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# Analysis of Stability in Couple-Stress Magneto-Fluid

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**Abstract:** *The aim of the present research was to study the effect of magnetic field on the layer of electrically conducting couple-stress fluid heated from below in porous medium. Following the linearized stability theory, Boussinesq approximation and normal mode analysis, the dispersion relation is obtained. The stationary convection, stability of the system and oscillatory modes are discussed. For the case of stationary convection, it is found that the couple-stress parameter and magnetic field have stabilizing effect on the system whereas the medium permeability has a destabilizing effect on the system. The magnetic field introduces oscillatory modes in the system which was non-existent in its absence. A sufficient condition for the non-existent of overstability is also obtained.*

**Keywords**: *Couple-stress fluid, Heated from below, Linearized stability theory, Normal mode analysis method, Porous medium, Uniform magnetic field*

## 1. Introduction

A comprehensive account of thermal instability (Be′nard convection) in a fluid layer, in the absence and presence of magnetic field has been summarized in the celebrated monograph by Chandrasekhar [3]. The use of the Boussinesq approximation has been made throughout, which states that the variations of density in the equations of motion can safely be ignored everywhere except in its association with the external force. The approximation is well justified in the case of incompressible fluids. Abdul-Bari and Al-Rubai [1] have studied the influence of Rayleigh-number in turbulent and laminar region in parallel-plate vertical channels. The influence of radiation on the unsteady free convection flow of a viscous incompressible fluid past a moving vertical plate with Newtonian heating has been investigated theoretically by Narahari and Ishak [10]. Admon et al. [2] have considered the unsteady free convection flow near the stagnation point of a three-dimensional body.

The flow through porous media is of considerable interest for petroleum engineers, for geophysical fluid dynamicists and has importance in chemical technology and industry. An example in the geophysical context is the recovery of crude oil from the pores of reservoir rocks. The derivation of the basic equations of a layer of fluid heated from below in porous medium, using Boussinesq approximation, has been given by Joseph [5]. The study of a layer of fluid heated from below in porous media is motivated both theoretically and by its practical applications in engineering disciplines. Among the applications in engineering disciplines one can find the food process industry, chemical process industry, solidification and centrifugal casting of metals. The development of geothermal power resources has increased general interest in the properties of convection in porous medium. Lapwood [9] has studied the stability of convective flow in a porous medium using Rayleigh's procedure. The Rayleigh instability of a thermal boundary layer in flow through a porous medium has been considered by Wooding [16]. When the fluid slowly percolates through the pores of the rock, the gross effect is represented by the well-known Darcy's law. An extensive and updated account of convection in porous media has been given by Nield and Bejan [11]. The effect of a magnetic field on the stability of flow is of interest in geophysics, particularly in the study of Earth's core where the Earth's mantle, which consists of conducting fluid, behaves like a porous medium which can become convectively unstable as a result of differential diffusion. The other application of the results of flow through a porous medium in the presence of a magnetic field is in the study of the stability of a convective flow in the geothermal region. The fluid has been considered to be Newtonian in all the above studies.

The theory of couple-stress fluid has been formulated by Stokes [14]. One of the applications of couple-stress fluid is its use to the study of the mechanisms of lubrications of synovial joints, which has become the object of scientific research. A human joint is a dynamically loaded bearing which has articular cartilage as the bearing and synovial fluid as the lubricant. When a fluid is generated, squeeze-film action is capable of providing considerable protection to the cartilage surface. The shoulder, ankle, knee and hip joints are the loaded–bearing synovial joints of the human body and these joints have a low friction coefficient and negligible wear. Normal synovial fluid is a viscous, non-Newtonian fluid and is generally clear or yellowish. According to the theory of Stokes [14], couple-stresses appear in noticeable magnitudes in fluids with very large molecules.

