Approximation in the Application of the Integration

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Abstract

The current research discusses approximation and application while introducing the idea of integration. This study aims to determine the problem function when its derivatives are available, as well as to examine its application and explore the approximation of the definite integral using the trapezoidal rule. For the purpose of resolving practical issues and examining mathematical models, it is essential to comprehend the notion of integration, its applications, and the application of approximation techniques. For scientists, engineers, and researchers, integration is still an essential tool that helps improve a variety of professions.

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1.Introduction

Integration, a fundamental concept in mathematics, plays a crucial role in various fields such as physics, engineering, economics, and more. In real-world scenarios, exact integration can be challenging or even impossible due to the complexity of functions involved. Hence, approximation techniques become invaluable tools to tackle integration problems efficiently and effectively. This article delves into the application of approximation methods in integration and their significance across different domains.

Integral is used to calculate the functions that characterize area, displacement, and volume of a group of small data points that cannot be measured independently. The concept of limit is applied broadly in calculus when algebra and geometry are used. Limits are useful when analyzing the behavior of points on a graph, such as how they approach one another until they are nearly at the same distance (Young, 2009). Two main categories of calculus exist:

- Differential Calculus
- Integral Calculus

Finding the area under a curve or the cumulative change over a period of time is made possible by the fundamental mathematical idea of integration. It is extensively utilized in many disciplines, including computer science, engineering, physics, and economics. Finding a function's anti-derivative, which stands for the original function, is a step in the integration process.

Literature Review

A Bayes issue requiring a numerical approximation of a two-dimensional integration was addressed. They noted that the vast amounts of computer time required to answer the problem restrict the correctness of the result (Davis & Willian, 1971). A four-dimensional integration is needed to solve the Bayes issue that posed. These are real-world instances where a proficient numerical integration technique is required (Davis, Chester, & Lucien ,1973).

The word “approximation” denotes to more divisions in mathematical analysis, which was developed as a result of the contributions by Chebyshev (1853), Weierstrass (1885), Lebesgue (1898), Bernstein (1952, 1937), Nikolskij (1945), Kolmogorov (1985) and followed by many of their followers such as (Nikolskii, 1945, & Kolmogorov,1985).

Preliminaries

Basics of Integration

The opposite of distinction is integration. A function f(x) represent the rate of variation of another function, F(x) plus an integration constant (sometimes abbreviated as C) are obtained through the integration of f(x). This constant results from the loss of information about the constant term in the original function during differentiation (Sneddon, 2004).

∫ f(x) dx is the notation for the indefinite integral of a function f(x), also referred to as the anti-derivative. "f(x)" is the function to be integrated, "dx" is the variable of integration, and the sign "∫" denotes the integration. Integration yields a family of functions, each of which varies by a constant (Thomas, & Finney,1998).

Conversely, the definite integral has upper and lower bounds. The area under the curve between these two limitations is what it represents.

$\int_a^b f(x)dx$ is the definite integral of a function f(x). It provides a single value that sums up the amount or net change throughout the interval [a, b].

Applications of Integration

Area and Volume Calculations

Determining area of region of a curve is one of the basic uses of integration. Area under the curve between two points can be found by integrating the function that defines the curve. This idea is applied to many other domains, such figuring out the area of asymmetrical objects.

Cross-sectional areas can also be used to determine a solid's volume by integration. A solid's total volume may be calculated by integrating the area of its cross-sections along its symmetry axis. In engineering and science, this method is frequently employed to determine the volume of intricate objects (Sharma, 2019.

Physics and Engineering

In engineering and physics, integration is essential. It is useful for computing motion-related quantities including acceleration, velocity, and displacement. The displacement function can be found by integrating the velocity function, which can be found by integrating the acceleration function. By integrating the current function or the charge flow rate, integration is used in electrical engineering to calculate the total charge or current flowing in a circuit (Rathod,2017).

2.Objective

This article aims to answer the following categories of problems: -

(1) Identifying the issue function based on supplied derivatives.

(2) To study the application and approximation of definite integrals.

The notion of the "Integral Calculus," which consists of definite and indefinite integral, was developed as a result of these two issues. The Fundamental Theorem of Calculus connects the ideas of differentiating and integrating functions in the subject of calculus. The function that could have functions as a derivative are called anti-derivatives of the function. The process of integration can be considered as:

(i) The first in that the integration in the inverse process of differentiation
(ii) Secondly the literal meaning of the word integration is summation.

