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An Analytical Study of Thermal Instability in a Nanofluid Layer with a Vertical Magnetic Field

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Abstract: *The paper re-examines analytically the work of Yadav et al.* ([43]) (*referred to as Y hereafter*) *wherein they have examined the effect of vertical magnetic field with free-free, rigid-rigid and rigid-free boundaries on the onset of convection in an electrically conducting nanofluid layer heated from below. A number of sufficient conditions regarding the non-existence of oscillatory convection have been found.*

Keywords: *Nanofluid, thermophoretic diffusion, Brownian motion, Thermal instability, Vertical magnetic field, Oscillatory convection, Galerkin weighted residuals method.*

1. Introduction

The thermal instability of a fluid layer heated from below has been extensively studied since the work of Bénard [9]. After the work of Choi and Eastman [17], a considerable amount of work on thermal instability of nanofluids has been witnessed ([1-8], [10-16], [18-24], [27-42], [45] and [47-48]). The problem of magneto-convection (the thermal convection in a horizontal fluid layer in the presence of buoyancy force due to gravity and the Lorentz force due to magnetic field) becomes more important due to its applications in the study of earth's interior, atmospheric physics, oceanography and geophysics. Unfortunately, a very little work has been noticed on the stability of thermo-magneto convection in a nanofluid ([25-26], [43], [44], [46] and [49]).

The effect of a constant vertical magnetic field on the onset of convection in a nanofluid layer for free-free, rigid-rigid and rigid-free boundaries has been studied by Y. Their findings are based on the discussions for two cases, namely, $M=0$ and $M\neq 0$ [$M=L_e^{2}JP_{rM}^{2}(1+P_r)$]. In this paper, we have re-examined their study analytically for oscillatory convection and a number of sufficient conditions for the non-existence of oscillatory convection have been established.

2. Physical Problem and its Analysis

As we have re-examined the work of *Y*, therefore, in order to avoid the repetition, we have directly considered the final equations derived by *Y*. The real and imaginary parts of the characteristic equation for oscillatory convection are obtained as

$$J^2[J\omega^2(P_r+P_{rM}+P_rP_{rM})-P_r^2\{J^3+J\pi^2Q-a^2R_a-a^2R_n(L_e+N_A)\}]$$
$$+L_e\omega^2[J^3P_r(1+P_r+P_{rM})+QP_r^2\pi^2J-P_{rM}\{\omega^2J+a^2P_r(R_a+R_n)\}]=0 \qquad (2.1)$$

and

$$JL_e\omega^2(P_r+P_{rM}+P_rP_{rM})-J^3P_r(1+P_r+P_{rM})-J^3L_eP_r^2+J\omega^2P_{rM}-J\pi^2P_r^2Q$$
$$-JL_e\pi^2P_r^2Q+a^2R_aL_eP_r^2+a^2R_aP_rP_{rM}+a^2P_r\{L_eP_r+P_{rM}(L_e+N_A)\}R_n=0. \qquad (2.2)$$

Here $J = a^2 + \pi^2$ (a being the dimensionless wave number) and ω is real and dimensionless angular frequency of oscillations.

The non-dimensional parameters stated in the Eqs. (2.1) and (2.2) are as follows:

$L_e\left(=\dfrac{\alpha}{D_B}\right)$, nanofluid Lewis number

$P_r\left(=\dfrac{\mu}{\rho_0\alpha}\right)$, nanofluid Prandtl number

$P_{rM}\left(=\dfrac{\mu}{\rho_0\eta}\right)$, nanofluid magnetic Prandtl number

$N_A\left(=\dfrac{D_T(T_0^*-T_1^*)}{D_BT_1^*(\phi_1^*-\phi_0^*)}\right)$, modified diffusivity ratio

$R_a\left(=\rho_0 g\beta L^3\dfrac{(T_0^*-T_1^*)}{\mu\alpha}\right)$, Rayleigh number

$R_n\left(=\dfrac{(\rho_p-\rho_{f_0})(\phi_1^*-\phi_0^*)gL^3}{\mu\alpha}\right)$, concentration Rayleigh number