Many of the flow problems in fluids with couple-stresses, discussed by Stokes, indicate some possible experiments, which could be used for determining the material constants, and the results are found to differ from those of Newtonian fluid. Couple-stresses are found to appear in noticeable magnitudes in polymer solutions for force and couple-stresses. This theory is developed in an effort to examine the simplest generalization of the classical theory, which would allow polar effects. The constitutive equations proposed by Stokes [14] are:

$$T_{(ij)} = (-p + \lambda D_{kk})\delta_{ij} + 2\mu D_{ij} \ ,$$
$$T_{[ij]} = -2\eta \vec{W}_{ij.kk} - \frac{\rho}{2}\vec{\varepsilon}_{ijs} G_s \ ,$$

and

$$M_{ij} = 4\eta\vec{\omega}_{j,i} + 4\eta'\vec{\omega}_{i,j} \ ,$$

where

$$D_{ij} = \frac{1}{2}\left(V_{i,j} + V_{j,i}\right), \vec{W}_{ij} = -\frac{1}{2}\left(V_{i,j} - V_{j,i}\right)$$

and

$$\vec{\omega}_i = \frac{1}{2}\vec{\varepsilon}_{ijk} V_{k,j} \ .$$

Here $T_{ij}$, $T_{(ij)}$, $T_{[ij]}$, $M_{ij}$, $D_{ij}$ , $\vec{W}_{i,j}$, $\vec{\omega}_i$, $G_s$, $\vec{\varepsilon}_{ijk}$, V, $\rho$ and $\lambda$, $\mu$, $\eta$, $\eta'$, are stress tensor, symmetric part of $T_{ij}$, anti-symmetric part of $T_{ij}$, the couple-stress tensor, deformation tensor, the vorticity tensor, the vorticity vector, body couple, the alternating unit tensor, velocity field, the density and material constants respectively. The dimensions of $\lambda$ and $\mu$ are those of viscosity whereas the dimensions of $\eta$ and $\eta'$ are those of momentum.

Since the long chain hyaluronic acid molecules are found as additives in synovial fluids, Walicki and Walicka [15] modeled the synovial fluid as a couple-stress fluid. The synovial fluid is the natural lubricant of joints of the vertebrates. The detailed description of the joint lubrication has very important practical implications. Practically all diseases of joints are caused by or connected with a malfunction of the lubrication. The efficiency of the physiological joint lubrication is caused by several mechanisms. The synovial fluid is, due to its content of the hyaluronic acid, a fluid of high viscosity, near to a gel. Goel et al. [4] have studied the hydromagnetic stability of an unbounded couple-stress binary fluid mixture under rotation with vertical temperature and concentration gradients. Sharma et al. [12] have considered a couple-stress fluid with suspended particles heated from below. They have found that for stationary convection, couple-stress has a stabilizing effect whereas suspended particles have a destabilizing effect. Kumar et al. [8] have considered the thermal instability of a layer of a couple–stress fluid acted on by a uniform rotation, and have found that for stationary convection, the rotation has a stabilizing effect whereas couple-stress has both stabilizing and destabilizing effects. Kumar and Kumar [6] have investigated the transport of vorticity in couple-stress fluid in the presence of suspended particles. In another study, Kumar and Mohan [7] studied the thermosolutal convection in a couple-stress fluid in the presence of uniform rotation.

Keeping in mind the importance in geophysics, soil sciences, ground water hydrology, astrophysics and various applications mentioned above, the electrically conducting couple-stress fluid heated from below in porous medium in the presence of uniform magnetic field has been considered in the present paper.

## 2. Formulation of the Problem and Perturbation Equations

The present problem is studied using methods of linearized stability theory and normal mode analysis. First of all linearized perturbation equations relevant to the problem are obtained. Here we consider an infinite, horizontal, incompressible, electrically conducting couple-stress fluid layer of thickness $d$, heated from below so that the temperatures and densities at the bottom surface $z = 0$ are $T_0, \rho_0$ and at the upper surface $z = d$ are $T_d, \rho_d$ , respectively, and that a uniform adverse temperature gradient $\beta(= |dT/dz|)$ is maintained. This layer is acted on by a gravity field $\vec{g}(0,0,-g)$ and a uniform magnetic field $\vec{H}(0,0,H)$.

