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A Comparison of Advection Diffusion Equation Solutions, Observed Data, and Statistical Results

Prem Sagar Bhandari
Tribhuvan University, Birendra Multiple campus, Bharatpur, Chitwan
Premsagar61075@gmail.com

Abstract

In this paper, the statistical procedure is compared to the analytical solutions of an advection-diffusion equation describing air pollution spread in a limited atmospheric boundary layer from a continuous point source. The equation is formulated by assuming the eddy diffusivity and wind velocity as the constant and the power law of vertical height. It has been observed that the expected concentration for eddy diffusivity and wind velocity as constants has a worse agreement with the actual concentration than the expected concentration for wind speed and eddy diffusivity as variables.

Keywords: Advection-diffusion equation, Concentration of pollutants, Analytical solution, Statistical method, Wind velocity.

1. Introduction

Air pollution is a major issue on a global scale. Scientists and researchers have achieved major advancements in the study of atmospheric dispersion of air pollutants from point, line, and area source materials over the past few decades. Different advection diffusion equation solutions have been used by various authors under various circumstances.

The advection-diffusion equation has been solved by Moreira et al. under the assumption that eddy diffusivity varies up on x and z by subdividing the domain using the Laplace transform approach [7].

By applying the variable separable approach and assuming that the wind velocity is alinear relationship with vertical elevation and that diffusion of eddies also depends in vertical elevation as well as the two constants, Marrouf et al. have provided an analytical solution to the advection diffusion problem. Additionally, they compared the concentrations that were anticipated and actual for the nine tests that were carried out in Cairo [6].

In order to analyze the properties of steady state diffusion the movement of contaminants produced from a source in the ground, Naresh and Nath have provided a solution to the advection-diffusion equation [8]. For the air pollutants released from elevated point sources, Ermark has provided a model[4].

Eddy diffusivity and Wind velocity were used as variables in an advection diffusion equation, and Bhandari's answer was graphically compared with actual data gathered at Inshas Cairo[2].

By presuming that eddy diffusivity is a linear consequence of vertical elevation and downhill distance from the sourceand that the wind velocity is a power law characteristic of vertical component above surface. Oneanalytic framework to the crosswind incorporated concentrations released by a continuous source has been created by Sharan and Kumar [9].

In this topic, various authors have provided the various results of advection diffusion model with various parameters of wind velocity and eddy diffusivity: Kumar, P. and Sharan, M. [5], Verma, V.S., Srivastava, U. and Bhandari, P.S. [11], Agarwal, M., Verma, V.S. and Srivastava, S. [1], Verma, V.S., Srivastava, U. and Bhandari, P.S. [12], Wortmann et al. [13], Sharan and Modani [10].

In this paper, the statistical method is compared to the analytical solutions of the concept for distribution of atmospheric contaminants produced from a regular single source in a limited environmental outer side, which is acquired by assuming wind velocity and eddy diffusion coefficient as power law of height and constants. In order to do this, we have used the climatic parameters and observed data from the nine experiments carried out in Inshas of Egypt.

2. Model Formulation

Air pollution's spread in steady state condition in the atmosphere can be described by

$$v\frac{\partial C}{\partial x} = \frac{\partial}{\partial y}(k_y\frac{\partial C}{\partial y}) + \frac{\partial}{\partial z}(k_z\frac{\partial C}{\partial z}) \qquad (1)$$

Where,

C is the average pollutant concentration, k_y and k_z the eddy diffusivity in y and z directions respectively, x-axis is in direction of average velocity$v$ and $v$more higher than the wind velocity in the path of y- axis. Source pollutants is ignored and we neglect the diffusion term in the path of x-axis as the advection term in the way of x- axis is greater than the diffusion in the way of x- axis.

For the solution of solution of (1), we take boundary conditions:

$$\frac{\partial C(x,z)}{\partial z} = 0;\text{for points z=0 and z=H} \qquad (2)$$

$$c(x,y,z) = 0;\text{ for point z= H} \qquad (3)$$

Where,

source height is H.

$$v\mathrm{C} = Q\delta(z - z_s);\ \ point\ \mathrm{x} = 0 \qquad (4)$$

here,

δ be the Dirac's delta function and Q a point source strength.

$$c(x,y,z) = 0\ as\ the\ variables\ x,y,z\ \rightarrow \infty \qquad (5)$$

3. Solution of Model

The variables separable approach is used in two instances to acquire the model's solutions: If wind velocity and eddy diffusivity are assumed to be variables as

$$v = uz^n, z \neq 0\ and\ v = v_0\ at\ z = 0 \qquad (6)$$

And, $$k_z = u_1 z^n \qquad (7)$$

Where, u is frictional speed and u_1 is turbulence intensity, then the solution is given by[3]

$$c_y(x,z) = \frac{2Q_p z^{-1/2} z_s^{3/2} J_{1/2}(\eta_\beta z_s)}{uH^2[J_{1/2+1}(\eta_\beta H)]^2}[\sum_{\alpha=1}^{\infty} J_{1/2}(\eta_\alpha z)]e^{-\lambda^2 x} \qquad (8)$$

Where, $J_{1/2}$ is Bessel's function of a first kind in order $1/2$, λ^2 $is\ a\ constant$. In which $\eta_\beta\, H$ is given as $J_{1/2}(\eta_\beta H) = 0$ (9)

