

RETROFITTING OF TELECOMMUNICATION TOWER VIA TUNED VISCOUS MASS DAMPER

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Abstract

This paper proposes an innovative method to retrofit existing telecommunication towers by integrating Tuned Viscous Mass Dampers (TVMDs) into the tower. This method of retrofitting is designed to improve the seismic resilience of towers by controlling their lateral displacements and accelerations during ground motions. The TVMD includes a component that offers stiffness, connected with a ball screw device that can generate firm damping and inertial forces even with small deformations. In this system, TVMDs are connected in parallel with the existing V-shaped members of the tower. Such strategic TVMD configuration uses the advantage of the V-shaped braces in controlling the vertical displacement of the tower while focusing on the relative lateral displacement in both positive and negative directions. A single-mode tuning design method is presented here for the optimal design of the TVMDs, giving the ability to choose the tuning mode. Modal analysis is performed to determine the target tuning modes. After integrating the tower with TVMDs based on target tuning mode, time-history analyses are conducted to examine the seismic performance of the proposed system. The studies demonstrate the effective control of the TVMD integrated system when tuned to the third mode compared to the first mode.

Keywords: Tuned viscous mass damper, Telecommunication tower, Retrofitting, Seismic response control, Single-mode tuning

1. Introduction

Telecommunication towers are crucial infrastructure that plays a critical role in facilitating communication and connectivity by providing a platform for transmitting data and voice signals for various communication services, such as mobile phones, television, radio, and the Internet. These towers enable people to stay connected, informed, and entertained. During natural disasters, the seamless operation of critical infrastructure, such as dams and electric and fuel transmission stations, heavily depends on the timely and efficient information delivery through telecommunication towers. Thus, telecommunication towers are critical in ensuring the rapid and accurate transmission of information from affected regions to response centers (Amiri et

al., 2007). The survival of telecommunication towers during and after an earthquake is essential. Earthquakes are one of the most significant natural disasters that can cause severe damage to telecommunication towers. The impact of an earthquake can cause the towers to sway, bend, or even collapse, leading to significant service disruptions or complete communication breakdowns. Telecommunication towers have long been a concern during earthquakes due to their potential to collapse and cause damage to nearby structures (Albermani et al., 2009).

Szafran (2020) conducted a comprehensive evaluation of a telecommunication tower to determine the failure mechanism for asymmetrical and symmetrical towers. The investigation revealed that the failure took place at the local level in the bracing members and legs of the tower. Takeuchi (2016) has revealed that telecommunication towers are susceptible to significant seismic forces and thus require seismic retrofitting to guarantee their safety. However, conventional strength-based retrofitting techniques may not be feasible, as reinforcing weak components could fail

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other construction elements or connections, ultimately necessitating the reinforcement of all members. A seismic retrofitting method has been proposed by Oukouch et al. (2006) in which energy-dissipating members (EDMs), such as buckling restrained braces (BRBs), replaced the critical truss members. This approach was implemented after the method's efficiency was confirmed analytically and experimentally, and approximately 20 communication towers were retrofitted using this method.

Takeuchi et al. (2015) performed the time-history analysis of a telecommunication truss tower. They studied its structural response after optimal incorporation of the viscoelastic dampers (VEDs) using genetic algorithm as an optimization method.

This research presents an innovative approach for incorporating Tuned Vibration Mass Dampers (TVMD) into truss towers intending to simultaneously control floor displacement and floor accelerations while considering TVMD force and overall cost. The proposed system is developed with three key features. Firstly, TVMD devices are utilized because they can generate significant inertial and damping forces even under small deformations (Ikago, Saito, and Inoue, 2012; Saito et al., 2008). Secondly, the TVMDs are arranged in parallel with existing truss members to retrofit the structure, thereby enabling efficient use of horizontal relative displacements to generate TVMD forces. Finally, an optimal design of TVMD parameters, encompassing damping coefficient, apparent mass, and stiffness, is developed to ensure effective control of seismic responses at varying heights within the structure (Smith, 2002).

