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Certain Subclasses of Bi-Univalent and Meromorphic Functions Defined By Al-Oboudi Differential Operator

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Abstract: *In this paper, we introduce a subclass* $\sum_{M}^{h,p}(k,\delta,\mu,\lambda)$ *of bi-univalent and meromorphic functions by using Al-Oboudi differential operator on* $\Delta=\{z\in\mathbb{C}: 1<|z|<\infty\}$. *Also we obtain bounds of coefficients* $|b_0|$ *and* $|b_1|$ *for functions belongs to* $\sum_{M}^{h,p}(k,\delta,\mu,\lambda)$. *The results obtained in this paper are more better and generalized of previous results of various author.*

Keywords: *Harmonic functions*, *Univalent functions*, *Differential operator*, *Extreme points*.

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1. Introduction

Let Σ be the class of functions f of the form

$$f(z)=z+\sum_{l=0}^{\infty}\frac{b_l}{z^l}\ , \tag{1}$$

which are meromorphic univalent in the domain $\Delta=\{z\in\mathbb{C}: 1<|z|<\infty\}$. Since every function f belong to Σ has an inverse function f^{-1} exist and inverse function satisfies conditions:

$$f^{-1}(f(z))=z\ \ (z\in\Delta)$$

and

$$f(f^{-1}(w))=w,\ w\in\Delta\quad (M<|w|<\infty,\ M>0),$$

where

$$f^{-1}(w)=q(w)=w-b_0-\frac{b_1}{w}-\frac{b_2+b_0b_1}{w^2}-\frac{b_3+2b_0b_2+b_0^2b_1+b_1^2}{w^3}+\cdots. \tag{2}$$

A function $f\in\Sigma$ is said to be meromorphic bi-univalent in Δ if both f and f^{-1} are meromorphic univalent in Δ. The class of meromorphic bi-univalent functions of the form (1) in Δ is denoted by Σ_M.

Srivastava et al. [17], Safa Salehian and Ahmad Zireh [12], Hamidi et al. [7], Amol Patil and Uday Naik [11] and many other researchers (see [4, 5, 8, 9, 10, 14, 15, 18]) have introduced new subclasses of meromorphically bi-univalent functions and obtained estimates on the initial coefficients for functions in each of these subclasses.

Let $\mathcal{A}$ denote the class of analytic functions $h(z)$ of the form

$$h(z)=z+\sum_{l=2}^{\infty}a_l z^l \tag{3}$$

defined in the unit disc $\mathbb{U}=\{z\in\mathbb{C}: |z|<1\}$ with normalization $h(0)=h_z(0)-1=0$. Let the class of all normalized analytic univalent functions in the unit disc $\mathbb{U}$ is denoted by S. A function $h\in\mathcal{A}$ is said to be bi-univalent in $\mathbb{U}$ if both h and h^{-1} are univalent in $\mathbb{U}$. Let the class of analytic bi-univalent functions is denoted by Σ'. Brannan and Taha [2], Srivastava et al. [16] and many other researchers (see [3, 6]) introduced certain subclasses of bi-univalent function class Σ'.

Now, Al-Oboudi [1] introduced the Al-Oboudi operator $D_\delta^k: \mathcal{A} \to \mathcal{A}$ and defined as

$$D^k h(z) = D_\delta^k h(z) = z + \sum_{l=2}^{\infty} [1 + (l-1)\delta]^k a_l z^l, \quad k \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}, \delta \geq 0,$$

where $h \in \mathcal{A}$ of the form (3).

Amol Patil and Uday Naik [11] extend the Al-Oboudi operator $D_\delta^k: \Sigma \to \Sigma$ and defined as

$$D^k f(z) = D_\delta^k f(z) = z + (1-\delta)^k b_0 + \sum_{l=1}^{\infty} [1 - (l+1)\delta]^k b_l z^{-l}, \quad k \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}, \delta > 1,$$

where $f \in \Sigma$ of the form (1).

In 2019, Saideh Hajiparvaneh and Ahmad Zireh [13] define the subclass $\Sigma_M^{h,p}(\mu, \lambda)$ consisting of meromorphic functions $f(z)$ of the form (1) satisfies the following conditions:

$$f \in \Sigma_M, \left[(1-\lambda)\left(\frac{f(z)}{z}\right)^{\mu} + \lambda f'(z)\left(\frac{f(z)}{z}\right)^{\mu-1}\right] \in h(\Delta)$$

and

$$\left[(1-\lambda)\left(\frac{q(w)}{w}\right)^{\mu} + \lambda q'(w)\left(\frac{q(w)}{w}\right)^{\mu-1}\right] \in p(\Delta),$$

where q is function given by (2).

