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Lexicography in the Context of Evacuation Planning Issues

Iswar Mani Adhikari
Department of Mathematics, Prithvi Narayan Campus, Pokhara, Nepal

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Corresponding author: *Iswar Mani Adhikari, Email: adhikariim35@gmail.com*
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Abstract

Lexicography in evacuation planning refers to prioritizing and structuring the evacuation process in emergencies. It provides a systematic method for hierarchically addressing multiple objectives, ensuring that the most critical factors are resolved first. This work focuses on such an approach to the evacuation planning problem for the transit-based network. Lexicographically dynamic flow determines the optimal flow in a network like the maximum and minimum cost flow. The term lexicography is based on lexicographic ordering and refers to sorting or optimizing solutions in a way that respects a lexicographic priority. Such a priority can be considered based on the need of the network structures, objectives, demands, etc. Multi-objective optimization problems where secondary objectives are considered only after primary objectives are met also use such a technique.

Keywords: Evacuation, lexicography, lexicographically maximum flow, network flow, transit-based network

1. Introduction

Lexicography is a decision-making approach where priorities are ranked in order of importance. The most important criterion is addressed first, and subsequent criteria are considered only after higher-priority criteria are satisfied. In evacuation planning, the primary focus might be saving lives, minimizing evacuation time, and then reducing damage or discomfort, like, ensuring safety, such as maximizing the number of people evacuated safely. We must also minimize evacuation time or distance, reduce congestion, optimize resource utilization, or minimize cost. The problem is often modeled using graph theory, where nodes represent locations and edges represent pathways. Each pathway might be weighted according to travel time, capacity, or safety.

Algorithms like lexicographic linear programming or multi-objective optimization are used. These methods iteratively solve for the highest-priority objective before moving to the next. On the other hand, dynamic lexicographic strategies can adapt to changing conditions, such as road blockages or sudden influxes of evacuees, and adapt well to multi-objective problems where trade-offs are unacceptable at the highest levels of importance. Computational demands can be high in large-scale scenarios with many objectives and constraints. Priorities may shift dynamically, requiring flexible and responsive systems. Hence, by incorporating lexicography, evacuation planning becomes a structured and ethical process, addressing urgent needs first while striving for efficiency and equity. The performance and efficiency of strategies for evacuation planning problems depend upon the nature of evacuation network topology, density, and behavior of the population and many other factors as mentioned in Adhikari et al. (2020), Dhamala & Adhikari (2021), Adhikari & Dhamala (2020), Adhikari & Dhamala (2020a), and Adhikari (2023).

Dynamic network flows are directed graphs in which an arc has a non-negative flow capacity and transit times. Such problems were first studied and introduced by Ford and Fulkerson to send the maximal amount of flow from a source to a sink for the fixed time horizon T, as in Ford and Fulkerson (1958) and (1962). The evacuation planning problem is fundamental in the dynamic network. For the time minimization, it is to get the minimum time limit θ for all the supplies that can be sent to the sinks within time. Disastrous events are uncertain like in an earthquake, there is no prior knowledge of the damage that can occur. In such a case, it is more desirable to simultaneously maximize the number of supplies that have reached the sinks at every time unit and are provided by the earliest arrival flows. Minieka (1973) and Wilkinson (1971) developed a pseudo-polynomial time algorithm in the discrete-time model with a network having a single source and a single sink. Hoppe and Tardos (2000) have provided a fully polynomial-time approximation scheme for such a problem.

Lexicographic dynamic flow problems are optimization problems that combine dynamic flows with lexicographic optimization. As mentioned above, these problems arise in contexts where multiple objectives must be addressed in a specific order of priority while considering the temporal aspects of flow in a network. For example, firstly, minimize the primary objective, then optimize the secondary one without worsening the primary, and so on. This is particularly useful when objectives are not commensurable or are hierarchically important.

Transit-based evacuation planning problems are to minimize the duration of evacuation by routing and scheduling a fleet of homogeneous and capacitated vehicles say buses, which were initially located at one or more depots. The number of evacuees at each pickup location could exceed the capacity of a bus, which signifies the necessity of a split delivery service. Moreover, the number of available buses is insufficient to transport all the evacuees without multiple trips and each shelter has a capacity that limits the number of evacuees it can serve. Such situations also demand a split delivery service. In such situations, two alternative models for the multi-depot, multi-trip, bus-based evacuation planning problems were proposed and analyzed by Bish (2011). The first model simultaneously identifies the optimal route construction and assignment of the vehicle, whereas the next identifies the optimal route assignment from a set of feasible routes. Goerigk et al. (2013) have developed a simplified version of the bus-based evacuation planning model for the evacuation of a region from a set of collection points to a set of capacitated sinks with the help of buses in minimum time if the bus pick-ups the same number of people that equals its capacity and hence, no need of split delivery service.