The TVMD is a specialized type of inerter-based vibration absorber (IVA) that utilizes an inerter - a mechanical component that generates a resisting force based on the relative acceleration of its terminals. This force is proportionate to the component's inertance (Smith, 2002), expressed in mass units. Inerters can be created using various methods, including mechanical devices such as ball screw systems (Ikago, Saito, and Inoue, 2012; Saito et al., 2008), rack-and-pinion flywheel systems (Papageorgiou and Smith, 2005), and hydraulic (Wang et al., 2011) and electromagnetic devices that use capacitors (S. Zhu et al., 2012).

Significant progress has been made by researchers in developing efficient techniques to fine-tune the design of inerter-based vibration control methods and accurately establish their parameters. To achieve the H_∞ optimization of the transfer function of a single-degree-of-freedom primary structure, Ikago, Saito, and Inoue (2012) proposed a fixed-point tuning design method for a TVMD. Meanwhile, Marian and Giaralis (2014) suggested a TMDI design method based on the H_2 optimization of the white-noise enthusiastic response of an SDOF primary structure. These optimal methods provide explicit design equations and can be extended to tune a single-mode response of a multi-degree-

of-freedom structure (Ikago, Sugimura, et al., 2012; Marian and Giaralis, 2014).

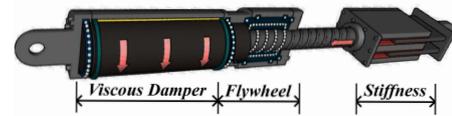


Figure 1. Schematic drawing of TVMD (Ji et al., 2020)

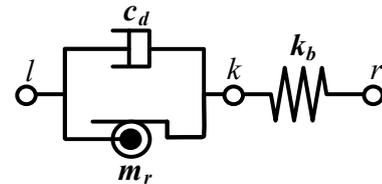


Figure 2. Mechanical model of TVMD.

The TVMD is a type of damper that uses a rotary damping tube (RDT) filled with viscous fluid and a rotary cylinder acting as a flywheel to provide amplified damping and inertial forces (Arakaki et al., 1999). Adding a spring connected in series to the RDT creates the TVMD, which can tune the vibration of a structure similar to a tuned mass damper (TMD) (Ikago, Saito, and Inoue, 2012; Saito et al., 2008), which is shown in Figure 1 (Ji et al., 2020). As shown in Figure 2, the TVMD consists of an inerter element, a viscous damper element connected in parallel which are connected in series with a stiffness element, with parameters such as apparent mass m_r , damping coefficient c_b , and stiffness k_b , respectively, that can be designed to tune the vibration of the primary structure. The apparent mass of a TVMD can be several hundred times its actual mass m_p due to the amplification factor of the ball screw mechanism, which is related to the outer and inner radii of the flywheel and the lead length of the ball screw (Ikago, Saito, and Inoue, 2012).

This study aims to introduce the concept of TVMDs in truss towers and suggest a single-mode tuning technique for achieving the best TVMD design. The paper comprises multiple sections, beginning with the Finite Element Method (FEM) modeling of a retrofitted telecommunication tower in Japan and the FEM modeling of TVMDs. The third section outlines a single-mode tuning method for the optimal integration of TVMDs into the truss tower design. Finally, the FEM model of the tower, incorporating TVMDs, is simulated with various ground motions to analyze the results.