And, As wind velocity v and the eddy diffusivity k_z are supposed as constants, then the solution of the model (1) is given by [3]

$$c_y(x,z) = \frac{Q}{v_{z_s}} e^{-\lambda^2 x} cos(\lambda\sqrt{\frac{v}{K}})z\ sec(\lambda\sqrt{\frac{v}{K}})z_s \qquad (10)$$

4. Results, Statistical Method and Discussion

Using information gathered at a 30 m multi-level micrometeorological tower's vertical distance, the concentration of pollutants was calculated. The calculated values for nine runs are shown in Table 1 along with the observed concentrations using the analytical solution equations (8) and (10):

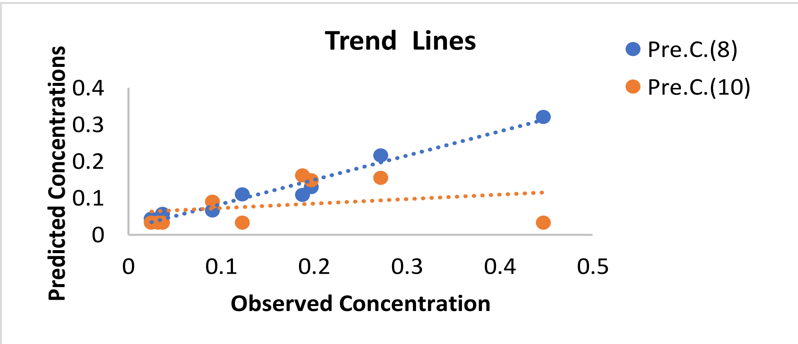
Table 1

Predicted concentrations from analytical equations (8) and (10) and observed (in run nine experiments conducted at Inshas, Cairo):

Experiment Numbers	Predicted pollutants concentration equation(8)	The downwind distance	Predicted pollutants Concentration by equation(10)	Observed pollutants	The vertical distance
1.	0.043	100.0	0.032	0.025	5.0
2.	0.057	98.0	0.033	0.037	10.0
3.	0.066	115.0	0.090	0.091	5.0
4.	0.130	135.0	0.148	0.197	5.0
5.	0.216	99.0	0.155	0.272	2.0
6.	0.108	184/0	0.162	0.188	11.0
7.	0.320	165.0	0.032	0.447	12.0
8.	0.110	134.0	0.033	0.123	7.5
9.	0.037	96.0	0.032	0.032	5.0

In the study of Bhandari [3], the difference of expected and observed concentrations of nine typical experiments to downwind distance is depicted graphically. He discovered that the anticipated concentration given by equation (8) and the observed concentration have a better agreement than the predicted concentration supplied by equation (10). Also, the calculated concentration from equation (10) and the measured concentration do not correlate well.

Using the trend lines we have also observed the predicted concentrations given by (8) and (10) with observed data. The figure below shows the trend of the concentrations.



We compare the analytical, observable, and statistical outcomes using the statistical technique. The following common statistical performance indicators that describe the agreement are used by usbetween predictions (c_p) and observations (c_o) :

i) Normalized Mean Square Error (NMSE): -It is defined as NMSE=$\frac{\overline{(c_o-c_p)^2}}{\overline{c_o}\,\overline{c_p}}$. This equation calculates the overall differences between expected and actual concentrations. Better model performance is indicated by smaller values of NMSE.

ii) Fractional bias (FB): -It is defined as FB =$\frac{(\overline{c_o}-\ \overline{c_p})}{0.5(\overline{c_o}-\ \overline{c_p})}$. It gives the model information about its propensity to either overestimate or underestimate the observed concentrations. Its value ranges from -2 to +2. FB is 0 for an idea model.

iii) Correlation Coefficient(R): - It is defined as R= $\frac{\overline{(c_o-\overline{c_o})((c_p-\overline{c_p})}}{\sigma_o\sigma_p}$ .It gauges how closely expected and actual concentrations are related.

iv) The Fraction within the factor of two (FAC2): -The data in percentage for which the following relation holds is what it is considered to be:0.5≤ $\frac{c_p}{c_o} \leq 2$.

In the above performance measures formulas, $\sigma_p$and σ_o are standard deviations of c_p and c_o respectively and over bars represent averagesfor measurements.When the performance metrics COR=FAC2=1 and NMSE=FB=0, the model is thought to be a perfect model. The performance results from the standard statistical performance measures are shown in table 2. For this two predicted concentration equations (8) and (10) are used with Inshas observed data.

Table 2

Performance comparisons between predicted with measured data.

Functions	NMSE	COR	FB	FAC2
Predicted concentration given by equation(8)	0.18	0.98	0.25	0.96
Predicted concentration given equation(10)	1.75	0.29	0.65	0.74

According to Table 2, the anticipated concentrations from Equations (8) and (10) are within a factor of 2 of the actual values. We conclude that the predicted concentration equation (8) is superior to the anticipated concentration equation (10) using the observed data based on the measurements NMSE, FB, COR, and FAC2.

7. Conclusion

This research makes use of an existing mathematical model for the dispersion of air contaminants that includes wind speed and eddy diffusivity as variables and constants. Using statistical methods, the analytical answers are compared to original data taken from Inshas experiments carried out at Cairo. We discovered that the predicted pollutant concentration given by the equation (10) with the constants wind speed and eddy diffusivity is less accurate than the concentration given by the equation (8) with the variables wind speed and eddy diffusivity.

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