In this work, we focus on the lexicographic approach to the integrated evacuation planning problem. Such an approach seems applicable for the formulation and implementation of an evacuation planning problem, as different priorities can also be assigned to various attributes in the network. The rest of the paper is organized as follows: In section 2, we discuss the preliminaries on network flow problems. The lex-max dynamic flow approach and its consequences are presented in Section 3. It also presents a simple numerical example with the necessary demonstration. Section 4 presents its importance and applications in evacuation planning problems. Finally, in Section 5, we present some concluding remarks.

1. Preliminaries on network flows

A dynamic network $N = (G, u_e, \tau_e, b_v, S)$ consists of a directed graph $G = (V, E)$ with a nonnegative capacity u_e and an integral transit time τ_e associated with each edge $(y, z) = e \in V$, and a set of terminals S, a set of edges E, where the terminal is either a source or sink. A set of sources and that of the sinks are denoted by S^+ and S^- respectively. For all $v \in V$, a set of edges entering and leaving a node v are denoted by $\delta^+(v)$ and $\delta^-(v)$, respectively. The edges having positive capacity are denoted by E^+which is used for sending flow with non-negative transit times in the network.
The excess in each node is given by the difference of the flow reaching the node and leaving from the node and is given by,

$$Ex_{f(v)} = \sum_{e \in \delta^-(v)} f(e) - \sum_{e \in \delta^+(v)} f(e) \tag{1}$$

Where the flow conservation is constraint by,

$$Ex_{f(v,)} = 0 \;\; for \;\; v \in V \backslash (S^+ \cup S^- \tag{2}$$

$$Ex_{f(v,)} \geq 0 \;\; for \;\; v \in V \backslash S^- \tag{3}$$

$$Ex_{f(v,)} \leq 0 \;\; for \;\; v \in V \backslash S^+ \tag{4}$$

In such a static network as above, the maximum flow is a flow that maximizes the flow value denoted by,

$$|f| = \sum\nolimits_{s \in S} Ex_f(S^-) \tag{5}$$

Wide varieties of flow models are used on different flow problems related to evacuation planning issues. For details, we prefer Adhikari (2020) and the references therein.

A dynamic network consists of a network with transit times $\tau_e: E \rightarrow$IR. We may have supplies and demands b_v for all nodes $v \in V$. Dynamic flow models are categorized fundamentally as discrete-time models and continuous-time models. Here we use the discrete dynamic flows where time steps are discretized as {1,2,3,…, T}. On the other hand, the continuous flow specifies the flow rate for every moment in time for $\theta \in [0, T)$. In the discrete-time model, the dynamic flow is a mapping $fE \times [0, T) \rightarrow R^+$. No flow is left in the edges after the time horizon T. These are to satisfy the capacity constraints as well as the flow conservation constraints. For the capacity constraints,

$$0 \leq f(e, \theta) \leq u_e \forall\, e \in E \tag{6}$$

for time $\theta \in \{1, 2, 3, .., T\}$. The excess of flow in each node v for two such discrete time settings is given by,

$$Ex_{f(v,\theta)} = \sum_{e \in \delta^-(v)} \sum_{\epsilon=0}^{\theta-\tau_e} f(e, \xi) - \sum_{e \in \delta^+(v)} \sum_{\epsilon=0}^{\theta} f(e, \xi) \tag{7}$$

Then the flow conservation constraints should be satisfied,

$$Ex_{f(v,\theta)} = 0 \;\; for \;\; v \in V \backslash (S^+ \cup S^- \tag{8}$$

$$Ex_{f(v,\theta)} \geq 0 \;\; for \;\; v \in V \backslash S^- \tag{9}$$

$$Ex_{f(v,\theta)} \leq 0 \;\; for \;\; v \in V \backslash S^+ \tag{10}$$

We have $Ex\,(v) = Ex(v,\; T)$ where $Ex\,(v) \leq -b_v \;\; for \;\; v \in V \backslash S^-$ and $Ex\,(v) \geq b_v \;\; for \;\; v \in V \backslash S^+$

In such a dynamic network, a maximum flow maximizes the flow value given by Equation (7). Whereas the total amount of flow sent to sinks until the time horizon θ is given by,

$$|f|_\theta = \sum_{s^- \in S} Ex_f(S^-, \theta) \tag{11}$$

1. Lexicographically maximum dynamic flow

Lexicographically dynamic flow determines the optimal flow in a network like the maximum and minimum cost flow. The term lexicography is based on lexicographic ordering and refers to sorting or optimizing solutions in a way that respects a lexicographic priority. In simpler terms, solution A is preferred over solution B if the first differing component of A is smaller (or larger, depending on the ordering) than the corresponding component of B. The solutions are evaluated lexicographically, meaning that preference is given to solutions based on a pre-defined order of priorities where such priorities are dynamically adjusted. Multi-objective optimization problems where secondary objectives are considered only after primary objectives are met also use the lexicographically maximum dynamic flow algorithm.

