NCCS Research Journal, Volume 3, No. 1 October 2024, *ISSN NO: 2822-1605*

Received Date: June 2024 *Revised: August 2024* *Accepted: October 2024*

Application of Mathematics in Economics

Nand Kishor Kumar, PhD

Assistant Professor, Tri-Chandra Campus, Tribhuvan University, Nepal

Email:

nandkishorkumar2025@gmail.com

https://orcid.org/0000-0003-3844-809X

Abstract

Mathematics is adopted in economics to provide a precise, logical, and systematic framework for analyzing and solving problems related to resource allocation, production, consumption, and distribution. This article demonstrates how some economic principles are translated into mathematical form and solves various economic issues using basic mathematical methods. Mathematics is used to create economic models. The Cobb-Douglas production model, IS-LM model, supply and demand model, game theory, Solow growth model, and others are examples of concrete economic models. Mathematics serves as the backbone of modern economics, providing rigor and a structured approach to understanding complex systems. By leveraging mathematical tools and models, economists can analyze data, predict outcomes, and design policies to address real-world challenges.

Keywords: Mathematics model Econometrics, Business and Commerce, Business Mathematics

Application of Mathematics in Economics

Mathematics has significantly influenced social science, particularly economics. It broadened economics' scope and provided a scientific methodology for its breakthrough development. This paper explores the importance of applying mathematics to economics, rationalizing research, and its role in economics.

Historical Background

Mathematics played a crucial role in economics during the "Marginalist Revolution," replacing classical production, growth, and distribution. The focus lifted from economy and social classes to leading to Augustin Cournot's systematic application of mathematics in economics. This shift has led to macroeconomics emerging as a separate mathematical system from microeconomics, heavily reliant on mathematics (Muto, 2023). Standard economic theory assumes decision makers, including consumers, firms, and governments, are rational, ordering outcomes based on preference actions and choosing the preferred one (Muto, 2023).

Literature Review

The application of mathematical methods to economic problems—often known as mathematical economics—is what Salunkhe called methodological potential. Through the application of mathematical techniques to intricate financial and economic ideas and issues, economists are able to develop sound hypotheses on difficult subjects (Salunkhe, 2020). Calculus and matrix algebra are two analytical tools used in mathematical economics that are essential to contemporary economic theory. Problems are solved using differential equations, real analysis, matrix algebra, and mathematical modeling (Ebele, 1996). In economics, the application of mathematics is becoming increasingly important for communicating and representing concepts. This is relevant on several levels, such as when educators choose their courses and when lawmakers understand the policy recommendations they make. Because of its many variables, mathematics is the main tool used to solve economic problems. A change in the discipline's content results from economics' growing reliance on sophisticated statistical techniques and mathematical tools (Dowlin, 2001).

Objectives

(i) To explore the application of Mathematics in Economic theory and analysis.

Material and Method

Role of Mathematics in Economics

Mathematics in economics involves translating economic theory into mathematical language using tools like ratios, equations, derivatives, anti-derivatives, logarithms, and determinants. This helps express models and theories in concrete form, making them more precise and practical. The uses of mathematical tools in economic analysis are presented below:

(i) Economics is a positive science that demonstrates cause-and-effect relationships between variables, such as quantity supplied and price. Mathematical terms like Q and f(P) help express functional relationships between economic variables like demand, price, saving, and investment.

(ii) Graphs can display relationship between variables, but for problems with multiple variables, mathematical tools like simultaneous equation solutions and matrixes can be used for analysis.

(iii) Mathematical symbols can be used to express concepts like average values, marginal values, and elasticity, making conclusions easier to interpret and reach faster than graphical and verbal methods.

(iv) Input output analysis is a recent development in economics. It is a technique for analyzing inter-industry relations and it can also be used in economic planning. Matrix algebra forms the basis of input output analysis.

Mathematics in Economics

Stock Market: Mathematical methods are utilized by investors and financial companies to create profitable portfolios and automate trading, reducing risk and saving time during busy market hours.

