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Review and Thermal Calculation of a Shell and Tube Heat Exchanger with Baffles in Cross - Counter Flow Arrangement

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***Abstract* - In one pass shell and tube heat exchanger with baffles one fluid flows inside of parallel tubes and other fluid flows outside of tubes in perpendicular to tube axis changing alternately its direction. Passing the window of the baffles the shell side fluid deviates its flow direction rapidly. This creates just behind the window of each baffle a so-called "dead volume" which renders a stagnation of flow. This dead volume assumed to be triangular in shape causing the flow behind the window to be conical.**

In this thermal model the "dead volume" taken into account and change of temperature of the shell side fluid during its flow from tube to tube is considered. The thermal analysis of the cross- counter flow arrangement for any number of tube in bundle with progressive changes in tube length has been carried out. The numerical computations with different tube and baffle numbers are performed. It is found that at the beginning the effectiveness of the heat exchanger increases distinctly with increasing tube number and baffle number. But after certain number of tube and baffles the increasing rate weakens and it becomes very small. Due to the presence of "dead volume" there is reduction in the thermally active heat transfer surface area which leads to a smaller value of Number of Transfer Units (NTU). Hence, the results of computation obtained in this paper with the consideration of the "dead volume" will provide lower value of the effectiveness than that of without the consideration of it.

Keywords– Shell and Tube Heat Exchanger, Baffles, Cross – Counter Flow, Effectiveness, Number of Transfer Units, Dead Volume

Introduction

For an effective heat transfer process, the cross flow heat exchangers have found great demand in technology. Shell and tube heat exchangers in their various construction modifications are the most widespread and commonly used heat exchanger configuration in the process industries. They are used in the conventional energy transfer as condensers, feed water heaters as well as steam generators for pressurized water reactor plants. They are installed in many alternative energy transfer applications and used in some refrigeration or air conditioning services. There are still many unsolved problems related to the thermal theory of these exchangers which are to be clarified. Thus, it needs a continuous development in the theory of these heat exchangers for the purpose of applications by the designer. The first papers regarding thermal theory of cross flow arrangement were published by Smith [1] and Hausen [2]. Further development was done mainly by Braun [3] and Nicole [4].

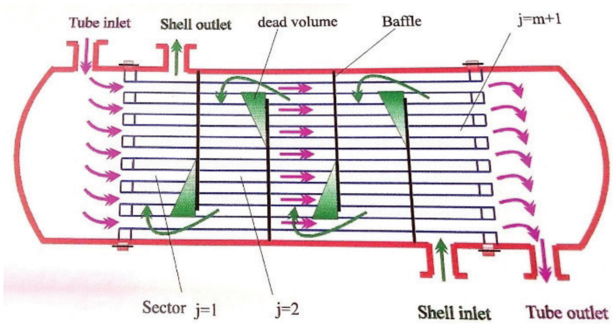


Fig. 1. One pass baffled shell and tube heat exchanger in cross counter flow arrangement

A shell and tube exchanger consists of a bundle of circular tubes fastened into a tube sheet at each end in a closed - packed arrangement of equilateral triangular or square pattern as shown in the fig. 1. The tube bundle is contained within a close – fitting shell. The clearance between the outside of the tube bundle and the inside of the shell is kept to a minimum but it varies from baffle to baffle and is a function of the design of the floating tube sheet. Baffles are used to control the fluid flow path i.e. to direct the shell side fluid to flow across the tube bundle, to increase its velocity and as a consequence the heat transfer coefficient, to provide support for the tubes to assure their unchanging space and prevent vibration. It causes the fluid a co- directed cross flow in a single sector between two neighboring baffles and a cross - counter flow regarding the whole heat exchanger.

The analysis of temperature fields and the effectiveness of heat transfer in a co-directed cross flow arrangement for any number of tubes in bundle has been carried out in the paper [5] on the basis of a new and exact method developed in the paper [6] and described in the paper [7]. But a clearly formulated thermal theory for a codirected cross flow in tube bundle of a shell and tube heat exchanger with baffles is desirable. Therefore, the main aim of this paper is to present the proposal of such theory accurately and in simple form from mathematical point of view.

1.2 Survey of Literature

1.2.1 Cross Flow Arrangement

The thermal theory of the co-directed cross-flow tube bundles is only supporting the problem under consideration, however, it is decisive by getting useful tool for straight-forward analysis of the shell and tube heat exchanger since it is the basic element of the cross-flow arrangement theory. That is why a survey of literature for the cross flow arrangement is presented here.

The publications regarding temperature fields and heat fluxes in single cross-flow tube originated in the mid-30th. For this flow arrangement where one fluid is cross mixed and the other unmixed, Smith formulated in 1934 the problem analytically and gave the method for its solution [3]. Later on, in 1937 Hausen proposed different ways of solving the problem of rectification-trays [4] which was then adopted to an analysis of cross flow tube bundles.