$Q\left(=\dfrac{\mu_e H_0^{*2}}{4\pi\rho_0\nu\eta}L^2\right)$, nanofluid magnetic number,

where μ is the viscosity of nanofluid, μ_e is the magnetic permeability of nanofluid, ρ_0 is the reference nanofluid density, ρ_{f_0} is the density of the base fluid, ρ_p is the density of nanoparticles, α is thermal diffusivity, ν is kinematic viscosity, η is electrical resistivity, β is thermal expansion coefficient of the fluid, ϕ_0^* is nanoparticle volume fraction at the lower plate, ϕ_1^* is nanoparticle volume fraction at the upper plate, g is gravitational acceleration, L is the depth between two parallel plates, T_0^* is temperature at the lower plate, T_1^* is reference temperature, H_0^* is uniform vertical magnetic field component along z-axis, D_B is Brownian diffusion coefficient and D_T is thermophoretic diffusion coefficient of nanoparticles. Asterisks are used to denote the dimensional variables.

On eliminating R_a from Eqs. (2.1) and (2.2), we get

$$M\lambda^2 + N\lambda + O = 0, \quad \text{where } \lambda = \omega^2, \tag{2.3}$$

$$M = L_e^2 J P_{rM}^2(1+P_r), \tag{2.4}$$

$$N = J^3(L_e^2P_r^2 + P_{rM}^2)(1+P_r) + L_e^2J\pi^2P_r^2Q(P_r - P_{rM}) + a^2L_eP_rP_{rM}^2R_n(L_e+N_A-1) \tag{2.5}$$

$$O = J^2P_r^2[J^3(1+P_r) + J\pi^2Q(P_r-P_{rM}) + a^2L_eP_rR_n(L_e+N_A-1)]. \tag{2.6}$$

3. Results and Discussions

Y have obtained the necessary and sufficient conditions for the non-existence of oscillatory convection under two situations namely, $M = 0$ and $M \neq 0$. However, the authors revised their study and, in the erratum [44] concluded that the conditions obtained by them in [43] are only the sufficient conditions and not the necessary conditions.

Here it is to mention that the case $M=0$ is possible if either $L_e=0$ or $P_{rM}=0,$ but this is not possible on physical as well as mathematical grounds. We have re-examined the work of Y for $M\neq 0.$ It is stated in *Y* that the sufficient conditions for the non-existence of oscillatory convection for $M\neq 0,$ are

$$\frac{N}{M}>0 \quad \text{and} \quad \frac{O}{M}>0. \tag{3.1}$$

These conditions ultimately convert into the following conditions:

$$P_r>P_{rM},\ R_n>0 \text{ and } L_e+N_A>1. \tag{3.2a, b, c}$$

It means that the oscillatory convection exists if at least one of the three conditions given by (3.2) is not satisfied.

We have extended the study with the help of following theorems and have shown that there are sets of conditions other than stated in (3.2 a, b, c) which are the sufficient conditions for the non-existence of the oscillatory modes.

Theorem 3.1: If $R_n>0$ and $L_e+N_A>1,$ then (QP_{rM},π^2) law ensures the non-existence of oscillatory convection.

Proof: Rewrite the expressions for N and O as

$$N=J^3\left\{\frac{L_e{}^2P_r^2}{\pi^2}(\pi^2-QP_{rM})+P_{rM}{}^2+P_r(L_e{}^2P_r^2+P_{rM}{}^2)+\frac{L_e{}^2P_r^3Q}{\pi^2}\right\}+a^2L_eP_rR_nP_{rM}{}^2(L_e+N_A-1) \tag{3.3}$$

and

$$O=J^2P_r^2\left[J^3\left\{1-\frac{\pi^2QP_{rM}}{J^2}\right\}+J^3P_r(1+\frac{\pi^2Q}{J^2})+a^2L_eP_rR_n(L_e+N_A-1)\right]. \tag{3.4}$$

Let the conditions (3.2b) and (3.2c) hold, then both N and O become positive if the (QP_{rM},π^2) law which states that $\pi^2-QP_{rM}>0$ is satisfied irrespective of any conditions on P_r and P_{rM}.

Thus $R_n>0,\ L_e+N_A>1$ and $QP_{rM}<\pi^2$ provide another set of conditions, satisfying conditions in (3.1) ensuring the non-existence of oscillatory convection.