Modeling: Most companies check for earnings using intricate mathematical models. These models are highly precise and may be used to determine the actual amount of profits earned by the firm after deducting all potential costs since they incorporate data on inflation and interest rates.

Machine Learning: Financial data is used by many companies to train the system, monitor potential earnings, identify problems, and make any necessary adjustments to the business plan. A corporation may train a computer to be quite accurate at predicting the preferences, dislikes, and potential commercial opportunities of its consumers because to the vast quantity of user data it possesses.

Blockchain: Blockchain, a distributed ledger system that employs sophisticated arithmetic for extremely secure encryption, is the way of the future for transactions. Math is also employed in bitcoin transactions, such as determining if the sender has enough cryptocurrency to deliver to the recipient.

Tax Calculation: Basic math may be used by both individuals and businesses to determine their taxes and potential deductions. Additionally, it aids in their assessment of potential tax breaks before they fill out the tax form. The optimal tax and investment choices to minimize taxes or optimize tax returns are then calculated with the use of a few mathematical formulae.

Econometrics: The use of statistical and mathematical techniques to examine economic data and evaluate economic ideas is known as econometrics. In order to comprehend and quantify the links between economic variables, estimate parameters, and make forecasts or policy recommendations based on empirical evidence, it entails employing mathematical models. In econometrics various type of econometrics models are used:

Simple Linear Economic Model

An equilibrium in economics is a state where demand and supply are equal, and the equilibrium price is the price when the demand and supply are equal, without any tendency to change.

Partial Equilibrium Market Model

In partial equilibrium analysis, the price of a commodity and the total sales are determined in a market which is supposed to be perfectly competitive. It is also assumed that the prices of other commodities and the demand for them do not change. We consider three micro economic variables the quantity demanded Q_d the quantity supplied Q_s and the unit price P in the model and we make the following assumptions to construct it:

(i) At the equilibrium price, the quantity offered for sale must be equal to the amount demanded. In other words, the market is cleared at that point.

(ii) Demand for the commodity increases as the price decreases and supply increases as the price increases.

Then the model can be written in the form of the following equations:

Equilibrium Condition: $Q_d = Q_s$ (i)

Demand function: Q_d = a - bp (ii)

Supply function: $Q_s = -\alpha + \beta p$

On solving,

a-bp = $-\alpha + \beta p$ or, a + α = (b+ $\beta)p$

So, p = $\frac{a+\alpha}{b+\beta}$

Now, putting the value of p in equation (ii), then, Q = a – $\frac{b(a+\alpha)}{b+\beta}$

Static Macroeconomic Model

In Keynesian model for the equilibrium of national income, the total expenditure is the sum of consumption expenditure (C) and investment expenditure (I). The consumption expenditure (C) itself is presumed to have two components.

(i) expenditure on the minimum requirements irrespective of income i.e. autonomous consumption (i.e. a) and

(ii) A fraction of every unit of income (Y) allocated to consumption. Also, for equilibrium, total expenditure or the aggregate demand (E) and national income Y are equal. Thus, the equations of the model are

E = C+ I (i)

C = a+ b Y (ii)

E = Y (iii)

Substituting Y = E in equation (i), Y = C +I or, C = Y -I

Substituting, C = Y – I in (ii), Y – I = a +bY

Or, Y-by = a +I

Or, Y = $\frac{1+a}{1-b}$ and C = Y-1 = $\frac{a+1}{1-b}$ – 1= $\frac{1+bI}{1-b}$

So, Y and C can be found out when I is given and b is marginal propensity to consume.

Application of Derivatives in Economics

Let C, P, Q, R and a represent the total cost, price, quantity produced, total revenue and profit respectively. Let AC, AQ and AR represent the respective small changes in C, Q and R respectively.

Marginal cost function

Let C = C(Q) be the cost function. Let C (Q_1) and C(Q_2) be the total cost when the volume of productions is Q_1 and Q_2 respectively.