In 1940 the method was extended by Bowman, Mueller and Nagel [8] to the case of two tube rows and in 1957 by Stevens, Fernandez and Woolf [9] to three tube rows in bundle. All results achieved were presented summarily by Gardner and Taborek [10] in 1977 and by Schedwill [11] in 1968 who provided the analytical relation for the effectiveness of the tube bundles fed with fluids of uniform temperatures.

A further publication [3] which consists of considerable generalisation in comparison to previous methods is one presented by Braun in 1975. For the prior assumed conditions and for the stream flowing at the outside crosswise to the bundle of tubes, Braun analysed the problem solving the system of partial differential equations with augmented boundary conditions. There, the desirable accuracy of solution was achieved at the cost of complicated mathematical procedures.

In 1972 Nicole [4] collected and discussed numerous variations which deal with the thermal effectiveness of different flow arrangements and different connections of the fluids flowing inside the tubes. Moreover, he added solutions for some new cases regarding the conditions of the inlet- and outlet fluids. His considerations are limited to first few number of tubes in bundle.

As an improvement over the previous methods Bes and Roetzel in 1983 proposed new simplified energy balance equations [6]. The simplification consists only in the notation but not in the representation of the exact energy balance. The demanded accuracy of the thermal analysis was achieved by proper selection of a weighted mean temperature for the outside fluid and it was defined as the average temperature determined on two control passes i.e. in front and behind of single tube. This method is universal since it makes the thermal calculation easy for any number of tube in bundle with consideration of different flow directions in subsequent tubes and different boundary conditions. This is an additional advantage over all of previous methods.

The analysis of temperature fields and the effectiveness of heat transfer in a codirected cross flow arrangement for any number of tubes in bundle has been carried out in a paper [5] on the basis of a new and exact method developed in 1983 [6] and described in the paper [7]. The point of differences between mentioned papers persists in an assumption regarding the overall heat transfer coefficient that in [7] was taken as constant and in [5] it can vary from tube row to tube row. In both cases lengths of the tubes were considered to be constant.

Now and then it happens that in the bundle of tubes, due to perturbation of fluid flow- and heat transfer conditions, these tubes are partly excluded from an intensive heat transfer process. Then, it "works" as if the tubes were (progressively) shorter in comparison to their real length and as if the "thermally active" tube bundle were "conical" in shape. A solution to this problem in case of the any length of tubes in bundle for cross-flow arrangement was presented by Bes, Roetzel & Koirala [12]. Moreover, due to change in length of tubes, the alternations in the overall heat transfer coefficients and as a consequence in the Number of Transfer Units (NTU) any tube are taken into account. Other publications on this subject are more or less complicated alternatives of previously discussed methods regarding the flow arrangements and boundary conditions.

1.2.2 Shell and Tube Heat Exchanger with Baffles

The most general work regarding design of shell and tube heat exchangers including subsequent improvements was developed at the University of Delaware and presented by Bell in 1963 [13]. This method is based mainly on the experimental studies, however, theoretical analysis is applied as well. But there is no sufficient theoretical evidence, how repeated cross flow arrangement affects the effectiveness of the baffled heat exchanger.

The first theoretical method which deals with the problem under consideration was developed by Gaddis and Schlünder in 1975 [14]. In this method the energy balances were used to determine the local effectiveness of cells into which the exchanger was divided.

A new two-dimensional model of thermal calculation for one pass baffled heat exchanger was proposed by Bes & Roetzel in 1994 [15] which can be evaluated as a more realistic alternative to the cell method of Gaddis & Schlünder [14] described above. In this model the cell is selected in a way other than that in [14]. As a principle each cell consists of only one cross-flow tube row between two neighbouring baffles. The shell side fluid is now considered to be mixed only in front of the first tube row following the baffle window. Between the subsequent rows the shell side fluid is unmixed until it is mixed again in the next baffle window. So that in the method developed in [15] in contrary to the method of Gaddis & Schlünder [14] the real distribution of shell side fluid temperature at the entrance to next tube except first is taken into account. Due to turbulence the tube side fluid is only mixed across each tube, but not along it and not from tube to tube.

Kalpesh P. D. & Chopra M. in 2013 [16] presented the mathematical modelling of cross-counter flow heat exchanger. An analysis of the performance of a shell and tube type cross counter flow heat exchanger was performed by changing the various parameters such as both hot and cold fluid flow rate, direction of fluid flow. After changing the various parameters, the maximum performance was obtained and for that the mathematical model of counter flow heat exchanger was adopted.

Gao et al in 2015 [17] solved a dynamic thermal model for a cross flow heat exchanger numerically in order to predict the transient response under step changes in the fluid mass flow rate and the fluid inlet temperature. Transient responses of both the primary and secondary fluid outlet temperatures are characterized under different scenarios, including fluid mass flow rate change and a combination of changes in the fluid inlet temperature and the mass flow rate. The numerical procedure and transient response are investigated in detail in this study. A review and comparison of several journal articles related to the similar topic are performed. Several sets of data available in the literatures which are in error are studied and analyzed in detail.