Let $p,\ \rho,\ T\ and\ \vec{q}(u,v,w)$ denote the fluid pressure, density, temperature and filter velocity, respectively. Then the momentum balance, mass balance and energy balance equations of couple-stress fluid through porous medium (Stokes [9], Joseph [5]) are

$$\frac{1}{\varepsilon}\left[\frac{\partial\vec{q}}{\partial t}+\frac{1}{\varepsilon}(\vec{q}.\nabla)\vec{q}\right] = -\frac{1}{\rho_0}\nabla p+\vec{g}\left(1+\frac{\delta\rho}{\rho_0}\right)-\frac{1}{k_1}\left(\nu-\frac{\mu'}{\rho_0}\nabla^2\right)\vec{q}+\frac{\mu_e}{4\pi\rho_0}\left(\nabla\times\vec{H}\right)\times\vec{H} \tag{1}$$

$$\nabla.\vec{q} = 0, \tag{2}$$

$$E\frac{\partial T}{\partial t}+(\vec{q}.\nabla)T = \kappa\nabla^2 T. \tag{3}$$

The Maxwell's equations are

$$\varepsilon\frac{\partial\vec{H}}{\partial t} = curl\left(\vec{q}\times\vec{H}\right)+\varepsilon\eta\nabla^2\vec{H}, \tag{4}$$

$$\nabla.\vec{H} = 0, \tag{5}$$

and the equation of state is

$$\rho = \rho_0[1-\alpha(T-T_0)], \tag{6}$$

where the suffix zero refers to values at the reference level $z = 0$. In writing equation (1), use has been made of the Boussinesq approximation. The kinematic viscosity $\nu$, thermal diffusivity $\kappa$, electrical resistivity $\eta$, couple-stress viscosity $\mu'$ and coefficient of thermal expansion $\alpha$ are all assumed to be constants. $E = \varepsilon + (1-\varepsilon)\rho_S c_S/(\rho_0 c_V)$, is constant where $\rho_S, c_S$ and $\rho_0, c_V$ stand for density and heat capacity of solid (porous matrix) material and fluid, respectively.

The basic motionless solution is

$$\vec{q} = (0,0,0), T = T_0-\beta z,\ \rho = \rho_0(1+\alpha\beta z). \tag{7}$$

Consider small perturbations around the basic solution and let $\vec{q}(u,v,w), \theta, \delta\rho, \delta p$ and $\vec{h}\left(h_x, h_y, h_z\right)$ denote, respectively, the perturbations in fluid velocity (0,0,0), temperature $T$, density $\rho$, pressure $p$ and magnetic field $\vec{H}(0,0,H)$. The change in density $\delta\rho$, caused mainly by the perturbation $\theta$ in temperature, is given by

$$\delta\rho = -\alpha\rho_0\theta. \tag{8}$$

The linearized hydromagnetic perturbation equations of the couple-stress fluid become

$$\frac{1}{\varepsilon}\frac{\partial\vec{q}}{\partial t} = -\frac{1}{\rho_0}\nabla\delta p-\vec{g}\alpha\theta-\frac{1}{k_1}\left(\nu-\frac{\mu'}{\rho_0}\nabla^2\right)\vec{q}+\frac{\mu_e}{4\pi\rho_0}\left(\nabla\times\vec{h}\right)\times\vec{H}, \tag{9}$$

$$\nabla.\vec{q} = 0, \tag{10}$$

$$E\frac{\partial\theta}{\partial t} = \beta w+\kappa\nabla^2\theta, \tag{11}$$

$$\varepsilon\frac{\partial\vec{h}}{\partial t}=(H.\nabla)\vec{q}+\varepsilon\eta\nabla^2\vec{h}, \tag{12}$$