Then, Change in total costs = ΔC = C(Q_2)-C(Q_1)

and Change in productions ΔQ= $Q_2 - Q_1$

$$\frac{\Delta C}{\Delta Q} = \frac{C(Q_2)-C(Q_1)}{Q_2-Q_1}$$

When $\lim_{\Delta Q \to 0} \frac{\Delta C}{\Delta Q} = \frac{dC}{dQ}$

gives the rate of change of cost concerning Q and is known as the marginal cost function.

Marginal Revenue functions

Let P be the price per unit output and Q, the output, then R = Revenue function = PQ. If ΔR and ΔQ represent the small changes in revenue and the output,

then $\frac{\Delta R}{\Delta Q} = \frac{R(Q_2)-R(Q_1)}{Q_2-Q_1}$

gives the average rate of change of revenue with respect to the output. This ratio is known as the average marginal revenue function.

When $\lim_{\Delta Q \to 0} \frac{\Delta R}{\Delta Q} = \frac{dR}{dQ}$ gives the rate of change of revenue with respect to Q and is known as the marginal revenue function.

Marginal profit function

Profit = Revenue - Cost

π= R-C

Or, $\frac{d\pi}{dQ} = \frac{dR}{dQ} - \frac{dC}{dQ}$

i.e. Marginal profit function = Marginal revenue function – Marginal cost function.

Marginal product

A production function Q = f(z) denotes the required quantity of labour or capital or some raw material for a production Q and we defined marginal product as

Marginal product = $\frac{dQ}{dZ} = \frac{dQ}{dL}$

Marginal propensity to Consume

P be the consumption expenditure of a person when his disposable income is x. Then P is a function of x

i.e. P = f(x)

Average propensity to consume (A.P.C) = $\frac{P}{X}$ and the marginal propensity to consume (M.P.C) = $\frac{dP}{dx}$.

Elasticity of demand

The elasticity of demand is the value of the ratio of the proportionate change in demand to the proportionate change in price. Let Q be the quantity of a commodity demanded when it's price is p. Let ΔQ be the change in Q when the change in p is Δp, then proportionate change in price f($\Delta p, p$) and corresponding proportionate change in demand is $\frac{\Delta Q}{Q}$.

The average elasticity of demand over the price range from p to (p+ Δp) = $\frac{\frac{\Delta Q}{Q}}{\frac{\Delta P}{P}} = \frac{P\Delta Q}{P\Delta P}$.

The elasticity of demand at price p is

$\lim\limits_{\Delta p \to 0} \frac{P\Delta Q}{P\Delta P} = \frac{P}{Q}.\frac{dQ}{dP}$

Generally, the demand falls when the price rises so that $\frac{dQ}{dP}$ is negative, in order to avoid the negative sign, we put a minus sign before it and define elasticity of demand as $-\frac{P}{Q}.\frac{dQ}{dP}$. It is also denoted by E_d.

i.e. $E_d = -\frac{P}{Q}.\frac{dQ}{dP}$

It should be understood that the negative sign is inserted for the sake of convenience.

Note: (i) If $E_d = 0$ then demand is perfectly inelastic.

(i) If $0 < E_d < 1$ then demand is inelastic

(ii) If $E_d = 1$ then demand is inversely proportional to price.

(iii) If $E_d < 1$then demand is elastic.

Marginal Revenue and elasticity of demand

The elasticity of demand $E_d = -\frac{P}{Q}.\frac{dQ}{dP}$ (i)

The total revenue R = PQ (ii)

And the marginal revenue $R' = \frac{dR}{dQ} = p + Q\frac{dp}{dQ}$ (iii)

From (i), $Q\frac{dp}{dQ} = -\frac{P}{Q}$

∴ From (iii), $R' = p - \frac{P}{E} = P\,(1-\frac{1}{E})$

Also, $\frac{1}{E} = 1 - \frac{R'}{P} = \frac{P-R'}{P}$

∴ $E = \frac{p}{p-R'}$

Thus, elasticity of demand can be obtained if we know the price and the margin revenue.