Aujla et al in 2017 [18] focused on calculation of effectiveness of shell and tube heat exchanger with variation in temperature by considering with three case of various combination of temperature change. A fixed end assembly receives at one end of the tube set and provides mainly folding for directing multi pass tube fluid flow through the tube set and opposite floating end assembly receives and seals fluid tube ends to accommodate temperature induced expansion and contraction.

Napitupulu et al in 2020 [19] determined whether the oil temperature exit the shell and tube heat exchanger one shell and two tube pass, by calculating the effectiveness of the cooling oil on the tube using water as cooling fluid flowing through the shell using Number of Transfer Units (NTU) and experimental studies. Shell and tube heat exchanger one shell and two tubes pass effective used to cooling oil, reducing oil temperature up to 32%.

Ravelo-Mendivelso et al in 2022 [20] applied methodology to increase the thermal efficiency of a heat exchanger in real operating conditions which was based on the AHP (Analytic Hierarchy Process) multi-criteria method. Three relevant criteria were identified: Thermodynamic, Hydrodynamic, and Economic. Additionally, energy and exergetic analysis of the process, analysis of the thermodynamic properties of the fluids, pressure drop, volumetric flow of hot and cold fluids, energy costs, maintenance, operation and geometry of the heat exchanger were performed to study their effects on the thermal efficiency of the heat exchanger.

1.3 Assumptions

Within of each tube the overall heat transfer coefficient is constant, but it can vary from tube to tube. The heat capacity rate of each stream is constant throughout the exchanger and independent of temperature. The temperature of the tube-side fluid changes continuously along the path of the stream. The flow of fluid outside of the tubes is turbulent. Due to well mixing the temperature of the tube-side fluid is uniform at any cross-section of the tube. The temperature of the fluid outside the tubes changes in both directions: along the perimeter of the tube as well as along the tube axis. In the window between two consecutive baffles the shell side fluid is thoroughly mixed so that its temperature at the inlet to each sector is uniform. Since there is almost no movement of shell side fluid in the "dead volume" of each sector, the heat exchange between the fluids in this part is negligible. The temperature of the shell side fluid at the inlet of the heat exchanger is uniform. But this condition can be replaced by another weaker condition for which any temperature at the inlet to tubes is assumed to be known. Neither stream undergoes a change of phase. Heat losses from the system are negligible. No external work is done and the effect of gravitational potential energy is negligible.

The Analytical Method

To determine the temperature distributions of the tube side- and shell side fluid it is necessary to calculate the parameters such as overall heat transfer coefficient and number of transfer units for the both fluids in a single tube. Because of different character of these parameters in the conical flow channel and in the rectangular flow channel the temperature distributions of the tube side and shell side fluid in these both flow channels are derived by using two different equations separately.

2.1 Overall Heat Transfer Coefficient for Outside Fluid in Single Tube

Let us consider first the conical flow channel in any sector of the heat exchanger under consideration in which fluid 1 flows inside of parallel tubes and fluid 2 flows outside of tubes as shown in fig. 2. The amount of fluid passing the bundle of tubes from outsides is distributed uniformly along the "thermally active" tube length and it is valid for each tube row. But due to increase in the length of tubes row to row in the conical flow channel its velocity decreases accordingly. This causes the stepwise change in the local heat transfer coefficients for the fluid 2 as well as in the overall heat transfer coefficients from tube to tube. The local heat transfer coefficients $\alpha_{2,i}$ for each tube can be calculated by using the relation given in paper [12] and putting n = 1 for the first tube as

$$\alpha_{2,i} = \alpha_{2,1}\left(\frac{l_1}{l_i}\right)^{0.8} \qquad (1)$$

The heat transfer coefficients for the ith tube can be now determined by using last equation when heat transfer coefficients for the first tube $\alpha_{2,1}$ and length ratios l_1/l_i for all tubes are known.

Similarly, the relations for overall heat transfer coefficients k_i is defined from the following well known relation [21] a

$$\frac{1}{k_i} = \frac{1}{\alpha_{1f}} + \frac{1}{\alpha_{2,i}} + \frac{\delta}{\lambda} \quad (2)$$

Further, the equation (2) can be noted in following simplified notation,

$$\frac{1}{k_i} = \frac{1}{\alpha_1} + \frac{1}{\alpha_{2,i}} \quad (3)$$

where

$$\frac{1}{\alpha_1} = \frac{1}{\alpha_{1,f}} + \frac{\delta}{\lambda} \quad (4)$$

Combination of the equations (1) and (3) leads to required relation for overall heat transfer coefficients k_i for each tube i in the conical flow channel of any sector as

$$\frac{1}{k_i} = \frac{1}{\alpha_1} + \frac{1}{\alpha_{2,1}}\left(\frac{l_i}{l_1}\right)^{0.8} \quad (5)$$