$$\nabla.\vec{h}=0. \tag{13}$$

Writing the scalar components of equation (9) and eliminating $u, v, h_x, h_y, \delta p$ between them by using (10) and (13), we obtain

$$\frac{1}{\varepsilon}\frac{\partial}{\partial t}\nabla^2 w+\frac{1}{k_1}\left(\nu-\frac{\mu'}{\rho_0}\nabla^2\right)\nabla^2 w-g\alpha\left(\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right)-\left(\frac{\mu_e H}{4\pi\rho_0}\right)\frac{\partial}{\partial z}\nabla^2 h_z=0, \tag{14}$$

$$\left(E\frac{\partial}{\partial t}-\kappa\nabla^2\right)\theta=\beta w, \tag{15}$$

$$\varepsilon\left(\frac{\partial}{\partial t}-\eta\nabla^2\right)h_z=H\frac{\partial w}{\partial z}. \tag{16}$$

Here we consider the case in which both the boundaries are free and are maintained at constant temperatures and the medium adjoining the fluid is a perfect electrical conductor. The case of two free boundaries is slightly artificial, except in stellar atmospheres (Speigel [13]) and in certain geophysical situation where it is most appropriate. However, the case of two free boundaries allows us to obtain analytical solution without affecting the essential features of the problem. Then the boundary conditions appropriate to the problem are

$$w=\frac{\partial^2 w}{\partial z^2}=0, \theta=0, and\ h_z=0\ at\ z=0\ and\ z=d. \tag{17}$$

## 3. Dispersion Relation Using Normal Mode Analysis Method

We now analyze the disturbances into normal modes, assuming that the perturbation quantities are of the form

$$[w,\ \theta,\ h_z]=[W(z),\Theta(z),K(z)]exp\left(ik_x x+ik_y y+nt\right), \tag{18}$$

where $k_x, k_y$ are wave numbers along the $x-$direction and $y-$direction, respectively, $k\left[=\left(k_x^2+k_y^2\right)^{1/2}\right]$is the resultant wave number and $n$ is, in general, a complex constant. Using expression (18), equations (14) - (16) in non-dimensional form become

$$(D^2-a^2)\left[\frac{\sigma}{\varepsilon}+\frac{1}{P_l}-\frac{F}{P_l}(D^2-a^2)\right]W+\frac{g\alpha d^2}{\nu}a^2\Theta-\frac{\mu_e H d}{4\pi\rho_0\nu}(D^2-a^2)DK=0, \tag{19}$$

$$(D^2-a^2-Ep_1\sigma)\Theta=-\left(\frac{\beta d^2}{\kappa}\right)W, \tag{20}$$

$$(D^2-a^2-p_2\sigma)K=-\left(\frac{Hd}{\varepsilon\eta}\right)DW, \tag{21}$$

where we have put $a=kd, \sigma=nd^2/\nu, x^*=x/d, y^*=y/d, z^*=z/d\ and\ D=d/dz$.

$p_1=\nu/\kappa$ is the Prandtl number, $p_2=\nu/\eta$ is the magnetic Prandtl number, $P_l=k_1/d^2$ is the dimensionless medium permeability and $F=\nu'/(\rho_0 d^2\nu)$ is the dimensionless couple-stress parameter. We shall suppress the stars $(*)$ for convenience hereafter.

Using (18), the boundary conditions become

$$W=D^2W=0, \Theta=0, K=0\ for\ z=0\ and\ z=1. \tag{22}$$

Using the boundary conditions (22), it can be shown that all the even order derivatives of $W$ must vanish for $z=0$ and 1 and hence the proper solution of $W$ characterizing the lowest mode is

$$W=W_0 sin(\pi z), \tag{23}$$

where $W_0$ is a constant.