A flow value is lexicographic if it is compared according to the rank of the terminals in the available network. Let $N = \{V, A, u, \tau, S, Z, T\}$ be a prioritized network with a priority $t_1, t_2, \ldots, t_n$ such that $t_i \in S \cup Z$, for S to be a source and Z to be a sink, then

$$|f|_t = \begin{cases} \sum_{a \in A_i^+} f_a \ , \mathrm{t} \in \mathrm{S} \\ \sum_{a \in A_i^-} f_a \ , \mathrm{t} \in \mathrm{Z} \end{cases} \tag{12}$$

This gives the out/in flow value from/in the source and sink, respectively. Let $f^1, \ f^2$ be the terminal respecting flows, f^1 is said to be lexicographically bigger than f^2 and written as $f^1 \geq_L \ f^2$. The maximum flow respecting the rank of the terminals is said to be lex-max flow.

The lex-max flow respects the prioritized network. A prioritized network is a multiterminal network that consists of prioritized terminals. Under the given priority of terminals, flows can be compared according to the departure or arrival flows from or in the sources or sinks, in respective order.

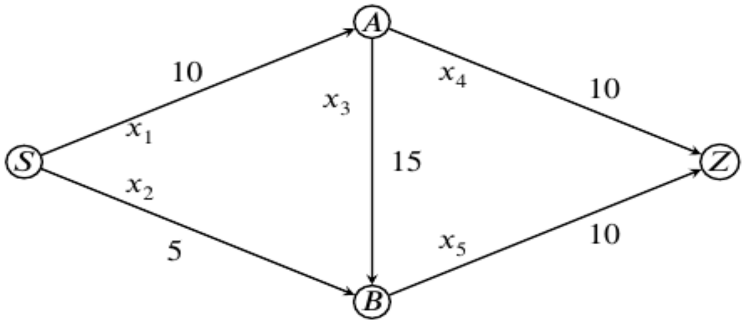
In evacuation planning problems, for the lexicographically maximum dynamic flow, an ordering of the k sources and sinks is given where we need to have a feasible dynamic flow within the specified time horizon T which lexicographically minimizes the amount of flow entering the sinks in that order, equivalent to say that it lexicographically maximizes the amount of flow entering the sink in such specified order and can be computed in polynomial time, as mentioned in Hoppe and Tardos (2000).

A super source can be connected to the network by infinite-capacity zero-transit time with artificial arcs for all sources. In such a resulting network, consider the zero flow in the artificial arcs of the network. Beginning with zero flow, the lexicographically maximum dynamic flow algorithm computes the successive layers of minimum cost circulations of static flows in residual graphs of their previous layers. The minimum-cost circulation and the maximum-cost flow computation are carried out in the residual network to achieve the lexicographically maximum dynamic flow, as designed by Hoppe and Tardos (2000).

Lexicographic optimization is a multi-objective optimization that deals with optimization problems with two or more objective functions to be optimized simultaneously. Often, the different objectives can be ranked in order of importance to the decision-maker, so that an objective f^1 is the most important, and f^2 is the next most important, and so on. For example, consider a firm that puts safety above all. It wants to maximize the safety of its workers and customers. Subject to attaining the maximum possible safety, it wants to maximize profits. This firm performs lexicographic optimization, where f^1 safety and f^2 denotes profits.

For instance,

Find the lexicographically maximum flow in the network provided as in the figure where the capacity of the arcs $S \to A$ *,* $S \to B$ *,* $A \to B$*,* $A \to T$*, and* $B \to T$ *are 10, 5, 15, 10, and 10 respectively.*



A simple weighted network

As mentioned above, the Lexicographic maximum flow refers to finding a flow through a network that not only maximizes the overall flow but also prioritizes the flow through arcs in a given lexicographical order. To find such a flow that maximizes the total flow from the source S to sink T. Among all the maximum flows, we need to choose the lexicographically maximum one, i.e., prioritizing the flow through the edges in lexicographic order: $S \to A$, $S \to B$, $A \to B$, $A \to T$, and $B \to T$ respectively.

Let the flows assigned to them be x_1 , x_2, x_3, x_4, and x_5 respectively. To maximize the flow, consider x_1 and x_2 be as much as possible, for this we get $x_1 = 10$ and $x_1 = 5$. Then the required flows are $f: S \to A = 10$, $f: S \to B = 5$, $f: A \to T = 10$, $f: B \to T = 5$, and $f: A \to B = 0$. Such a flow of 15 units is both the maximum flow and lexicographically maximum flow in the network provided.