Distribution of the shell side fluid in rectangular flow channel of each sector is assumed to be uniform. Moreover, the length of the tubes in this part of each sector is the same from row to row. Therefore, the overall heat transfer coefficient of the shell side fluid in rectangular flow channel of each sector can be taken as constant for each tube and it is equal to

$$\frac{1}{k} = \frac{1}{\alpha_1} + \frac{1}{\alpha_2} \quad (6)$$

2.2 Number of Transfer Unit for Outside Fluid in Single Tube

Due to the change in the overall heat transfer coefficients as well as change in the tube lengths from tube to tube the number of transfer units in the case of the outside-fluid also differs from tube to tube. As given in reference [22] the relation between number of transfer units (*NTU*) for whole exchanger and the same for a single tube is:

$$NTU_2 = \sum_{i=1}^{n+n_1} ntu_{2,i} = \sum_{i=1}^{n+n_1} \frac{k_i b_0 l_i}{C_2} \quad (7)$$

Combining this relation with the equation (6), we get following relation,

$$\left[\frac{n.NTU_2}{(n+n_1)(m+1)}\right] = \frac{\alpha_1 b_0 l_1}{C_2} \sum_{i=1}^{i=n} \frac{\frac{l_i}{l_1}}{1+\frac{\alpha_1}{\alpha_{2,1}}\left(\frac{l_i}{l_1}\right)^{0.8}} \quad (8)$$

where n, n_1 and m are number of tubes in the conical flow channel, number of tubes in the rectangular flow channel and number of baffles in the heat exchanger respectively.

After simple algebraic manipulations one can write:

$$ntu_{2,1} = \left[\frac{n.NTU_2}{(n+n_1)(m+1)}\right] \Big/ 1+\left(1+\frac{\alpha_1}{\alpha_{2,1}}\right)\sum_{i=2}^{i=n}\left(\frac{l_i}{l_1}\right) \Big/ \left[1+\frac{\alpha_1}{\alpha_{2,1}}\left(\frac{l_i}{l_1}\right)^{0.8}\right]$$

for the first tube (9)

and

$$ntu_{2,i} = ntu_{2,1}\left(1+\frac{\alpha_1}{\alpha_{2,1}}\right)\left(\frac{l_i}{l_1}\right) \Big/ \left[1+\frac{\alpha_1}{\alpha_{2,1}}\left(\frac{l_i}{l_1}\right)^{0.8}\right]$$

for the tubes i = 2, 3,………n – 1, n. (10)

The relations for the tube length ratios l_i/l_1 can be expressed in term of given parameter κ are:

$$\frac{l_i}{l_1} = 1 + (i-1)\kappa \quad (11)$$

where $\kappa = \Delta l/l_1$ is the ratio of tube length difference to length of first tube known as relative deviation.

Due to the same length of the tubes in the rectangular flow channel of the heat exchanger the number of transfer units for each tube can be taken as constant and it is calculated as follows:

$$ntu_2 = \frac{NTU_2}{(n+n_1)(m+1)} \quad (12)$$

where m is the number of baffles present in the heat exchanger.

2.3 Energy Balance for Fluids Flowing Inside and Outside a Single Tube

A simplified model of the flows in a single tube selected from the bundle is shown in figure 3. Let one consider a single tube i selected from the bundle in any one sector of the heat exchanger.

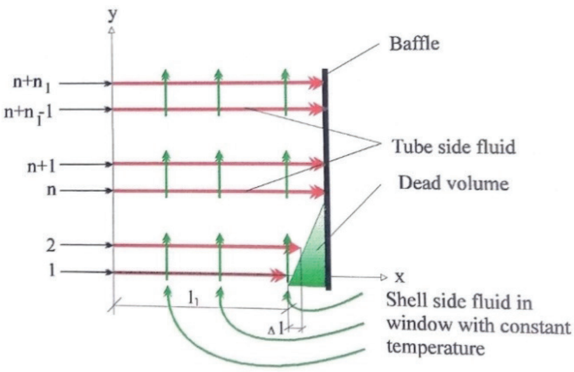


Fig. 2. Schematic representation of codirected cross flow arrangement in any sector of shell and tube heat exchanger with baffles.

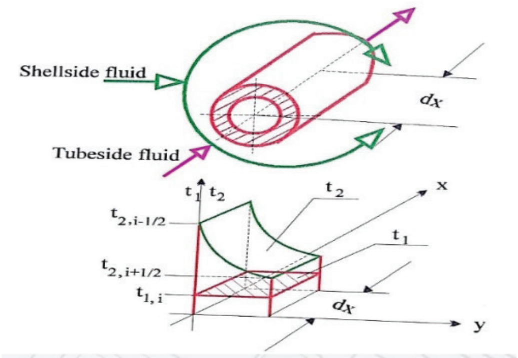


Fig. 3. Visual demonstration of flow arrangement in cross flow for i[th] tube

A simplified model of the flows in a single tube selected from the bundle is shown in figure 3. Let one consider a single tube i selected from the bundle in any one sector of the heat exchanger.