Theorem 3.2: If (QP_{rM},π^2) law fails and $P_r<P_{rM},\ R_n>0,\ L_e+N_A>1,$ then the non-existence of oscillatory convection is ensured under the condition

$$Q<\frac{\pi^2(1+P_r)}{P_{rM}-P_r}.$$

Proof:

Rewrite the expressions for N and O as

$$N=\frac{J^3L_e{}^2P_r^2(P_{rM}-P_r)}{\pi^2}\left(Q_1-Q\right)+a^2L_eP_rR_nP_{rM}{}^2(L_e+N_A-1) \tag{3.5}$$

and

$$O=J^3P_r^2\pi^2(P_{rM}-P_r)\left[Q_2-Q+\frac{a^2L_eP_rR_n(L_e+N_A-1)}{J}\right], \tag{3.6}$$

where

$$Q_1=\frac{\left(L_e{}^2P_r^2+P_{rM}{}^2\right)\left(1+P_r\right)\pi^2}{L_e{}^2P_r^2\left(P_{rM}-P_r\right)} \tag{3.7}$$

and $Q_2 = \dfrac{\pi^2(1+P_r)}{P_{rM} - P_r}$. (3.8)

Since $Q_1 > Q_2$, therefore, under the given conditions, the additional condition $Q < Q_2$ makes both N and O positive. Thus if (QP_{rM}, π^2) law fails then the sufficient conditions for the non-existence of oscillatory convection are

$$P_r < P_{rM},\ R_n > 0,\ L_e + N_A > 1 \text{ and } Q < \frac{\pi^2(1+P_r)}{P_{rM} - P_r}.$$

Theorem 3.3: If $P_r < P_{rM}$ and $R_n > 0$, then the non-existence of oscillatory convection is ensured under the conditions

$$L_e + N_A > 1 + P \text{ if } S \le 1 \text{ and } L_e + N_A > 1 + PS \text{ if } S > 1, \text{ where } S = \frac{P_{rM}^{\ 2}}{L_e^{\ 2} P_r^2}.$$

Proof: Rewrite the expressions for N and O as

$$N = J^3\left(L_e^{\ 2}P_r^2 + P_{rM}^{\ 2}\right)\left(1+P_r\right) + a^2 L_e P_r P_{rM}^{\ 2} R_n [L_e + N_A - 1 - P] \tag{3.9}$$

and

$$O = J^2 P_r^3 a^2 L_e R_n \left[\frac{J^3\left(1+P_r\right)}{a^2 L_e P_r R_n} + L_e + N_A - \left(1+PS\right)\right], \tag{3.10}$$

where $P = \dfrac{\pi^2 J L_e P_r Q\left(P_{rM} - P_r\right)}{a^2 P_{rM}^{\ 2} R_n}$ (3.11)

and $S = \dfrac{P_{rM}^{\ 2}}{L_e^{\ 2} P_r^2}$. (3.12)

It is clear from Eqs. (3.9) and (3.10) that for $P_r < P_{rM}$ and $R_n > 0$, N and O both are positive under the conditions

$$L_e + N_A > 1 + P \text{ if } S \le 1 \qquad \text{and } L_e + N_A > 1 + PS \text{ if } S > 1.$$

This completes the proof of the theorem.

Theorem 3.4: If $P_r > P_{rM}$, $R_n > 0$ but $L_e + N_A < 1$, then a sufficient condition for the non-existence of oscillatory convection is given by

$$\frac{L_e P_r R_n (1 - L_e - N_A)}{\pi^4 (1+P_r)} < 1. \tag{3.13}$$

Proof: Rewrite the expressions for N and O as

$$N = J^3(1+P_r)(L_e^{\ 2}P_r^2 + P_{rM}^{\ 2}) - a^2 L_e P_r R_n P_{rM}^{\ 2}(1 - L_e - N_A) + Q L_e^{\ 2}\pi^2 J P_r^2 (P_r - P_{rM}) \tag{3.14}$$

and

$$O = J^{\ 2} P_r^2 \left\{J^3(1+P_r) - a^2 L_e P_r R_n (1 - L_e - N_A) + Q\pi^2 J (P_r - P_{rM})\right\}. \tag{3.15}$$