Eliminating $\Theta$ and $K$ from equations (19)-(21), we obtain

$$(D^2-a^2)(D^2-a^2-Ep_1)\left[\left\{\frac{\sigma}{\varepsilon}+\frac{1}{P_l}-\frac{F}{P_l}(D^2-a^2)\right\}\{D^2-a^2-p_2\sigma\}+\frac{Q}{\varepsilon}D^2\right]W$$
$$=Ra^2(D^2-a^2-p_2\sigma)W, \tag{24}$$

where $R=g\alpha\beta d^4/(\nu\kappa)$ is the Rayleigh number and $Q=\mu_e H^2 d^2/(4\pi\rho_0\nu\eta)$ is the Chandrasekhar number.

Substituting the proper solution (23) in equation (24), we obtain the dispersion relation

$$R_1=\frac{1+x}{x}\left[(1+x+iEp_1\sigma_1)\left\{\frac{i\sigma_1}{\varepsilon}+\frac{1}{P}+\frac{F\pi^2}{P}(1+x)\right\}+\frac{Q_1(1+x+iEp_1\sigma_1)}{\varepsilon(1+x+ip_2\sigma_1)}\right], \tag{25}$$

where

$$R_1=R/\pi^4, Q_1=Q/\pi^2, x=a^2/\pi^2, i\sigma_1=\sigma/\pi^2 \; and \; P=\pi^2 P_l.$$

## 4. Important Theorems and Discussion

**Theorem 1**: The system is stable or unstable.

**Proof**: Multiplying equation (19) by $W^*$, the complex conjugate of $W$, and using (20) and (21) together with the boundary conditions (22), we obtain

$$\frac{F}{P_l}I_1+\left(\frac{1}{P_l}+\frac{\sigma}{\varepsilon}\right)I_2-\frac{g\alpha\kappa a^2}{\nu\beta}(I_3+Ep_1\sigma^* I_4)+\frac{\varepsilon\eta}{4\pi\rho_0\nu}(I_5+p_2\sigma^* I_6)=0, \tag{26}$$

where

$$I_1=\int_0^1(|D^2W|^2+2a^2|DW|^2+a^4|W|^2)dz,\; I_2=\int_0^1(|DW|^2+a^2|W|^2)dz,$$
$$I_3=\int_0^1(|D\Theta|^2+a^2|\Theta|^2)dz,\; I_4=\int_0^1|\Theta|^2dz,\; I_5=\int_0^1(|D^2K|+2a^2|DK|^2+a^4|K|^2)dz,$$
$$I_6=\int_0^1(|DK|^2+a^2|K|^2)dz, \tag{27}$$

which are all positive definite. Putting $\sigma=\sigma_r+i\sigma_i$ and then equating real and imaginary parts of equation (26), we obtain

$$\sigma_r\left[\frac{I_2}{\varepsilon}-\frac{g\alpha\kappa a^2}{\nu\beta}Ep_1I_4+\frac{\varepsilon\eta}{4\pi\rho_0\nu}p_2I_6\right]=-\left[\frac{F}{P_l}I_1+\frac{1}{P_l}I_2-\frac{g\alpha\kappa a^2}{\nu\beta}I_3+\frac{\varepsilon\eta}{4\pi\rho_0\nu}I_5\right] \tag{28}$$

and

$$\sigma_i\left[\frac{I_2}{\varepsilon}+\frac{g\alpha\kappa a^2}{\nu\beta}Ep_1I_4-\frac{\varepsilon\eta}{4\pi\rho_0\nu}p_2I_6\right]=0. \tag{29}$$

It is clear from equation (28) that $\sigma_r$ is positive or negative. The system is, therefore, stable or unstable.

**Theorem 2**: The modes may be oscillatory or non-oscillatory in contrast to case of no magnetic field, where modes are non-oscillatory.