Motivated by the aforecited works, we introduce new subclasses of bi-univalent and meromorphic functions by using Al-Oboudi Differential operator. Also obtain the coefficient bounds $|b_0|$ and $|b_1|$ for functions in this new subclasses.

2. Coefficient Estimates

Definition 2.1 Let the analytic functions $h, p: \Delta \to \mathbb{C}$ be

$$h(z) = 1 + \frac{h_1}{z} + \frac{h_2}{z^2} + \frac{h_3}{z^3} + \cdots, \qquad p(z) = 1 + \frac{p_1}{z} + \frac{p_2}{z^2} + \frac{p_3}{z^3} + \cdots,$$

such that

$$\min\{\Re(h(z)), \Re(p(z))\} > 0 \quad (z \in \Delta).$$

Definition 2.2 A function $f(z)$ of the form (1) is said to be in the class $\Sigma_M^{h,p}(k, \delta, \mu, \lambda)$, $k \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}, \lambda \geq 1, \lambda > \mu, \mu \geq 0$ and $\delta > 1$ if satisfies the following conditions :

$$f \in \Sigma_M, \left[(1-\lambda)\left(\frac{D_\delta^k f(z)}{z}\right)^{\mu} + \lambda\left(D_\delta^k f(z)\right)'\left(\frac{D_\delta^k f(z)}{z}\right)^{\mu-1}\right] \in h(\Delta) \tag{4}$$

and

$$\left[(1-\lambda)\left(\frac{D_\delta^k q(w)}{w}\right)^{\mu} + \lambda\left(D_\delta^k q(w)\right)'\left(\frac{D_\delta^k q(w)}{w}\right)^{\mu-1}\right] \in p(\Delta), \tag{5}$$

where q is the function given by (2).

For $k = 0$, the class $\Sigma_M^{h,p}(k, \delta, \mu, \lambda)$ become $\Sigma_M^{h,p}(\mu, \lambda)$, studied by Saideh Hajiparvaneh and Ahmad Zireh [13].

Remark 2.1 For various choices of h and p, we get various subclasses of class $\Sigma_M^{h,p}(k, \delta, \mu, \lambda)$ as follows:

If take $h(z) = p(z) = \left(\frac{1+\frac{1}{z}}{1-\frac{1}{z}}\right)^{\alpha} = 1 + \frac{2\alpha}{z} + \frac{2\alpha^2}{z^2} + \cdots \quad (0 < \alpha \leq 1, z \in \Delta)$ in Definition 2.2, then we get subclass $\Sigma_M^{h,p}(k, \delta, \mu, \lambda) = \Sigma_M^*(k, \delta, \mu, \lambda, \alpha)$, studied by Bobalade and Sangle [4].

Definition 2.3 [4] A function $f(z) \in \Sigma_M^{h,p}(k, \delta, \mu, \lambda)$ of the form (1) belongs to the class $\Sigma_M^*(k, \delta, \mu, \lambda, \alpha)$ if

$$f \in \Sigma_M, \left|\arg\left[(1-\lambda)\left(\frac{D_\delta^k f(z)}{z}\right)^{\mu} + \lambda\left(D_\delta^k f(z)\right)'\left(\frac{D_\delta^k f(z)}{z}\right)^{\mu-1}\right]\right| < \frac{\alpha\pi}{2} \quad (z \in \Delta)$$

and

$$\left|\arg\left[(1-\lambda)\left(\frac{D_\delta^k q(w)}{w}\right)^\mu + \lambda\left(D_\delta^k q(w)\right)'\left(\frac{D_\delta^k q(w)}{w}\right)^{\mu-1}\right]\right| < \frac{\alpha\pi}{2} \quad (w \in \Delta),$$

where $k \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}, \lambda \geq 1, \lambda > \mu, \mu \geq 0, \delta > 1$ and $0 < \alpha \leq 1$.

Remark 2.2 If take $h(z) = p(z) = \frac{1+\frac{(1-2\beta)}{z}}{1-\frac{1}{z}} = 1 + \frac{2(1-\beta)}{z} + \frac{2(1-\beta)}{z^2} + \cdots \quad (0 \leq \beta < 1, z \in \Delta)$ in Definition 2.2, then we get subclass $\Sigma_M^{h,p}(k,\delta,\mu,\lambda) = \Sigma_M^*(k,\delta,\mu,\lambda,\beta)$, studied by Bobalade and Sangle [4].