Increasing and Decreasing Functions

A function f(x) is called increasing on an interval if, for any two points x_1 and x_2 in that interval where $x_1 < x_2$,the inequality $f(x_1) \le f(x_2)$ holds. If $f(x_1) < f(x_2)$ for all $x_1 < x_2$ then f(x) is said to be strictly increasing.

A function f(x) is called decreasing on an interval if, for any two points x_1 and x_2 in that interval where $x_1 < x_2$,the inequality f(x_1) ≥ f(x_2) holds. If f(x_1) > f(x_2) for all x_1> x_2 then f(x) is said to be strictly increasing.

How to Determine Increasing or Decreasing

Derivative Test: If a function if is differentiable on an interval, you can use its derivative f′(x) to determine if the function is increasing or decreasing:

Increasing: If f′(x) > 0 for all x in the interval, then if is increasing on that interval.

Decreasing: If f′(x) < 0 for all x in the interval, then if is decreasing on that interval.

Critical Points: To find intervals where a function changes from increasing to decreasing or vice versa, you can:

Find the critical points of the function where f'(x) = 0 or f′(x) does not exist.Use the first derivative test by checking the sign of f′(x) around these critical points to determine if the function is increasing or decreasing on the intervals.

Maxima and Minima

Business and manufacturing aim for maximum profit and revenue, requiring economic analysis students to tackle problems such as maximization of profit, revenue, and minimization of production costs. This involves finding maximum or minimum values of functions using first and second order derivatives.

Maxima

A function y = f(x) is said to have a maximum value at a point x = a if its value at x = a i.e. the ordinate f(a) is the greatest of all the ordinates at the points other than x = a in a small neighborhood of this point. The corresponding maximum point on the curve represented by y = f(x) is A (a, f(a)) at which the tangent to the curve is parallel to the x-axis. The quantity f(a) is said to be maximum value of f(x).

Minima

A function y = f(x) is said to have a minimum value at the point x = b if its value at x=b i.e. the ordinate f(b) is the smallest of all the ordinates at the points other than x = b in a small neighborhood of this point.

The corresponding minimum point on the curve represented by y = f(x) is (b, f(b)) at which also the tangent to the curve is parallel to the x-axis. The quantity f(b) is said to be the minimum value of f(x).

Procedure of Finding Maxima and Minima

To find the maximum or the minimum values of a function, we follow the following steps:

(a) We at first find $\frac{dy}{dx}$ (or f'(x) of (the function y = f(x) and then find those values of x for which$\frac{dy}{dx}$ = f'’(x)= 0. Let these values be a, b,----

(b) At the point of maximum, the function changes its nature from increasing to decreasing so that $\frac{dy}{dx}$ or f’’(x) changes its sign from positive to negative. So, we calculate fla-h) and fla + h) where h is an arbitrarily small positive number. If we find that f'(a-h) > 0 and f(a-h) <0, then f(x) has a maximum at x=a and its maximum value is f(a) which is obtained by putting a in f(x).

(c) At the point of minimum, the function changes its nature from decreasing to increasing so that f'(x) changes its sign from negative to positive. So, if f'(a-h) <0 and f'(a + h)>0, then f(x) has a minimum at x= a and its minimum value is f(a) which is obtained by putting x=a in f(x).

(d) Although$\frac{dy}{dx}$ or f'(x)=0 at x = a, if f’(x) does not change the sign, when x is changed from a-h to a +h i.e. if f'(a-h) and f'(a + h) are of the same sign, then the function f(x) has neither maximum nor minimum at x =a.

(e) The same steps are repeated with the other points x=b etc. at which $\frac{dy}{dx}$=0.

Order Condition for Maxima and Minima (Extremum)

These conditions are known as order conditions.

Table 1

Condition for Maxima and Minima

Conditions	Maximum	Minimum	Neither Max. Nor. Min.
1st order	$\frac{dy}{dx} = 0$	$\frac{dy}{dx} = 0$	$\frac{dy}{dx} = 0$
2nd order	$\frac{d^2y}{dx^2}<0$	$\frac{d^2y}{dx^2} >0$	$\frac{d^2y}{dx^2} = 0$
3rd order	--	--	$\frac{d^3y}{dx^3} \neq 0$

Economic Application of maxima and minima

The economists frequently called upon to help a firm maximize profits and levels of physical output and productivity, as well as to minimize costs, levels of pollution and the use of scarce nature resources. This is done with the help of the following techniques.

