
	2011	1652	1416	959	954	605	287		
	2012	1831	1634	1124	1087	725			
	2013	2074	1919	1330	1240				
	2014	2434	2263	1661					
	2015	2810	2108						
	2016	3072							

Source: Al-Atar (2017)

The claims in incremental run-off triangle above are summed up to obtain the cumulative run-off triangle.

Table 6 : *Cumulative Run-Off Triangle*

		Development Year *j*							
		0	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Accident Year	2009	1232	2178	2698	3420	3736	3901	3949	3963
	2010	1469	2670	3378	4223	4684	4919	4975	
	2011	1652	3068	4027	4981	5586	5873		
	2012	1831	3465	4589	5676	6401			
	2013	2074	3993	5323	6563				
	2014	2434	4697	6358					
	2015	2810	4918						
	2016	3072							

Source: own computations

The development factors are computed below to forecast the unknown part of the triangle

$$\hat{\theta}_0 = \frac{\sum_{i=0}^{6} S_{i1}}{\sum_{i=0}^{6} S_{i0}} = \frac{2178 \times 2670 \times 3068 \times 3465 \times 3993 \times 4697 \times 4918}{1232 \times 1469 \times 1652 \times 1831 \times 2074 \times 2434 \times 2810} = 1.85 \tag{14}$$

$$\hat{\theta}_1 = \frac{\sum_{i=0}^{5} S_{i2}}{\sum_{i=0}^{6} S_{i1}} = \frac{2698\times3378\times4027\times4589\times5323\times6358}{2178\times2670\times3068\times3465\times3993\times4697} = 1.31 \quad (15)$$

$$\hat{\theta}_2 = \frac{\sum_{i=0}^{4} S_{i3}}{\sum_{i=0}^{4} S_{i2}} = \frac{3420\times4223\times4981\times5676\times6563}{2698\times3378\times4027\times4589\times5323} = 1.24 \quad (16)$$

$$\hat{\theta}_3 = \frac{\sum_{i=0}^{3} S_{i4}}{\sum_{i=0}^{3} S_{i3}} = \frac{3736\times4684\times5586\times6401}{3420\times4223\times4981\times5676} = 1.16 \quad (17)$$

$$\hat{\theta}_4 = \frac{\sum_{i=0}^{2} S_{i5}}{\sum_{i=0}^{2} S_{i4}} = \frac{3901\times4919\times5873}{3736\times4684\times5586} = 1.05 \quad (18)$$

$$\hat{\theta}_5 = \frac{\sum_{i=0}^{1} S_{i6}}{\sum_{i=0}^{1} S_{i5}} = \frac{3949\times4975}{3901\times4919} = 1.01 \quad (19)$$

$$\hat{\theta}_6 = \frac{\sum_{i=0}^{1} S_{i7}}{\sum_{i=0}^{1} S_{i6}} = \frac{3963}{3949} = 1.00 \quad (20)$$

However, the final development factors $\hat{G}_j$ are estimated as follows

$$\hat{G}_0 = \hat{\theta}_0\times\hat{\theta}_1\times\hat{\theta}_2\times\hat{\theta}_3\times\hat{\theta}_4\times\hat{\theta}_5 = 1.85\times1.31\times1.24\times1.16\times1.05\times1.01 = 3.70 \quad (21)$$

$$\hat{G}_1 = \hat{\theta}_1\times\hat{\theta}_2\times\hat{\theta}_3\times\hat{\theta}_4\times\hat{\theta}_5\times\hat{\theta}_6 = 1.31\times1.24\times1.16\times1.05\times1.01\times1.00 = 2.00 \quad (22)$$

$$\hat{G}_2 = \hat{\theta}_2\times\hat{\theta}_3\times\hat{\theta}_4\times\hat{\theta}_5\times\hat{\theta}_6 = 1.24\times1.16\times1.05\times1.01\times1.00 = 1.53 \quad (23)$$

$$\hat{G}_3 = \hat{\theta}_3\times\hat{\theta}_4\times\hat{\theta}_5\times\hat{\theta}_6 = 1.16\times1.05\times1.01\times1.00 = 1.23 \quad (24)$$

$$\hat{G}_4 = \hat{\theta}_4\times\hat{\theta}_5\times\hat{\theta}_6 = 1.05\times1.01\times1.00 = 1.06 \quad (25)$$

$$\hat{G}_5 = \hat{\theta}_5\times\hat{\theta}_6 = 1.01\times1.00 = 1.01 \quad (26)$$

$$\hat{G}_6 = \hat{\theta}_6 = 1.00 \quad (27)$$

Table 7 : *Development Factors*

Development Year j	0	1	2	3	4	5	6
$\hat{\theta}_j$	1.85	1.31	1.24	1.16	1.05	1.01	1.00
$\hat{G}_j$	3.70	2.00	1.53	1.23	1.06	1.01	1.00

Source: own computations

The final loses S_{i6} and reserves R_i for years 2009 to 2016 are computed below. The final reserves describe the changes occurring between the final loss and the last known claim so that the sum of the computed final reserves equates to the total chain ladder reserve.