**Proof**: Equation (29) yields that $\sigma_i=0 \; or \; \sigma_i\neq 0$, which means that the modes may be non-oscillatory or oscillatory. In the absence of magnetic field, equation (29) reduces to

$$\sigma_i\left[\frac{I_2}{\varepsilon}+\frac{g\alpha\kappa a^2}{\nu\beta}Ep_1I_4\right]=0, \tag{30}$$

and the terms in brackets are positive definite. Thus $\sigma_i=0$ which means that oscillatory modes are not allowed and the principle of exchange of stabilities is valid. The oscillatory modes are introduced due to the presence of magnetic field which were non-existent in its absence.

**Theorem 3**: The system is stable for $\frac{g\alpha\kappa}{\nu\beta} \le \frac{27\pi^4}{4}$ and under the condition $\frac{g\alpha\kappa}{\nu\beta} > \frac{27\pi^4}{4}$, the system becomes unstable.

**Proof**: From equation (29), it is clear that $\sigma_i$ is zero when the quantity multiplying it is not zero and arbitrary when this quantity is zero.

If $\sigma_i \neq 0$, then equation (29) gives

$$\frac{I_2}{\varepsilon} = -\frac{g\alpha\kappa a^2}{\nu\beta} E p_1 I_4 + \frac{\varepsilon\eta}{4\pi\rho_0\nu} p_2 I_6$$

Substituting this in equation (28), we get

$$\frac{2\sigma_r I_2}{\varepsilon} + \frac{F}{P_l} I_1 + \frac{1}{P_l} I_2 + \frac{\varepsilon\eta}{4\pi\rho_0\nu} I_5 = \frac{g\alpha\kappa a^2}{\nu\beta} I_3 \,, \tag{31}$$

Equation (31) on using Rayleigh-Ritz inequality gives

$$\frac{(\pi^2 + a^2)^3}{a^2} \int_0^1 |W|^2 dz + \frac{(\pi^2 + a^2)}{a^2} \left\{ \frac{\varepsilon\eta}{4\pi\rho_0\nu} I_5 + \frac{1}{P_l} I_2 + \frac{2\sigma_r}{\varepsilon} I_2 + \frac{F}{P_l} I_1 \right\} \le \frac{g\alpha\kappa}{\nu\beta} \int_0^1 |W|^2 dz \; . \tag{32}$$

Therefore, it follows from equation (32) that

$$\left[\frac{27\pi^4}{4} - \frac{g\alpha\kappa}{\nu\beta}\right] \int_0^1 |W|^2 dz + \frac{(\pi^2 + a^2)}{a^2} \left\{ \frac{\varepsilon\eta}{4\pi\rho_0\nu} I_5 + \frac{1}{P_l} I_2 + \frac{2\sigma_r}{\varepsilon} I_2 + \frac{F}{P_l} I_1 \right\} \le 0, \tag{33}$$

since minimum value of $\frac{(\pi^2+a^2)^3}{a^2}$ with respect to $a^2$ is $\frac{27\pi^4}{4}$.

Now, let $\sigma_r \ge 0$, we necessary have from (33) that

$$\frac{g\alpha\kappa}{\nu\beta} > \frac{27\pi^4}{4} \; . \tag{34}$$

Hence, if

$$\frac{g\alpha\kappa}{\nu\beta} \le \frac{27\pi^4}{4} \quad , \tag{35}$$

then $\sigma_r < 0$. Therefore, the system is stable. Thus, under condition (35), the system is stable and under condition (34) the system becomes unstable.

**Theorem 4**: For stationary convection case:

**(I)** The couple-stress parameter and magnetic field have stabilizing effects on the system.

**(II)** The medium permeability hastens the onset of convection i.e. has destabilizing effect on the system.

**Proof**: When instability sets in as stationary convection, the marginal state will be characterized by $\sigma = 0$. Putting $\sigma = 0$, the dispersion relation (25) reduces to

$$R_1 = \frac{1+x}{x} \left[ \frac{1+x}{P} (1 + \pi^2 F(1+x) + \frac{Q_1}{\varepsilon} \right]. \tag{36}$$

which expresses the Rayleigh number $R_1$ as a function of the dimensionless wave number $x$ and the parameters $F, Q_1$ and $P$. To investigate the effects of couple-stress parameter, magnetic field and permeability, we examine the behaviour of $\frac{dR_1}{dF}, \frac{dR_1}{dQ_1}$ and $\frac{dR_1}{dP}$ analytically.