Condition for cost minimization and minimum average cost:

Let a firm's costs function be C = C (Q)
(i)

where Q denotes the quantity. Then average cost represents the cost per unit of production. Then,

average cost (AC = $\frac{C}{Q} = \frac{C(Q)}{Q}$.

To find minimum cost we must apply first and second order conditions.

(i) MC = $\frac{dC}{DQ}$=0; and

(ii) (ii) $\frac{d^2C}{dQ^2} > 0$.

For average cost (AC) to be minimum, the conditions are:

(i) $\frac{d}{dQ}(AC)$ = 0; and (ii) $\frac{d^2}{dQ^2}$ (AC) $>$ 0.

Condition for maximum revenue

For any demand function p = f(Q), and revenue function R= R(Q), the total revenue (TR) is the product of demanded (Q) and the price per unit of output.

TR = P$\times Q$= f(Q)$\times Q$

Total revenue TR to be maximum, the conditions are

MR = $\frac{dR}{dQ}$ = 0; and $\frac{d^2R}{dQ^2}$ <0.

Conditions of profit maximization

Let a firm's total revenue and total cost function are given as R(Q)and C(Q) respectively, where Q denotes the quantity or output. Then R= R(Q) and C= C(Q)

Let P be the price at which a quantity Q is sold Then total receipt or revenue (R)= P $\times$Q.

If π be the total profit, then

n=R-C = R(Q)-C(Q) = PQ-C

For π to be maximum, the following conditions are holds:

First order condition

$\frac{d\pi}{dQ} = \frac{dR}{dQ}-\frac{dC}{dQ} = 0$

$\frac{dR}{dQ} = \frac{dC}{dQ}$

MR=MC

For maximum π, marginal revenue should be equal to marginal cost for maximum profit.

Second order condition

$\frac{d^2\pi}{dQ^2} = \frac{d^2R}{dQ^2} - \frac{d^2C}{dQ^2} <0$

For π to be maximum, the conditions are

$\frac{d\pi}{dQ}$ = 0 and $\frac{d^2\pi}{dQ^2}$ <0.

Economic Application of Partial Derivative

Partial derivatives are a crucial economic tool for analyzing the impact of small changes in one variable on another, thereby aiding in problem-solving. As an application of partial derivative, we deal with the production function, utility function and elasticity of demand etc.

Production function

For firms, partial derivatives are used to analyze production functions, which describe how inputs (like labor L and capital K) are converted into output Q. Various factors such as raw materials, land, labour, capital etc. are essential in order to produce the goods. Thus, in producing the goods, we need labour (L), and the capital (K). Hence output is the function of the labour (L) and the capital (K) i.e.

Q = f (L, K)

A production function, mostly used in economic analysis is the Cobb-Douglas production function which is expressed as

Q = A $L^{\alpha} \; K^{\beta}$

where A is constant and 0<α<1, 0<β<1, L>0, K>0.

Returns to scale

As of the Cobb-Douglas production function Q = A$L^{\alpha}K^{\beta}$ if proportionate changes occur in the inputs L and K; proportionate change in the output Q can be obtained.

Utility function

Let X and Y be the two goods that a consumer wants to purchase. If the consumer has a satisfaction when he purchases x quantity of X and y quantity of Y, then the total utility function denoted by U is defined by

U= f(x, y)

A most commonly used utility function in economic analysis is Cobb-Douglas utility function which is expressed in the form

U = A$x^{\alpha}y^{\beta}$

where A is constant and 0<α< 1, 0<β<1, x>0, y> 0. and x, y being the respective quantities of goods X and Y consumed.