The process involves determining the upper bound of a sum of a given number of terms, let's say n. Later on, it will be demonstrated how integral calculus may be used to determine a generic approach for calculating the area of a region surrounded by two (curved lines) by dividing the territory into an endless number of tiny stripes (Kumar, 2023).

Meaning of Integration

The integration as the reverse method of differentiation that can be defined (Sharma, 2019).

if F(x) is the differential coefficient of f(x) is called the integral that is (anti-derivative) of F(x).

Symbolically, if the derivative of F(x) i.e. $\frac{d\,F(x)}{dx}$ = f(x)

(i)

Then indefinite integration of f(x) i.e. $\int f(x)$ = F(x)

Equation (i) can be written $\frac{d\{F(x)+C\}}{dx}$ = f(x), where c is constant. Then

$\int f(x)dx = F(x)$, then we also have

$\int f(x)dx$ = F(x) + c is a general value of indefinite integral. Here c is constant of integration.

3.Fundamental Properties of Integral

(a) The integral of sum and difference of two functions equals the sum of the difference of the integrals of two functions, as shown below:

$$\int (f_1(x) \pm f_2(x))\,dx = \int f_1(x) \pm f_2(x)dx$$

(ii)

(b) The integral of a product of a constant and a function equals the product of the constant and the integral of the function.

$$\int K\,f(x)dx = K \int f(x)dx$$

(iii)

3.1 Riemann Integral

A function defined on [a, b] and it's partition p of [a, b] into a collection of subintervals defined as $[x_0,x_1]$, $[x_1,x_2]$,$----$$[x_{n-1}, x_n]$, and for for each I = 1,2,3,-----n:

$x_i \in [x_{i-1}, x_i]$, then we can write

$$\sum_{i=1}^{n} f(x\ )(x_i - x_{i-1}\ ) = \sum_{i=1}^{n} f x_1\ \Delta x_1$$

(iv)

is called Riemann sum.

If $\lim_{n\to\infty} \sum_{i=1}^{n} f x_i\ \Delta x_i$ exists, then it is called the Riemann integral and denoted by

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f x_1\ \Delta x_1$$

(v)

(length of the largest sub interval) and the function f is called Riemann integral on [a, b].

3.2 Definite Integral

$\int_a^b f(x)dx$ is called the definite integral, where a as a lower limit and b is the upper limit of integration. Now, two fundamental theorems of integral calculus.

(a)The first Fundamental Theorem of Integral Calculus

Let f be a continuous real-valued function on a closed interval [a, b] and F be the function defined for all x in [a, b] by

F(x) = $\int_a^x f(t)dt$, F is continuous on [a, b] and differential on (a, b) and

F'(x)= f(x) for all x in [a, b].

This theorem establishes the relation between integration and differentiation. Here F'(x) is called anti-derivative of f(x) (Nikolskii,1985).

(b)Second Fundamental Theorem of Integral Calculus

Let f be real valued continuous function on [a, b] and F is an anti-derivative then we may define

$$\int_a^b f(t)dt = F(b) - F(a) \qquad \text{(vi)}$$

3.3 Calculation of Definite Integral

When we introduce the limits for in the indefinite integration, then it becomes definite integral. As we have already known that $\int f(x)dx$ is called indefinite integral and when f is integral area the interval [a, b], then

$\int_a^b f(x)$ dx is called definite integral, we have already stated on the second fundamental theorem of integral calculus:

$$\int_a^b f(x)\,dx = [\,F(x)\,]_a^b = F(b) - F(a) \qquad \text{(vii)}$$

where F(x) is ant- derivatives.

Thus, in the definite integral, we proceed as

(i) Evaluate $\int f(x)$ dx. Denote it by F(x)

(ii) Substitute x = b and x = a in F (x) and find F(b) and F(a)

(iii) Then finally, we evaluate $\int_a^b f(x)dx = F(b) - F(a)$.

4.Applications of Definite Integrals

(a)Quadrature

Quadrature is the method of finding the area bounded by any portion of a curve. The equation of the curve is given in Cartesian, parametric or polar form.

Area in Cartesian Coordinates

If y = f(x) is the equation of a continuous curve defined in (a, b), the area of the region bounded by a curve y = f(x), the x- axis, and at x= a and y = b is

$\int_a^b f(x)dx.$

(b)Rectification

Rectification is the process of determining the length of an arc along a given curve. The curve's equation can be written in Cartesian, parametric, or polar form. However, the term "rectification" refers to the process of finding a straight line with the same length as a given arc.