Definition 2.4 [4] A function $f(z) \in \Sigma_M^{h,p}(k,\delta,\mu,\lambda)$ of the form (1) belongs to the class $\Sigma_M^*(k,\delta,\mu,\lambda,\beta)$ if

$$f \in \Sigma_M, \Re\left[(1-\lambda)\left(\frac{D_\delta^k f(z)}{z}\right)^\mu + \lambda\left(D_\delta^k f(z)\right)'\left(\frac{D_\delta^k f(z)}{z}\right)^{\mu-1}\right] > \beta \quad (z \in \Delta)$$

and

$$\Re\left[(1-\lambda)\left(\frac{D_\delta^k q(w)}{w}\right)^\mu + \lambda\left(D_\delta^k q(w)\right)'\left(\frac{D_\delta^k q(w)}{w}\right)^{\mu-1}\right] > \beta \quad (w \in \Delta),$$

where $k \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}, \lambda \geq 1, \lambda > \mu, \mu \geq 0, \delta > 1$ and $0 \leq \beta < 1$.

If we put $k = 0$ in the classes $\Sigma_M^*(k,\delta,\mu,\lambda,\alpha)$ and $\Sigma_M^*(k,\delta,\mu,\lambda,\beta)$, then we get two classes $\Sigma_M^*(\mu,\lambda,\alpha)$ and $\Sigma_M^*(\mu,\lambda,\beta)$ respectively, studied by Orhan et al. [10].

Theorem 2.1 *Let $f(z)$ of the form* (1) *belong to the class $\Sigma_M^{h,p}(k,\delta,\mu,\lambda)$. Then*

$$|b_0| \leq min\left[\sqrt{\frac{|h_1|^2+|p_1|^2}{2(\mu-\lambda)^2(1-\delta)^{2k}}}, \sqrt{\frac{|h_2|+|p_2|}{|(\mu-2\lambda)(\mu-1)(1-\delta)^{2k}|}}\right] \tag{6}$$

and

$$|b_1| \leq min\left[\frac{|h_2|+|p_2|}{2|(\mu-2\lambda)(1-2\delta)^k|}, \sqrt{\frac{|h_2|^2+|p_2|^2}{2(\mu-2\lambda)^2(1-2\delta)^{2k}} + \frac{(\mu-1)^2(|h_1|^2+|p_1|^2)^2}{16(\mu-\lambda)^4(1-2\delta)^{2k}}}\right]. \tag{7}$$

Proof. From conditions (4) and (5), we have

$$(1-\lambda)\left(\frac{D_\delta^k f(z)}{z}\right)^\mu + \lambda\left(D_\delta^k f(z)\right)'\left(\frac{D_\delta^k f(z)}{z}\right)^{\mu-1} = h(z) \;\; (z \in \Delta) \tag{8}$$

and

$$(1-\lambda)\left(\frac{D_\delta^k q(w)}{w}\right)^\mu + \lambda\left(D_\delta^k q(w)\right)'\left(\frac{D_\delta^k q(w)}{w}\right)^{\mu-1} = p(w) \;\; (w \in \Delta), \tag{9}$$

where $h(z)$ and $p(w)$ are functions such that it's real part positive in Δ and have forms

$$h(z) = 1 + \frac{h_1}{z} + \frac{h_2}{z^2} + \frac{h_3}{z^3} + \cdots \tag{10}$$

and

$$p(w) = 1 + \frac{p_1}{w} + \frac{p_2}{w^2} + \frac{p_3}{w^3} + \cdots. \tag{11}$$

Implies

$$(1-\lambda)\left(\frac{D_\delta^k f(z)}{z}\right)^\mu + \lambda\left(D_\delta^k f(z)\right)'\left(\frac{D_\delta^k f(z)}{z}\right)^{\mu-1} = 1 + \frac{h_1}{z} + \frac{h_2}{z^2} + \frac{h_3}{z^3} + \cdots, (z \in \Delta) \tag{12}$$

and

$$(1-\lambda)\left(\frac{D_\delta^k q(w)}{w}\right)^\mu + \lambda\left(D_\delta^k q(w)\right)'\left(\frac{D_\delta^k q(w)}{w}\right)^{\mu-1} = 1 + \frac{p_1}{w} + \frac{p_2}{w^2} + \frac{p_3}{w^3} + \cdots (w \in \Delta). \tag{13}$$