Price elasticity of demand

If Q = f(P, Y, P_A) denoted by E_d is the price elasticity of demand is the percentage change in quantity demanded Q by the percentage change in the respective price P of the goods A, keeping Y and P_A constant. E_d is given by

$$E_d = \frac{dQ}{dP} . \frac{P}{Q}$$

Application of Integration in Business and Economics

If C) x) R(x) and π(x) be the cost function, revenue function and profit function respectively.

Marginal Cost Function = (MC) = $\frac{dC}{dx}$ = C'(x)

Marginal Revenue Function (MR)= $\frac{dR}{dx}$ = R'(x)

Marginal Profit Function (MR)= $\frac{d\pi}{dx}$ = π'(x)

Since integration is the reverse is the reverse process of differentiation, so find C(x), R(x) and π(x) by integration C'(x), R'(x) and π'(x) respectively.

cost function =$\int \frac{dC}{dx}$. dx =$\int C'(x)dx = C(x) + k$

Revenue function = $\int \frac{dR}{dx}$. Dx = $\int R'(x)dx = R(x) + k$

And profit function = $\int \frac{d\pi}{d\pi}$. dx = $\int \pi'(x)dx = \pi(x) + k$

Where k is the constant of integration and its value will be obtained by the use of given condition. **Economic application of differential equation**

Differential equations are utilized in various economic branches to identify functions, determine dynamic stability conditions, and track growth paths. They estimate demand functions based on point elasticity and growth rate.

Dynamic analysis

For a one commodity market model, the demand and supply functions are given by

$$Q_d = a - bP \quad (a, b > 0)$$
$$Q_s = -c + dP \quad (c, d > 0) \qquad \text{(i)}$$

where Q_d = quantity demanded, Q_S = quantity supplied, P = price.

Here, each of Q_d, Q_s and P are the functions of time t.

Let us assume that rate of price change with respect to time (t) at any moment is always directly proportional to the excess of demand over supply $(Q_d - Q_s)$, prevailing at that moment. Such a pattern of change can be expressed symbolically as

$$\frac{dP}{dt} = k(Q_d - Q_s) \ (k > 0) \qquad \text{(ii)}$$

where k = positive constant called adjustment coefficient.

From (i), and (ii), we get

$$\frac{dP}{dt} = k[(a - bP) - (-c + dP)]$$
$$= k[(a + c) - (b + d)P]$$
$$= k(a + c) - k(b + d)P$$
$$\frac{dP}{dt} + k(b + d)P = k(a + c)$$

This is the first-order linear differential equation with constant coefficient and constant.

$$P(t) = P_c + P_p \qquad \text{(iii)}$$

To find P_c

Let homogeneous differential equation of (iii) be

$$\frac{dP}{dt} + k(b + d)P = 0$$

or $\frac{dP}{dt} = -k(b + d)P$

or $\frac{dP}{P} = -k(b + d)dt$

Integrating both sides, we get

$$\int \frac{dP}{P} = -k(b + d) \int dt$$

or $\ln P = -k(b + d)t + \ln C$

or $\ln P = \ln e^{-K(b+d)r} + \ln \ln \because \ln e^x = x$

or $\ln P = \ln\left[Ce^{-K(b+d)t}\right]$

or $P = Ce^{-K(b+d)t}$

This is the required complementary function (P_c) of the homogeneous differential equation. For equilibrium price P,

$$Q_d = Q_s$$

or $a - bP = -c + dP$

or $P = \frac{a+c}{b+d}$.

This is the required equilibrium price.

To find P_p

Let $P = \bar{P}$ be the particular integral of (iv). Then, $\frac{dP}{dt} = 0$. Substituting these values in (iv), we get

$$0 + k(b+d)\bar{P} = k(a+c)$$

or $\bar{P} = \frac{a+c}{b+d}$, which is the required particular integral.

Hence, the general solution of (iii) is given by

$$P(t) = P_c + P_p = Ce^{-k(b+d)t} + \frac{a+c}{b+d} \quad \text{(iv)}$$

where $P_p =$ the particular integral which gives the equilibrium price. $P_c =$ the complementary function which gives the deviations from the equilibrium.