The temperature distribution along the flow paths of streams inside and outside a tube can be calculated after setting up the energy balance equations and by using relation for the heat flux transferred between the fluids. Finally, the energy balance for the tube-side fluid 1 flowing in the i[th] tube using some dimensionless quantities leads to the differential equations in the following form [12]:

$$-\frac{dT_1}{ds} = ntu_{1,i}\frac{l_i}{l_1}\left(T_i - \vartheta_i\right) \qquad (13)$$

where T, ϑ and s are dimensionless temperature of tube side fluid, dimensionless temperature of shell side fluid and dimensionless path of fluid for the n^{th} tube respectively.

Similarly, the energy balance for the fluid outside the tube using some dimensionless quantities leads to an equation as follows [12]:

$$\vartheta_{i+1/2} - \vartheta_{i-1/2} = ntu_{2,i}\left(T_i - \vartheta_i\right) \qquad (14)$$

Equations (13) and (14) can be solved by introducing an average temperature ϑ_i defined as the weighted average of the temperatures $\vartheta_{i+1/2}$ and $\vartheta_{i-1/2}$ given in the paper [11] as

$$\vartheta_i = \omega_i \vartheta_{i+1/2} + \left(1-\omega_i\right)\vartheta_{i-1/2} \qquad (15)$$

where w is the weighing factor.

Eliminating the temperatures ϑ_i from equations (14) and (15) gives

$$\vartheta_{i+1/2} - \mu_i \vartheta_{i-1/2} = \left(1-\mu_i\right)T_i \qquad (16)$$

Where,

$$\mu_i = 1 - ntu_{2,i} / \left(1 + \omega_i ntu_{2,i}\right) \qquad (16a)$$

Eliminating the temperature ϑ_i from equations (13) and (15) then using the equation (16), we get

$$-\frac{dT_i}{ds} = B_i\left(T_i - \vartheta_{i-1/2}\right) \qquad (17)$$

where $B_i = \left(l_n / l_i\right) ntu_{1,i} / \left(1 + \omega_i ntu_{2,i}\right)$ (17a)

for i = 1, 2, ……………….., $n+n_1$

Now, let one consider a cross-section of any selected tube and the fluid belonging to it. Due to well mixing of the inside fluid across the tube its temperature at any location between x and $x+dx$ is locally constant. Indeed, the slice of the tube under consideration with the appropriate volume of fluids should be treated as a small recuperator with the heat transfer surface area equal to *bo.dx*.

If in this simple recuperator the temperature of one fluid is constant, then for solving the problem the flow direction of this inside-fluid is immaterial. This is an essential fact in the theory of heat exchangers. Thus, the mean difference of temperature should be calculated as the logarithmic mean of the temperature differences at the inlet and outlet. This leads to the relation:

$$\vartheta_i - T_i = \frac{\left(\vartheta_{i-1/2} - T_i\right) - \left(\vartheta_{i+1/2} - T_i\right)}{\ln\left(\dfrac{\vartheta_{i-1/2} - T_i}{\vartheta_{i+1/2} - T_i}\right)} \qquad (18)$$

Evaluating the equation (14) for $ntu_{2,i}$ and the equation (16) for μ_i, and putting these two parameters in the equation (18), we get,

$$\mu_i = e^{-ntu_{2,i}} \qquad (19)$$

By eliminating μ_i from equations (16a) and (19), we get a required expression for the parameter ω_i of the outside fluid temperature as

$$\omega_i = \frac{1}{1-e^{-ntu_{2,i}}} - \frac{1}{ntu_{2,i}} \qquad (20)$$

Now, a system of difference-differential equations (16) and (17) with only two unknown functions T_i and $\vartheta_{i\pm1/2}$ can be easily solved with help of the boundary conditions adjusted to the heat exchanger under consideration.

2.4 The Temperature Distribution of Tube-Side- and Outside Fluid

In order to calculate the temperature distribution of the fluids flowing inside and outside the tubes, one refers to the set of the difference-differential equations (16) and (17) which was derived in section 2.3. According to the mathematical way of classification it is a system of *2(n+n$_l$)* linear difference-differential equations with constant coefficients. Because of the different flow conditions in the conical flow channel and rectangular flow channel the solution to this problem is divided into two parts.

The set of difference – differential equations (16) and (17) can be solved by using two different ways: either by the Laplace transformation or by the recursive method of is integration i.e. integration of the differential equations one by one. In this paper the second way chosen.

2.4.1 Conical Flow Channel

In case of conical flow channel, the starting equation for solving the problem is a differential equation for i = 1 from the set of *n* linear differential equations (17). This equation can be solved by using the assumption that $\vartheta_{1-1/2}(s) = \overline{\vartheta}$. Finally, it can be proved as in [12] that the step by step procedure of integration of the equations (16) and (17) leads to the following relation for the tube side-fluid temperature as:

$$T_i(s) = \overline{\vartheta} - c_i e^{-B_s s} - \sum_{j=1}^{j=i-1} a_{i-1,j} b_{i,j} c_j (1-\mu_j) e^{-B_j s}$$

for i = 1,2,……………, n-1, n. (21)

where $$a_{i-1,j} = \prod_{k=j+1}^{k=i-1} B_k - (\mu_k B_j)/(B_k - B_j) \qquad (21a)$$

($a_{i-1,i-1} = 1$ is defined)

$$b_{i,j} = B_i/(B_i - B_j) \qquad (21b)$$

are auxiliary parameters.