Now, equating the coefficients in equation (12) and (13), we obtain

$$(\mu-\lambda)(1-\delta)^k b_0 = h_1, \tag{14}$$

$$(\mu-2\lambda)\left[(1-2\delta)^k b_1 + \left(\frac{\mu-1}{2}\right)(1-\delta)^{2k} b_0^2\right] = h_2, \tag{15}$$

$$-(\mu-\lambda)(1-\delta)^k b_0 = p_1 \tag{16}$$

and

$$(\mu-2\lambda)\left[-(1-2\delta)^k b_1 + \left(\frac{\mu-1}{2}\right)(1-\delta)^{2k} b_0^2\right] = p_2. \tag{17}$$

From equation (14) and equation (17), we get

$$h_1 = -p_1 \tag{18}$$

and

$$2(\mu-\lambda)^2(1-\delta)^{2k} b_0^2 = h_1^2 + p_1^2. \tag{19}$$

By adding equation (15) to equation (17), we get

$$(\mu-2\lambda)(\mu-1)(1-\delta)^{2k} b_0^2 = h_2 + p_2. \tag{20}$$

Therefore, From equation (19), we get

$$b_0^2 = \frac{h_1^2+p_1^2}{2(\mu-\lambda)^2(1-\delta)^{2k}} \tag{21}$$

and from equation (20), we get

$$b_0^2 = \frac{h_2+p_2}{(\mu-2\lambda)(\mu-1)(1-\delta)^{2k}}. \tag{22}$$

Hence, from (21) and (22), we find that

$$|b_0|^2 \le \frac{|h_1|^2+|p_1|^2}{2(\mu-\lambda)^2(1-\delta)^{2k}}$$

and

$$|b_0|^2 \le \frac{|h_2|+|p_2|}{|(\mu-2\lambda)(\mu-1)(1-\delta)^{2k}|}.$$

Hence

$$|b_0| \le min\left[\sqrt{\frac{|h_1|^2+|p_1|^2}{2(\mu-\lambda)^2(1-\delta)^{2k}}}, \sqrt{\frac{|h_2|+|p_2|}{|(\mu-2\lambda)(\mu-1)(1-\delta)^{2k}|}}\right].$$

Now, subtracting equation (17) from equation (15), we obtain

$$2(\mu-2\lambda)(1-2\delta)^k b_1 = h_2 - p_2. \tag{23}$$

By squaring and adding equations (15) and (17), we get new equation. Using equation (19) in new equation, we obtain

$$b_1^2 = \frac{h_2^2+p_2^2}{2(\mu-2\lambda)^2(1-2\delta)^{2k}} + \frac{(\mu-1)^2(h_1^2+p_1^2)^2}{16(\mu-\lambda)^4(1-2\delta)^{2k}}. \tag{24}$$

By using equations (10), (11) in equation (23) and (24), finally we yield

$$|b_1| \le \frac{|h_2|+|p_2|}{2|(\mu-2\lambda)(1-2\delta)^k|} \tag{25}$$

and

$$|b_1| \le \sqrt{\frac{|h_2|^2+|p_2|^2}{2(\mu-2\lambda)^2(1-2\delta)^{2k}} + \frac{(\mu-1)^2(|h_1|^2+|p_1|^2)^2}{16(\mu-\lambda)^4(1-2\delta)^{2k}}}. \tag{26}$$

Hence

$$|b_1| \le min\left[\frac{|h_2|+|p_2|}{2|(\mu-2\lambda)(1-2\delta)^k|}, \sqrt{\frac{|h_2|^2+|p_2|^2}{2(\mu-2\lambda)^2(1-2\delta)^{2k}} + \frac{(\mu-1)^2(|h_1|^2+|p_1|^2)^2}{16(\mu-\lambda)^4(1-2\delta)^{2k}}}\right]. \tag{27}$$

This complete the proof.

3. Corollaries and Consequences

If we take $k = 0$ in Theorem 2.1, then obtain following Corollary.

Corollary 3.1 [13] *Let* $f(z)$ *of the form* (1) *be in the class* $\Sigma_M^{h,p}(\mu,\lambda)$, $\lambda \ge 1, \lambda > \mu, \ge 0$. *Then*

$$|b_0| \le min\left[\sqrt{\frac{|h_1|^2+|p_1|^2}{2(\mu-\lambda)^2}}, \sqrt{\frac{|h_2|+|p_2|}{|(\mu-2\lambda)(\mu-1)|}}\right]$$

and $$|b_1| \le min\left[\frac{|h_2|+|p_2|}{2|(\mu-2\lambda)|}, \sqrt{\frac{|h_2|^2+|p_2|^2}{2(\mu-2\lambda)^2}+\frac{(\mu-1)^2(|h_1|^2+|p_1|^2)^2}{16(\mu-\lambda)^4}}\right].$$

Remark 3.1 Corollary 3.1 is an improvement result of result obtained by Orhan [10] in Theorem 1.2.