Table 8 : *Estimation of Final Loses*

FINAL LOSES

$S_{16} = S_{15} \times \hat{G}_6 = 4975 \times 1.00 = 4975$, $S_{26} = S_{24} \times \hat{G}_5 = 5873 \times 1.01 = 5931.73$

$S_{36} = S_{33} \times \hat{G}_4 = 6401 \times 1.06 = 6785.06$, $S_{46} = S_{42} \times \hat{G}_3 = 6563 \times 1.23 = 8072.49$

$S_{56} = S_{51} \times \hat{G}_2 = 6358 \times 1.53 = 97272.74$, $S_{66} = S_{60} \times \hat{G}_1 = 4918 \times 2.00 = 9836.00$

$S_{76} = S_{70} \times \hat{G}_0 = 3072 \times 3.70 = 11366.40$

Total

Source: own computations

Table 9 : *Estimation of Final Reserve*

Reserves

$R_1 = S_{16} - S_{15} = 4975 - 4975 = 0$, $R_2 = S_{26} - S_{24} = 5931.73 - 5873 = 58.73$

$R_3 = S_{36} - S_{33} = 6785.06 - 6401 = 384.06$, $R_4 = S_{46} - S_{42} = 8072.49 - 6563 = 1509.49$

$R_5 = S_{56} - S_{51} = 9727.74 - 6358 = 3369.74$, , $R_6 = S_{66} - S_{60} = 9836.00 - 4918.00 = 4918.00$

$R_7 = S_{77} - S_{70} = 11366.40 - 3072.00 = 8294.40$

Total

Source: own computations

Therefore the total chain ladder reserve is

$$CL_{reserve} = \sum_{n=1}^{7} R_i = R_1 + R_2 + R_3 + R_4 + R_5 + R_6 + R_7 \quad (28)$$

$$CL_{reserve} = 0 + 58.73 + 384.06 + 1509.49 + 3369.74 + 8294.40 = 18{,}534.42$$

and equals the value of the total expected future claims for accidents that occurred between 2009 and 2016. From the foregoing, the chain ladder method seems computationally easy but suffers the following set-backs

(i) Following (Straub, 1997) $\hat{G}_j$ are the estimated parameter which could result in serious bias

(ii) It is responsive to the stimulus of variation of a single number S_{iy-i} values

(iii) The technique is superfluous where $X_{y0} = 0$ or $S_{iy-i} = 0$

(iv) It does not seem to give room to use information on the earned premium

(v) If the current claims amount approach zero or increases without bounds then, the Chain-Ladder prediction leads to inadequate results

Cape Cod Method

The cape-cod method is developed to deal with the deficiencies of the chain ladder method by embedding the lag factors and requisite information on earned premium. The mathematical principle behind the theory of cape-cod technique is to compare defined losses with used-up premiums. The reserve (R_i) for year i is obtained as follows

$$R_i = P_i \times \left(1 - \hat{L}_{k-1}\right) \times CF \tag{30}$$

Where R_i is the reserve in year i, P_i is the paid premium in year i, L_{K-1} is the lag parameters and CF is the correction factor. Technically speaking, the lag parameters measures how much of the expected final loss of a given accident year is obtained by the end of the development year j. Mathematically, the lag factor is estimated as follows

$$\hat{L}_j \hat{G}_j = 1 \Rightarrow \hat{L}_j = \frac{1}{\hat{G}_j} \tag{31}$$

The correction factor serves as the ratio of the experienced claims to the used premiums for the experienced development years. The cape-cod technically allows us to compute claims and premiums for many accident years making it more computationally robust in comparison to the chain ladder method. The correction factor for cape-cod method is:

$$CF = \frac{\sum_{i=1}^{y} S_{i,y-1}}{\sum_{i=1}^{y} \hat{L}_{f-1} \times P_i}$$

accident year i, then the reserve formula is defined as follows

$$R_i = P_i \times \left(1 - \hat{L}_{k-1}\right) \times \frac{S_{i,y-1}}{\hat{L}_{y-1} \times P_i} = \left(\frac{1}{\hat{L}_{k-1}} - 1\right) \times S_{i,y-i} = \left(\hat{G}_{y-i} - 1\right) \times S_{i,y-i} \tag{33}$$

It will be apparent that where only one accident year i is adopted, then the cape-cod reserve computations will equate to the chain ladder reserve computation. Consequently, following (Straub, 1997), the cape-cod technique defines a mathematical variant of the chain ladder technique which seems less dependent on the variations of a single observation. It is pertinent to observe that in cape-cod technique, the cumulative run-off triangle remains the same as in the previous discussions, however the information on the paid premium P_i may differ.