Similarly, the general equation for the shell side fluid temperature can be written as

$$\vartheta_{i-1/2}(s) = \overline{\vartheta} - \sum_{j=1}^{j=i-1} a_{i-1,j} c_j (1-\mu_j) e^{-B_j s} \qquad (22)$$

for i = 1, 2, …………, n-1, n.

The unknown parameters c_i are integration constants which will be found with the help of boundary conditions:

$T_i\ (s=0) = T_{n+n1+1-i}\ (s=l_i/l_n)$.

2.4.2 Rectangular Flow Channel

In case of rectangular shape of flow channel, the starting equation for solving the problem is a differential equation for *i = n+1* from the same set of *(n+n$_l$)* linear difference –differential equations (17). These equations can be solved to determine T_{n+1} by using the boundary condition that the temperature profile of the shell side fluid behind the last tube in the conical flow channel which at the same time is its inlet temperature in front of the first tube in the rectangular flow channel, i.e. $\vartheta_{n+1/2}(s) = \vartheta_{(n+1)-1/2}(s)$. Finally, the shell side fluid temperature is determined by using the equations in its final form as follows:

$$\vartheta_{n+1/2} = \overline{\vartheta} - \sum_{j=1}^{n} c_j d_j e^{-B_j s} \qquad (23)$$

where $$d_j = a_{n-1,j}(1-\mu_j)\left[\mu_n + (1-\mu_n) b_{n,j}\right] \qquad (23a)$$

for j = 1,2, ……….., n-2, n-1

$$d_j = (1-\mu_j) \qquad (23b)$$

for j = n

Similarly, it can be proved as in [15] that the integration of the equations (16) and (17) for *i = n+1, n+2, ..., n+n$_l$* by using the step by step procedure leads to the following general equation for the tube side fluid temperature in the rectangular flow channel of each sector as:

$$T_i(s) = \overline{\vartheta} - \mathbf{e}^{-B} \sum_{j=n+1}^{j=i} c_j^* p_{i-j}(s\sigma)\mu^{i-j} - B\sum_{k=1}^{k=n} \frac{(B-\mu B_k)^{i-n-1}}{(B-B_k)^{i-n}} c_k d_k \mathbf{e}^{-B_k s}$$

for i = n+1, n+2,………, n+n (24)

where

$$\sigma = B[(1/\mu) - 1] \qquad (24a)$$

and

$$p_i(s\sigma) = s\sigma \sum_{j=1}^{j=i} \frac{(i-1)!}{(j-1)!\,(i-j)!} \frac{(s\sigma)^{j-1}}{j!} \qquad (24b)$$

are auxiliary parameter and polynomials respectively.

Bes and Roetzel [23] pointed out that the polynomials $p_1(\sigma)$ for variable s =1 can be expressed and determined by using other polynomials known as the *Laguerre Polynomials* from literature of applied mathematics [24].

2.5 Effectiveness of Heat Exchanger

The heat exchanger effectiveness is defined as the ratio of the actual rate of heat transfer to the maximum possible rate of heat exchange between the fluids. The maximum possible rate would be obtained in a counter flow heat exchanger of infinite heat transfer area. The maximum heat transfer can never be attained in practice. To evaluate the performance of a heat exchanger the calculation of its effectiveness is used [21]. For the flow arrangements as in this paper it can be proved that the thermal effectiveness is equal to the arithmetical mean value of the tube side fluid temperatures at the outlet of the heat exchanger [15]. For the flow arrangement under consideration the general formula is given by

$$P = \frac{1}{\left(n + n_1\right)} \sum_{i=1}^{i=n+n_1} T_{i,m+1} \tag{25}$$

where $T_{i,m+1}$ are the tube side fluid temperatures at the exit from $(m+1)^{th}$ sector (outlet of heat exchanger).

On the other hand, the effectiveness of the heat exchanger can also be calculated by using the shell side fluid temperature at its outlet (such as first sector) as

$$P = \frac{\left(1 - \overline{\vartheta}_0\right)}{R_1} \tag{26}$$

In this relation $\overline{\vartheta}_0$ denotes the average temperature of the shell side fluid at its outlet and it is determined from the energy balance at the first sector and R_1 is the heat capacity rate ratio of the tube side fluid.

This relation allows one for construction of diagrams: the effectiveness of tube-side fluid versus the effectiveness of outside fluid in the way as reported in literature [25].