2. Importance of Lex-max flows in evacuation planning strategies

In evacuation planning, the lexicographically maximum dynamic flow problem focuses on optimizing the evacuation process such that the flow is prioritized across time steps in a lexicographic manner. This means ensuring that as many people (or as much flow) as possible are evacuated at each time step, prioritizing earlier time steps over later ones. This concept combines the idea of dynamic flow with lexicographic optimization where a dynamic flow evolves over discrete time steps, respecting capacities and transit times on the network edges and the lexicographic optimization ensures the solution prioritizes maximizing flow at earlier time steps before considering later steps. Such a lexicographic optimization maximizes the flow without compromising flow maximization in earlier time steps, at each step. Such optimization techniques solve the problem iteratively by treating the earlier flow values as fixed constraints for subsequent time steps. It helps in Urban optimization during emergencies in time-critical scenarios. Modern solvers like Gurobi, and CPLEX can handle lexicographic optimization for small to medium-sized evacuation networks. For large networks, heuristic or approximation methods may be required. Like others, the computational complexity increases with the planning horizon and such lexicographic priorities require careful constraint management.

Lexicographically maximal flows are a critical concept in evacuation planning because they prioritize maximizing the efficiency and fairness of resource allocation, which is essential in time-sensitive and high-stakes scenarios like evacuations. This ensures the most effective use of limited evacuation routes, minimizing bottlenecks. In emergencies, certain groups (e.g., the injured or vulnerable) or areas may require prioritized evacuation. Lex-max flows can incorporate prioritization by sequentially optimizing flows to reflect these needs. By balancing flow distribution, lex-max solutions reduce the risk of catastrophic failures if one route or resource becomes unavailable. This robustness is essential in dynamic and unpredictable evacuation contexts.

In real-time evacuations, changing conditions (e.g., blocked roads or increased crowd densities) can be addressed by recalculating lex-max flows. Lexicographically maximal flows provide a structured, equitable, and efficient framework for planning evacuations. Their ability to integrate fairness, adaptability, and optimal resource use makes them indispensable in emergency and disaster management scenarios. It is applicable in traffic management, telecommunication networks, resource allocation strategies, supply-chain management, event management, project management, crowd control, day-to-day transport scheduling, military logistics, etc.

Lex-max flows ensure that the smallest flow in the network is maximized first, then the second smallest, and so on. This results in a fair distribution of resources, ensuring no part of the network is disproportionately disadvantaged. In networks where different nodes or paths have varying priorities, lex-max flows provide a natural framework for considering these priorities. For example, in evacuation planning, lex-max flows can prioritize critical routes or populations without compromising overall efficiency. Their advantages are particularly significant in scenarios requiring equitable and efficient flow distribution. Traditional maximum flow algorithms focus solely on maximizing the total flow in the network. Lex-max flows go further by optimizing the flow distribution hierarchically. By prioritizing the smallest flows, lex-max flows help identify and mitigate bottlenecks in the network. This improves the robustness of the system, as critical weak points are addressed systematically.

In a city with multiple evacuation routes of varying capacities, a traditional maximum flow approach might prioritize maximizing total evacuees but leave smaller routes underutilized. A lex-max flow would ensure even the smallest-capacity route is utilized optimally before increasing flows on larger routes, reducing overall congestion and delays.

3. Conclusion

Handling large and complex networks with fine-grained time steps; Trade-offs or the balancing between the objectives that may conflict are the major challenges of the lexicographic evacuation strategies where such a lexicographic nature of the problem adds a layer of computational difficulty. In a lexicographic solution approach, the problem is solved iteratively by optimizing one objective at a time while treating previously optimized objectives as constraints. /In some cases, greedy approaches can be used to approximate solutions for specific objective types, particularly when network constraints are simple. Such approaches are frequently used in traffic management, telecommunication, logistics, and supply chain management.

The dynamic flow obtained in the integrated evacuation network is lexicographically maximum concerning the priorities in the sources and sinks simultaneously. Such lexicographic maximum dynamic flow with multiple sinks and in addition to multiple sources can be computed in polynomial time setting. Transit-vehicle assignment and routing in the integrated evacuation network to such lexicographic arrivals of evacuees at pickup locations become smooth and spontaneous for transit-based evacuation planning problems. In such cases, evacuees are collected at different pickup locations of the primary sub-network in a maximum dynamic lexicographic flow approach and are assigned simultaneously to the homogeneous transit vehicles in the subsequent connected secondary sub-network of the integrated evacuation network topology. The waiting delay at the pickup locations is almost negligible. This might be better suited for different variants of evacuation planning problems even for sufficiently large-scale evacuation scenarios as an asset for disaster management. This can be extended for the heterogeneous or mixed model transit vehicles and also for the disparate group of evacuees in a different network topology.

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