When $t = 0$, then $P(0) = Ce^{\circ} + \frac{a+c}{b+d}$

or $P(0) = C + \frac{a+c}{b+d}$

or $C = P(0) - \frac{a+c}{b+d}$

Putting the value of C in (iv), we get the definite solution as

$$P(t) = \left(P(0) - \frac{a+c}{b+d}\right)e^{-t(b+d)} + \frac{a+c}{b+d}$$

$$\Rightarrow P(t) = (P(0) - P)e^{-2(b+dt} + P,\ P = \text{ equilibrium price } = \frac{a+c}{b+d}$$

$$\Rightarrow P(t) = [P(0) - P]e^{-\beta t} + P \ \dots \text{ (v) where } \beta = k(b+d)$$

This is the required time path of the price.

In equation (v), as $t \rightarrow \infty$, $e^{-\sin'} = \frac{1}{e^{\pi r}} \rightarrow 0$

Hence, $\lim_{t\rightarrow\infty} P(t) = 0 + \overline{P} = \overline{P}$ = equilibrium price.

In other words, in the long-run, price will converge to the equilibrium price (P) and in this way the equilibrium is said to be dynamically stable and $P(t)$ is called inter-temporal price.

When $\beta = k(b + d) < 0$, then $e^{-\beta t} \rightarrow \infty$ as $t \rightarrow \infty$. In such case, $P(t)$ diverges and price will not be stable.

Economic Application of Difference Equation

The Cobweb Model

The basic Cobweb model assumes that today's demand for any commodity is a function of the present price (P_t), it means Q_d is an unlagged function of price, while today's supply depends upon yesterday's decision about the output, it means Q_s is a lagged function of price. Hence output is naturally influenced by yesterday's price (P_{t-1}).

$$\therefore\; Q_{st} = S(P_{t-1})$$
$$Q_{dt} = D(P_t)$$

Since both demand and supply functions are assumed to be linear, then,

Demand function: $Q_{dt} = a - bP_t, (a > 0, b > 0)$ (i)

Supply function: $Q_{st} = c + dP_{t-1}, (c > 0, d > 0)$ (ii)

For market to be in equilibrium,

$$Q_{dt} = Q_{st}$$
$$\therefore\quad a - bP_t = c + dP_{t-1}$$
$$\text{or,}\quad -bP_t - dP_{t-1} = c - a$$
$$\text{or,}\quad bP_t + dP_{t-1} = a - c$$

or, $P_t = \left(\frac{d}{b}\right) P_{t-1} = \frac{a-c}{b}$

Application of Linear Algebra

Sarus rule

In determinant, Sarus rule is the mathematical rule which is used to find out the values of three order determinant only.

Let $|A| = \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} = x_1y_2z_3 + y_1z_2x_3 + z_1x_2y_3 - x_3y_2z_1 - y_3z_2x_1 - z_3x_2y_1$

Sarus rule consists of the following steps:

(i) List the elements of the first three columns of the given determinants.

(ii) Repeat the first two columns.

(ii) Find the products of the elements lying on the diagonals from top to bottom containing three elements $x_1y_2z_3, x_3y_1z_2, x_2y_3z_1$.

(iii) Similarly, find the products of the elements lying on the off diagonals from bottom to top containing three elements: $x_3y_2z_1, x_1y_3z_2, x_2y_1z_3$.

(iv) The three products obtained in step (iii) is taken with positive signs and the three products obtained in step (iv) is taken with negative signs then the sum of six terms is the value of the determinant. Thus,

$$\mathrm{D} = x_1y_2z_3 + x_3y_1z_2 + x_2y_3z_1 - x_3y_2z_1 - x_1y_3z_2 - x_2y_1z_3$$

Cramer's Rule

It is a method of solving simultaneous equations with the help of determinants.

Let $a_1x + b_1y = c_1$ (i)

$a_2x + b_2y = c_2$ (ii)

be two simultaneous equations of two variables x and y.