Table 10: Cumulative Run-Off Triangle Including Paid Premiums

		Development Year j							
Accident Years	**Premiums**	0	1	2	3	4	5	6	7
2009	4572	1232	2178	2698	3420	3736	3901	3949	3963
2010	5397	1469	2670	3378	4223	4684	4919	4975	
2011	6192	1652	3068	4027	4981	5586	5873		
2012	6872	1831	3465	4589	5676	6401			
2013	7534	2074	3993	5323	6563				
2014	9219	2434	4677	6358					
2015	10328	2810	4918						
2016	12358	3072							

Source: own computations

The lag factors are estimated adopting the final development factors from the chain ladder method.

$$\hat{L}_j = \frac{1}{\hat{G}_j} \tag{34}$$

$$\hat{L}_0 = \frac{1}{\hat{G}_0} = \frac{1}{3.70} = 0.27 \tag{35}$$

$$\hat{L}_1 = \frac{1}{\hat{G}_1} = \frac{1}{2.00} = 0.50 \tag{36}$$

$$\hat{L}_2 = \frac{1}{\hat{G}_2} = \frac{1}{1.53} = 0.65 \tag{37}$$

$$\hat{L}_3 = \frac{1}{\hat{G}_3} = \frac{1}{1.23} = 0.81 \tag{38}$$

$$\hat{L}_4 = \frac{1}{\hat{G}_4} = \frac{1}{1.06} = 0.94 \tag{39}$$

$$\hat{L}_5 = \frac{1}{\hat{G}_5} = \frac{1}{1.01} = 0.99 \tag{40}$$

$$\hat{L}_6 = \frac{1}{\hat{G}_6} = \frac{1}{1.00} = 1.00 \tag{41}$$

Table 11: *Lag Factors*

Policy j	0	1	2	3	4	5	6
$\hat{L}_j$	0.27	0.50	0.65	0.81	0.94	0.99	1.00

Source: own computations

The lag parameters are subsequently used to estimate the correction factor,
The correction factor (CF) is defined as follows

$$CF = \frac{\sum_{i=1}^{y} S_{i,y-i}}{P_i \times \sum_{i=1}^{y} \left(\hat{L}_{y-1} \right)} \tag{42}$$

$$CF = \frac{4975 + 5873 + 6401 + 6563 + 6358 + 4918 + 3072}{(1.00 \times 5397) + (0.99 \times 6192) + (0.94 \times 6872) + (0.81 \times 7534) + (0.65 \times 9217) + (0.50 \times 10328) + (0.27 \times 12358)} \tag{43}$$

$$CF = 0.99 \tag{44}$$

The cape-cod (CC) reserves are estimated as a product of the correction factor and the residual premium with available future losses:

$$CC_{reserve} = P_i \times \left(1 - \hat{L}_{y-1} \right) \times CF \tag{45}$$

Table 12: *Cape Cod Reserves*

i	$\left(1 - \hat{L}_{y-1}\right)$	P_i	$R_i = P_i \times \left(1 - \hat{L}_{y-1}\right) \times CF$
1	0.00	5397	0.0000
2	0.02	6192	61.3008

3	0.06	6872	408.1968
4	0.19	7534	1417.1454
5	0.35	9217	3193.6905
6	0.48	10328	5112.36
7	0.73	12358	8931.1266
Total			**19,123.8201**

Source: own computations

Discussion of Result

Having computed the individual premium and collective premium, it is then sufficient to appraise the credibility parameter $\hat{\alpha}$. In order to obtain the credibility parameter, it is instructive that we investigate the values of the variances $\hat{V}$ and ϕ. As observed from the equations for credibility factors and credibility premium, the bigger the variance of the individual premium, the higher is denominator of credibility factor meaning lower is the credibility factor and consequently smaller weight is mapped to individual premium. Consequently the parameter $\hat{V}$ measures the noise in individual policy. However, the bigger the variance among the schemes, the lower is denominator of credibility parameter and the bigger is the credibility factor and hence a lower weight is mapped to the collective premium hence ϕ measures the noise among the schemes.