**(I)** Equation (36) yields

$$\frac{dR_1}{dF} = \frac{\pi^2(1+x)^3}{xP}, \tag{37}$$

which is positive. Thus, the couple-stress parameter has a stabilizing effect on the system for the case of stationary convection.

Also (36) gives

$$\frac{dR_1}{dQ_1} = \frac{1+x}{\varepsilon x}, \tag{38}$$

which is always positive. The magnetic field, therefore, has a stabilizing effect on the system for the case of stationary convection.

Thus, the couple-stress parameter and the magnetic field postpone the onset of convection i.e. have stabilizing effect on the system.

**(II)** It is evident from equation (36) that

$$\frac{dR_1}{dP} = -\frac{(1+x)^2}{xp^2}(1+\pi^2 F\overline{1+x}), \tag{39}$$

which is negative. Hence, the medium permeability hastens the onset of convection and hence has a destabilizing effect on the system for the case of stationary convection.

**Theorem 5**: The sufficient condition for the non-existence of overstability is

$$\kappa < \eta\left[\varepsilon + (1-\varepsilon)\frac{\rho_S c_S}{\rho_0 c_V}\right].$$

**Proof**: Here we discuss the possibility of whether instability may occur as overstability. Since for overstability, we wish to determine the critical Rayleigh number for the onset of instability via a state of pure oscillations, it suffices to find conditions for which equation (25) will admit of solutions with $\sigma_i$ real.

Equating real and imaginary parts of equation (25), and eliminating $R_1$ from the resulting equations, we obtain

$$\sigma_i^2\left[p_2^2\left(\frac{b}{\varepsilon} + \frac{Ep_1}{P}\overline{1+\pi^2 Fb}\right)\right] = -\left[b^2\left(\frac{b}{\varepsilon} + \frac{Ep_1}{P}\overline{1+\pi^2 Fb}\right) + \frac{Q_1 b}{\varepsilon}(Ep_1 - p_2)\right], \tag{40}$$

where we have put $b = 1 + x$.

Since $\sigma_i$ is real for overstability, the value of $\sigma_1^2$ is positive. Equation (40) shows that this is impossible if

$$Ep_1 > p_2, \tag{41}$$

which implies that

$$\kappa < \eta\left[\varepsilon + (1-\varepsilon)\frac{\rho_S c_S}{\rho_0 c_V}\right]. \tag{42}$$

The condition (42) is, therefore, a sufficient condition for the non-existence of overstability, the violation of which does not necessary imply the occurrence of overstability.

## 5. Conclusions

An attempt has been made to investigate the effect of uniform magnetic field on a layer of electrically conducting couple-stress fluid heated from below in porous medium. The investigation of thermal instability is motivated by its direct relevance to soil sciences, groundwater hydrology, geophysical, astrophysical and biometrices. The main conclusions from the analysis of this paper are as follows:

- It is found that the magnetic field introduces oscillatory modes in the system which was non-existent in its absence.

- It is observed that the system is stable for $\frac{g\alpha\kappa}{\nu\beta} \leq \frac{27\pi^4}{4}$ and under the condition $\frac{g\alpha\kappa}{\nu\beta} > \frac{27\pi^4}{4}$, the system becomes unstable.
- For the case of stationary convection, the couple-stress parameter and magnetic field are found to have stabilizing effects on the system whereas the medium permeability has a destabilizing effect on the system.
- The case of overstability is also considered. The condition

$$\kappa < \eta \left[\varepsilon + (1-\varepsilon)\frac{\rho_S c_S}{\rho_0 c_V}\right]$$

is the sufficient condition for the non-existence of overstability, the violation of which does not necessarily imply the occurrence of overstability.

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