Arc Length from Cartesian Form:

If y = f(x) be the equations of the curve, the length of its are included between two points whose abscissa arc a and b is

$$\int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2} \qquad \text{(viii)}$$

4.1. Volume of Solid of Revolution

If a curve provided by the equation y = f (x) is continuous in (a, b), then the volume of the solid created when the region defined by the curve, the x-axis, and the coordinates at

x = a and x = b, is revolved about the x – axis is

$$V = \int_a^b \pi y^2 \, dx. \qquad \text{(ix)}$$

4.2 Surface of Solid of Revolution

If a curve y = f (x) is continuous in the interval (a, b), then the surface of solid is formed by the area bounded by curve, the x – axis and the ordinates x = a and x = b is revolved about the x – axis. This surface of this solid is called surface of solid of revolution.

A curve y = f (x) is continuous in the interval (a, b), the surface area of the solid of revolution obtained by revolving the area bounded by given curve, ordinates x = a, x = b about x - axis, is defined as

$$\int_a^b 2\pi y \frac{ds}{dx}\, dx = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)}^{2} dx.$$

(x)

5. Approximation Techniques for Integration

Analytically determining the anti-derivative or expressing it in a closed-form solution is frequently impossible. Approximation techniques are used in these instances to estimate the integral's value. Typical techniques for approximation include:

Riemann Total: By splitting a curve into many subintervals and evaluating the function at particular locations within each subinterval, it approximates the area under the curve. An estimate of the integral can be obtained by adding these areas.

Trapezoidal Rule: This method estimates the area beneath a curve by considering the trapezoids that are created when joining the curve's successive points. An estimate of the integral can be obtained by adding the areas of these trapezoids.

Simpson's Rule: Fitting quadratic functions between three successive points on a curve approximates the area under the curve. An estimate of the integral can be obtained by adding these quadratic functions together.

6.Material and Method

When exact solutions are not possible, these approximation techniques are frequently employed in numerical analysis to assess integrals.

6.1 Limitations

While approximation methods in integration offer valuable tools for numerical solutions, they are subject to various limitations that can affect the accuracy and reliability of results. Understanding these limitations and their implications is essential for effectively applying approximation techniques in real-world problems. Researchers must carefully consider these constraints and apply appropriate strategies to mitigate their impact, ensuring the validity of numerical integration results in diverse applications.

6.2 Approximation of Definite Integral

We know that $\int_a^b f(x)$ is definite integrable. But, we keep in mind that all function is not integral on the interval and even if it is integral it is very difficult. For example, it is impossible to evaluate the following integrals exactly

$$\int_0^1 e^{x^2}\ dx\ ;\int_{-1}^1 \sqrt{1+x^3}\ dx$$

Also, in same time we have data from the scientific experiment through instrument readings which gives no any formula for function. In both above cases we need to approximate the definite integral by the calculation of approximate integral. Here we describe two methods for approximate integrals (Davis, Chester, & Lucien ,1973).

6.3 Trapezoidal Rule

Let we have to calculate $\int_a^b f(x)dx.$ It gives the area between the curve y = f(x) and x-axis between ordinate at x = a and x = b for f(x) ≥ 0. In this rule, we approximate this area by calculating the area of trapeziums having equal which h = $\frac{b-a}{h}$ and heights (Sneddon,2004).

f(a) = f(x_0), f(a + h) = f(x_1), (a +2h) = f(x_2) ……..f(a + n h) = f(b)

7. Future Directions

Development of Hybrid Methods: Combining different approximation techniques to influence their respective strengths and alleviate weaknesses.

Integration with Artificial Intelligence: Discovering the synergy between numerical integration methods and machine learning algorithms for improved problem-solving capabilities.

Quantum Computing: Investigating the potential of quantum algorithms for efficient numerical integration, promising exponential speedups over classical approaches.

Conclusions

We also explore certain applications and their approximation from the trapezoidal rule. A strong mathematical idea called integration is used to compute volumes, find the area under curves, and solve a variety of real-world issues. It finds use in a variety of disciplines, including computer science, economics, physics, and engineering. Although precise solutions for integrals might not always be achievable, approximation methods offer useful ways to approximate their values. Anyone exploring the field of mathematics and its applications has to have a solid understanding of integration and its applications.

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