Numerical Results and Discussion

Numerical results received according to the theory developed above are illustrated in figures 4, 5 and 6. The effectiveness of the tube side fluid P_1 is plotted against the effectiveness of the outside fluid P_2 to demonstrate how effective a heat exchanger will behave over a range of the Number of Transfer Units NTU and heat capacity rate ratio R. In order to compare the results of the heat exchanger under consideration with pure counter flow heat exchanger, it is also needed to calculate the log mean temperature difference correction factor F which is presented graphically in the diagrams of the effectiveness P_1 against P_2. From the diagrams of the effectiveness P_1 and P_2 one can abstract the followings:

- The numerical results have shown that for all values of relative deviation in conical flow channel the diagrams of effectiveness P_1 against P_2 (figures 4, 5 and 6) show almost symmetry referred to a line where heat capacity rate ratio R = 1.
- From these diagrams (figures 4, 5 and 6) it is clear that at the very beginning the effectiveness of the heat exchanger under consideration increases distinctly with increasing tube numbers $n + n_1$ (together in conical – and rectangular flow channel) and increasing baffle numbers m in heat exchanger. But after certain number of tube rows and baffles (e.g. $n+n_1 = 20$, $m = 5$) the increasing rate weakens and then it becomes very small. Finally, the effectiveness approaches its asymptotic value (e.g. fig. 6).
- All the log mean temperature correction factor F – curves begin at $P_1 = 1$, $P_2 = 0$ and end at $P_1 = 0$ and $P_2 = 1$ for all values of the relative deviations which occurs normally in cross flow arrangements. The trends of these F – curves (e.g. for F = 0.99) show clearly how the effectiveness increases with increasing the tube- and baffle numbers in the heat exchanger and how it differs from that of the pure counter flow heat exchanger (e.g. fig. 4 with two baffles and fig. 5 with five baffles).
- It should be emphasized that the consideration of "dead volume" in heat exchangers results the reduction in its effectiveness. This is because of the presence of "dead volume" where due to almost no movement of shell side fluid there is the reduction in the thermally active heat transfer surface area which dominates the increase in the local heat transfer coefficient. This leads to the smaller value of *NTU* (Number of Transfer Units with "dead volume") in comparison with the reference value of NTU_o (Number of Transfer Units without "dead volume"). When the velocity of the shell side fluid and therefore the size of "dead volume" grows up (i.e. for the higher value of relative deviation κ) the effectiveness of the exchanger decreases further. As the reference diagram for the previous figures the effectiveness for number of tube $n = 10$ rows and relative deviation $\kappa = 0$ is shown in the figure 7 [12].

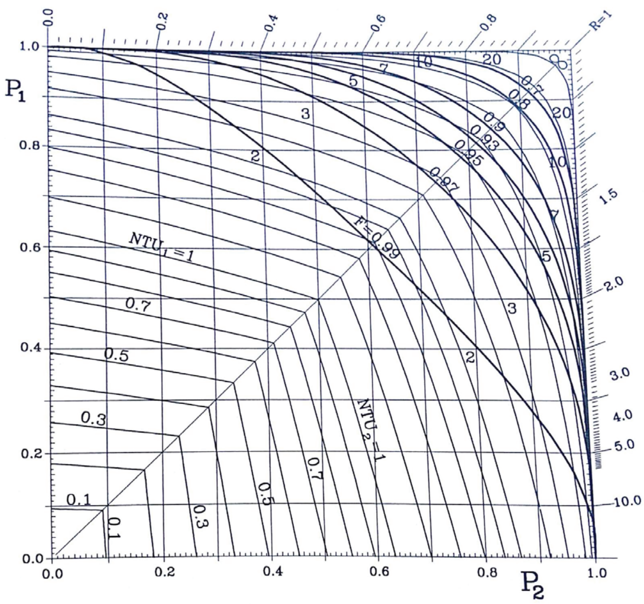
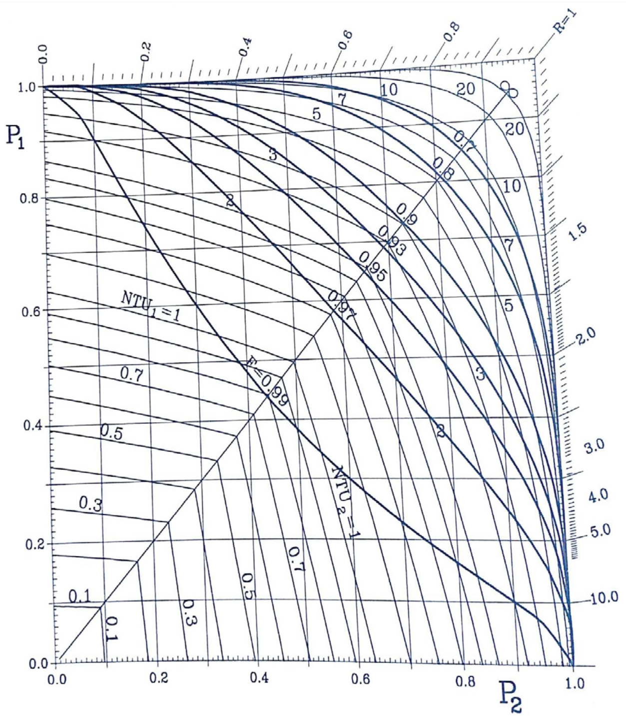


Fig. 4. Effectiveness of baffled shell and tube heat exchanger in cross counter flow arrangement with number of tube rows in conical flow channel n = 2, number of tube rows in rectangular flow channel n_1 = 5, number of baffles m = 2 and relative deviation κ = 0.05



.Fig. 5. Effectiveness of baffled shell and tube heat exchanger in cross counter flow arrangement with number of tube rows in conical flow channel n = 2, number of tube rows in rectangular flow channel n_1 = 5, number of baffles m = 5 and relative deviation κ = 0.05.