It is obvious that both N and O are positive if

$$1 > bigger\ of \left\{\frac{a^2 L_e P_r R_n P_{rM}^{\ 2}(1 - L_e - N_A)}{J^3(L_e^{\ 2}P_r^2 + P_{rM}^{\ 2})(1+P_r)}, \frac{a^2 L_e P_r R_n (1 - L_e - N_A)}{J^3(1+P_r)}\right\}. \tag{3.16}$$

As $\frac{a^2}{J^3} < \frac{1}{\pi^4}$, condition (3.16) can be written as

$$1 > bigger\ of \left[\frac{L_e P_r R_n (1 - L_e - N_A)}{\pi^4 (1 + P_r)} \left\{ \frac{P_{rM}{}^2}{(L_e{}^2 P_r^2 + P_{rM}{}^2)}, 1 \right\} \right],$$

or $\frac{L_e P_r R_n (1 - L_e - N_A)}{\pi^4 (1 + P_r)} < 1,$

which is a sufficient condition for the non-existence of oscillatory convection under the given conditions

Corollary: If condition (3.13) fails, then the theorem holds under the conditions

$Q > Q^*$ if $S \le 1$ and $Q > SQ^*$ if $S > 1$,

where, $$Q^* = \frac{L_e P_r R_n (1 - L_e - N_A)}{\pi^2 (P_r - P_{rM})} \qquad (3.17)$$

and S is given by Eq. (3.12) .

4. Conclusion

The work presented by Y i.e. thermal instability in a nanofluid layer with a vertical magnetic field is reinvestigated. We have obtained a number of sufficient conditions for the non-existence of oscillatory convection.

Theorem-3.1 proves that even if $P_r < P_{rM}$, then (QP_{rM}, π^2) law ensures the non-existence of oscillatory convection. Physically (QP_{rM}, π^2) law holds for large wave numbers or small nanofluid magnetic number. In other words, small wave numbers and large nanofluid magnetic numbers are more prone to the non-existence of oscillatory convection. Likewise, Theorem-3.3 establishes that under certain suitable conditions, the non-existence of oscillatory convection is ensured for $P_r < P_{rM}$ without any restrictions on nanofluid magnetic number. It is also shown that if the condition $L_e + N_A > 1$ is violated, then again under certain physically realistic restrictions on different physical parameters there is the possibility of non-existence of oscillatory convection. We have thus, established that under the violation of conditions given by *Y* in (3.2) for the non-existence of oscillatory convection or the existence of non-oscillatory convection, the convection will not convert to oscillatory.

Nomenclature

R_a	Rayleigh number
L_e	nanofluid Lewis number
N_A	modified diffusivity ratio
P_r	nanofluid Prandtl number
T_1^*	temperature at upper plate
T_0^*	temperature at lower plate
Q	nanofluid magnetic number
a	dimensionless wave number
R_n	concentration Rayleigh number
g	gravitational acceleration (m s^{-2})

P_{rM} nanofluid magnetic Prandtl number
D_B Brownian diffusion coefficient ($m^2 s^{-1}$)
D_T thermophoretic diffusion coefficient ($m^2 s^{-1}$)
k thermal conductivity of the nanofluid ($W m^{-1} K^{-1}$)
L dimensional depth between two parallel plates (m)
c specific heat of fluid at constant pressure ($J kg^{-1} K^{-1}$)
H_0^* uniform vertical magnetic field component along z-axis

Greek Symbols

α thermal diffusivity ($m^2 s^{-1}$)
ρ_{f_0} density of base fluid ($kg m^{-3}$)
ρ_p density of nanoparticles ($kg m^{-3}$)
μ dynamic viscosity of nanofluid (Pa s)
ρ_0 reference nanofluid density ($kg m^{-3}$)
ν kinematic viscosity of nanofluid ($m^2 s^{-1}$)
ϕ_0^* nanoparticle volume fraction at lower plate
ϕ_1^* nanoparticle volume fraction at upper plate
μ_e magnetic permeability of nanofluid ($H m^{-1}$)
η electrical resistivity of nanofluid ($kg m^3 s^{-3} A^{-2}$)
β thermal expansion coefficient of the fluid (K^{-1})
ω dimensionless angular frequency of oscillations
σ electrical conductivity of nanofluid ($kg^{-1} m^{-3} s^3 A^2$)

Subscripts

0 lower plate
1 upper plate

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