If we take $h(z) = p(z) = \left(\frac{1+\frac{1}{z}}{1-\frac{1}{z}}\right)^{\alpha} = 1 + \frac{2\alpha}{z} + \frac{2\alpha^2}{z^2} + \cdots (0 < \alpha \le 1, z \in \Delta)$, in Theorem 2.1, then we obtain following result.

Corollary 3.2 *Let* $f(z)$ *of the form* (1) *belong to the class* $\Sigma_M^*(k,\delta,\mu,\lambda,\alpha)$, *then*

$$|b_0| \le min\left[\frac{2\alpha}{|(\mu-\lambda)(1-\delta)^k|}, \frac{2\alpha}{\sqrt{|(\mu-2\lambda)(\mu-1)(1-\delta)^{2k}|}}\right]$$

and

$$|b_1| \le min\left[\frac{2\alpha^2}{|(\mu-2\lambda)(1-2\delta)^k|}, 2\alpha^2\sqrt{\frac{1}{(\mu-2\lambda)^2(1-2\delta)^{2k}}+\frac{(\mu-1)^2}{(\mu-\lambda)^4(1-2\delta)^{2k}}}\right].$$

Remark 3.2 Corollary 3.2 is an improvement result of result obtained by Bobalade and Sangle [4] in Theorem 2.6.

If we take $k = 0$ in Corollary 3.2, then we obtain following Corollary.

Corollary 3.3 [13] *Let* $f(z)$ *of the form* (1) *belong to the class* $\Sigma_M^*(\mu,\lambda,\beta)$, *then*

$$|b_0| \le min\left[\frac{2\alpha}{(\lambda-\mu)}, \frac{2\alpha}{\sqrt{|(\mu-2\lambda)(\mu-1)|}}\right]$$

and

$$|b_1| \le min\left[\frac{2\alpha^2}{(2\lambda-\mu)}, 2\alpha^2\sqrt{\frac{1}{(\mu-2\lambda)^2}+\frac{(\mu-1)^2}{(\mu-\lambda)^4}}\right].$$

If we take $h(z) = p(z) = \frac{1+\frac{(1-2\beta)}{z}}{1-\frac{1}{z}} = 1 + \frac{2(1-\beta)}{z} + \frac{2(1-\beta)}{z^2} + \cdots \quad (0 \le \beta < 1, z \in \Delta)$, in Theorem 2.1, then we obtain following result.

Corollary 3.4 *Let* $f(z)$ *of the form* (1) *be in the class* $\Sigma_M^*(k,\delta,\mu,\lambda,\beta)$, *then*

$$|b_0| \le min\left[\frac{2(1-\beta)}{|(\mu-\lambda)(1-\delta)^k|}, \sqrt{\frac{4(1-\beta)}{|(\mu-2\lambda)(\mu-1)(1-\delta)^{2k}|}}\right]$$

and

$$|b_1| \le min\left[\frac{2(1-\beta)}{|(\mu-2\lambda)(1-2\delta)^k|}, 2(1-\beta)\sqrt{\frac{1}{(\mu-2\lambda)^2(1-2\delta)^{2k}}+\frac{(1-\beta)^2(\mu-1)^2}{(\mu-\lambda)^4(1-2\delta)^{2k}}}\right].$$

Remark 3.3 Corollary 3.4 is an improvement result of result obtained by Bobalade and Sangle [4] in Theorem 2.3.

If we take $k = 0$ in Corollary 3.4, then we get following Corollary .

Corollary 3.5 [13] *Let* $f(z)$ *of the form* (1) *belong to the class* $\Sigma_M^*(\mu,\lambda,\beta)$, *then*

$$|b_0| \le min\left[\frac{2(1-\beta)}{(\lambda-\mu)}, 2\sqrt{\frac{(1-\beta)}{|(\mu-2\lambda)(\mu-1)|}}\right]$$

and

$$|b_1| \le min\left[\frac{2(1-\beta)}{(2\lambda-\mu)}, 2(1-\beta)\sqrt{\frac{1}{(\mu-2\lambda)^2}+\frac{(1-\beta)^2(\mu-1)^2}{(\mu-\lambda)^4}}\right].$$

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