Multiplying equation (i) by a_2 and equation (ii) by a_1 and subtracting (i) from (ii), we get

$$\begin{aligned} a_1a_2x + b_2a_1y &= c_2a_1 \\ a_1a_2x + b_1a_2y &= c_1a_2 \\ - \ - \ & \end{aligned}$$

$$(b_2a_1 - b_1a_2)y = c_2a_1 - c_1a_2$$
$$y = \frac{c_2a_1 - c_1a_2}{b_2a_1 - b_1a_2} = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} = \frac{D_2}{D}$$

Similarly, we can obtain, $x = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} = \frac{D_1}{D}$

where, $D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$ is coefficient matrix

$D_1 = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$ and $D_2 = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$. Provided that $D \neq 0$.

Similarly, for 3 variables x, y and z

$$\mathrm{x} = \frac{\mathrm{D_1}}{\mathrm{D}}, \mathrm{y} = \frac{\mathrm{D_2}}{\mathrm{D}}, \mathrm{z} = \frac{\mathrm{D_3}}{\mathrm{D}}, \text{ provided } \mathrm{D} \neq 0$$

Inverse Matrix Method

Consider the following system of simultaneous equations

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

It can be expressed in augmented matrix form as:

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

which is in the form of $A\, X = D$

where $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}, X = \left\{\begin{matrix} x \\ y \\ z \end{matrix}\right]$ and $D = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$

Pre-multiplying equation (i) by A^{-1}

then $A^{-1} \cdot AX = A^{-1}D$

$$IX = A^{-1}D$$
$$X = A^{-1}D$$

Thus, the solution is obtained by finding A^{-1} and multiplying it to D.

Conclusion

Many assumptions are made in the fields of econometrics, game theory, and mathematical economics. For instance, all Boolean topics are assumed to be homogenous and fully rational in game theory. Mathematics has been widely employed in the study of economics as a technique and as a tool.

Reference

Alpha, C.C., (1984). Fundamental methods of mathematical economics. Third Edition. McGraw-Hill International Book Company. New York

Birdetle, A.C (1971). Analytic geometry; London: Academic press Inc.

Burton, L. (1999). Learning mathematics: From hierarchies to network. Routledge, Falmer's

Cuiping, Wu. (2016). Analysis of The Application of Mathematics in The Economy. *Shanxi Agricultural Economics*. 83-87.

Duanduan, Li. (2021). The Relationship Between Mathematics and Economics. *Marketing Industry*. 173-174.

Dowlin, E.T (2001). *Introduction to mathematical economics*. 3, New-York: McGraw -Hill.

Ebele, A.N. (1996). *Application of mathematics to economics*. An unpublished B. Tech. project. Department of Mathematics, Faculty of Natural Science, University of Jos.

Kumar, N.K. (2022). *Derivative and it's Real Life Applications*. A Mini-Research Report Submitted to the Research Directorate, Rector's Office Tribhuvan University, Nepal

Kumar, N. K. (2023). Determination of the Eigen Values of Nurlund Matrix as an operator on Sequence Spaces C and C_0. *SSRN*: DOI: 10.2139/ssrn.4615935

Kumar, N., Yadav, D. P., & Sahani, S. K. (2024). Poincare's Theorem of Asymptotic Series and its Application. *Mikailalsys Journal of Mathematics and Statistics*, 2(1), 1-7. https://doi.org/10.58578/mjms.v2i1.2460

Muto, I. (2023). The Emergence of Mathematical Economics; Cournot's Economic Methodology. *Communications in Economics and Mathematical Sciences*, 1-13.

Owen, G. (1970). Finite mathematics. Philadelphia. W.B. Saunders Co.

Restivo, S. (1988). The social construction of mathematics. *Zentralblatt fur didaktik der Mathematik*, 20(1), 15-19.

Salunkhe, A.S. (2020). Application of Mathematics in Economics- A study. *Jetir*, 7(2).

Yunhua, Zhang. Han, & Zhang. (2018). Advanced Mathematics in The Field of Economics. *Journal of Harbin Institute of Vocational Technology*. 141-143

Yan, Cao. (2010). Examples of Application of Higher Mathematics in Economic Analysis. *Popular Science*. 138-139