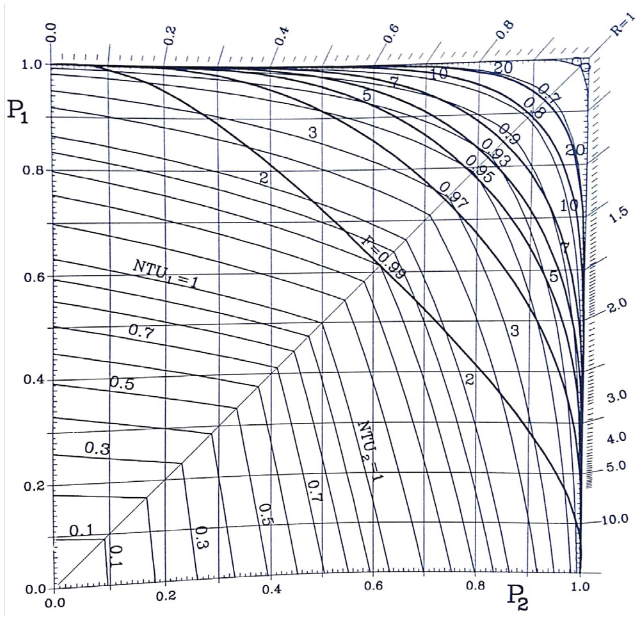


Fig. 6. Effectiveness of baffled shell and tube heat exchanger in cross counter flow arrangement with number of tube rows in conical flow channel n = 5, number of tube rows in rectangular flow channel n_1 = 15, number of baffles m = 5 and relative deviation κ = 0.03.

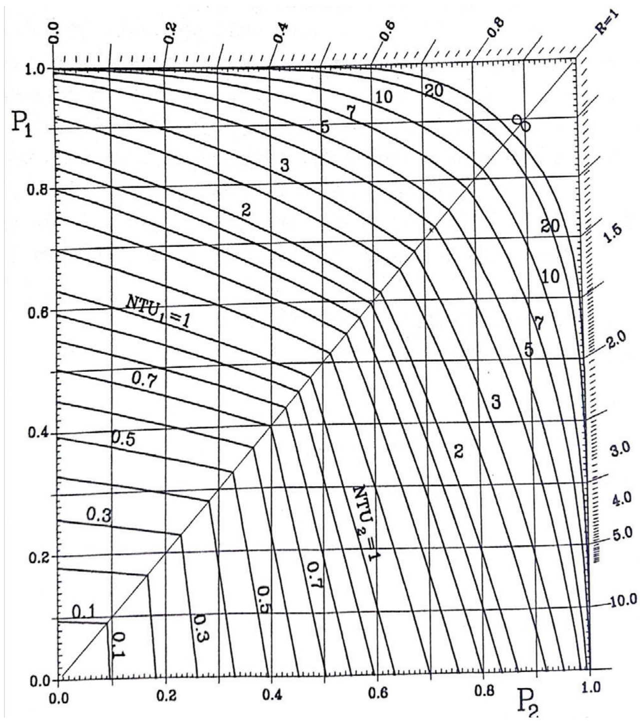


Fig. 7. Effectiveness of codirected cross flow heat exchanger with one pass and number of tube n = 10 rows with relative deviation κ = 0. [12]

Conclusions

- A new model is proposed for the thermal calculation of the one pass baffled shell and tube heat exchanger in cross counter flow arrangement in which so called "dead volume" just behind the window of each baffle is taken into account. This model allows to consider more realistically changes of temperature in each sector during the shell side fluid flow from tube to tube which is an improvement upon the previous model of Gaddis & Schleunder [14].

- The effectiveness P_1 and P_2 can be promptly calculated by using the relations derived in this paper for given number n of tubes in the conical flow channel, given number n_1 of tubes in the rectangular flow channel, Number of Transfer Units NTU, heat capacity rate ratio R of the fluids, ratio of heat transfer coefficients $\alpha_1/\alpha_{2,1}$ of the fluids and ratio κ of the tube length difference to length of first tube (relative deviation).
- It should be emphasized that the consideration of "dead volume" in heat exchangers results the reduction of its effectiveness in comparison with case where it was ignored in [15]. When the velocity of the shell side fluid and therefore the size of "dead volume" grows up (i.e. for the higher value of relative deviation) the effectiveness of the exchanger decreases further. This paper gives the numerical evidences of this fact and it should be taken into account by the designer.

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