In **table** 2, we obtained the collective premium. In the same table, the computed credibility factor is 0.7222 showing a bigger weight being mapped to the individual experience rather than to the all-encompassing experience.

In **table** 3, it is apparent that scheme holders 2, 3 and 4 would be bound to pay bigger premiums in comparison to their average claim while policyholders 1 and 5 will be bound to pay smaller premiums compared to their average claim. Underwriters can embark on extensive claims and premium data analytics so as to examine the insured's behaviour. Based on the premium paid and the average claims, an underwriter could possibly classify two different sets of insured as aggressive and defensive scheme holders.

Table 10 on cape-cod presents the run-off triangle containing cumulative data over the incurred claims in respect of accident year and development year.

Table 5, describes the source data while **table** 6 is the cumulative run-off triangle of the source data. From Table 7, it is apparent that the initial development factors $\left(D_{IN}\right)$ are

$$D_{IN} = \{1.85, 1.31, 1.24, 1.16, 1.05, 1.01, 1.00\} \tag{46}$$

while the final values of the computed development factors $\left(D_{FIN}\right)$ are obtained as follows

$$D_{FIN} = \{3.70, 2.00, 1.53, 1.23, 1.06, 1.01, 1.00\} \tag{47}$$

These development factors are adopted to forecast the unknown area of the run-off triangle. Note that the initial development factor is used to compute the final development factor.

Table 8 shows the numerical estimate of the final losses and actuarial reserve. From **table** 9, the chain-ladder reserve value is $18,534.42$ and represents the total value of the expected future claims for those accidents that evolved between the periods $2009-2016$.

In order to apply the cape cod technique efficiently in **table** 12, it is sufficient that we first compute an estimate of the lag factors in **table** 11. Usually, the lag factor is the reciprocal of the final development factor. Furthermore, in other to evaluate our reserve, it becomes necessary that we compute the correction factor using **table** 10. The computed correction factor is 0.99. In the final analysis, the total cape-cod reserve is $19,123.84$ as shown in **table** 12.

This has confirmed that the chain-ladder is a subset of cape-cod method. It is immediately apparent that the computational comparison from the evaluation of cape-cod and chain-ladder reveals that

$$CL_{RESERVE} = 18,534.42 < CC_{RESERVE} = 19,123.84 \tag{48}$$

4.Conclusion

Insurance provisions are part of legal requirements for general insurance business since they represent a higher fraction of underwriting liabilities. General insurers require a robust capital buffer because they are constantly exposed to high level uncertainty of claims. In view of the uncertainty and economic dimensions that reliability of reserves connotes, the valuation directly impacts on the fiscal strength of general business insurance firms and hence have a pervasive impact on their solvency capital requirements, consequently it is necessary to adopt the data analytics and decide which reserve technique will be preferred. From the analysis, it is apparent that the two actuarial methods of computing claims provisions were applied on the omnibus portfolio over the run-off triangles and then adopted to compute the claims reserve. The actuarial theory of development parameter and lag function characteristics in the run-off triangle are the basic tools to estimate accurate claims reserve. In practice, the classical equivalence principle that the present value of expected premiums must equate to the present value of the expected claims could be actuarially invalid in general insurance and as a result, the classical ruin theory assumes that pure risk premium without the requisite loading term will not be actuarially sufficient since at the long run, ruin may be unavoidably orchestrated even though the underwriter could have a sufficient initial reserve base. Underwriters adopt different premium mechanisms usually the expected and the variance premium hypothesis. However, premiums in general insurance business are also computed through premium principles but the classical ruin theory assumes that a requisite fixed loading ψ should be added in order to militate against ruinous conditions.

Underwriters, in general, business insurance attempt to classify insurance risks into homogeneous groups and advise identical premium on group members. Since uncertainties vary in form, underwriters advise and impose premium that is a function of collective and individual premium. Based on the previous experience, insurance firms could estimate the premium value as a weighted average function between the two premiums but if the variance of a typical scheme holder is bigger compared to the variance of the whole group, then a bigger weight will be mapped to the collective premium and vice versa.

The reliability of evaluating reserves directly impacts on the financial health of an insurance firm and therefore this study has demonstrated claims reserving estimation techniques in general insurance business based on classical Chain-Ladder methods that are normally employed in actuarial practice for the evaluation of outstanding claims reserves in non-life insurance business. Specifically, the development factors in Cape-Cod were computed based on the Lag parameters and correction factors. The clear advantage of these techniques lies in the expedience of their application in loss reserving. The results from this study under the two chain claims reserving application show varied differences between the classical Chain-Ladder and Cape-Cod in respect of the claims reserve level.
Future research work could be carried out using stochastic technique where relevant data is available.

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