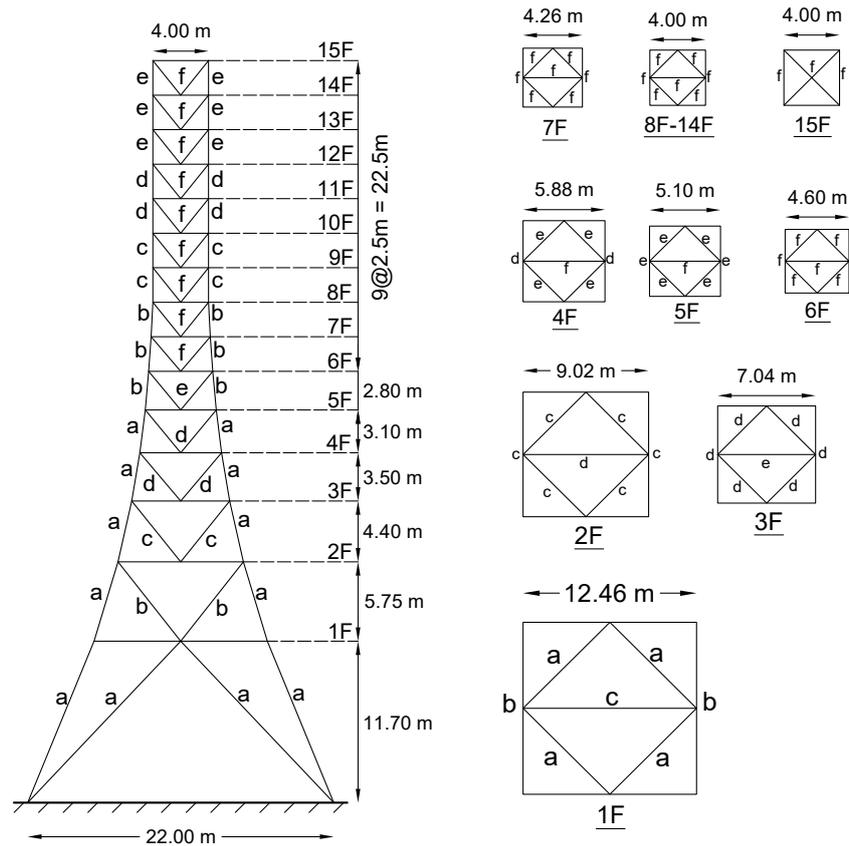


Figure 3. Schematics of telecommunication tower.

2. FE Model of Tower and TVMD

2.1. FE model of telecommunication tower

The analysis model is based on an existing lattice telecommunication truss tower located in Japan, consisting of 15 layers with total height of 53.75 meters (Inanaga et al., 2020). The structure was previously retrofitted by replacing members of the truss with elasto-plastic dampers as well as visco-elastic dampers (Takeuchi et al., 2015). The model is depicted in Figure 3. The truss members are grouped into 6 categories and their section properties are depicted in Table 1 (Inanaga et al., 2020). Circular hollow sections (CHS) of varying diameter and thickness have been used made up of materials STK540 and STK400 whose tensile strengths are 540 MPa and 400 MPa respectively while their modulus of elasticity is 200 GPa.

The telecommunication tower is modeled and considering Rayleigh damping of 2% for 1st and 2nd mode based on Chinese code for seismic design of electric installations GB50260-2013, modal analysis is performed in an open source FE analysis library available in python known as OpenseesPy (M. Zhu et al., 2018). The modal properties provide significant information on different structural dy-

Table 1. Section properties of telecommunication tower

Designation	CHS $D \times t$ (mm)	Material
a	$\varnothing 355.6 \times 7.9$	STK540
b	$\varnothing 318.5 \times 6.9$	
c	$\varnothing 267.4 \times 6.9$	
d	$\varnothing 216.3 \times 5.8$	
e	$\varnothing 165.2 \times 4.5$	STK400
f	$\varnothing 139.8 \times 4.5$	

namic parameters, and among different parameters fundamental frequency of vibration is one of the crucial parameter. The modal parameters are compared and verified with the study performed by Terazawa and Takeuchi (2018). Although 3D modeling is done in OpenseesPy, due to the model's symmetry, the analysis and results are focused in one direction only i.e., X-axis. The mode shapes of the respective modes are shown in Figure 4. The natural frequency of primary structure at 1st, 2nd and 3rd modes are 1.529 Hz, 5.464 Hz, and 9.523 Hz respectively.

2.2. FE model of TVMD

In this study, the TVMD is modeled in Openseespy which provides "twoNodeLink" elements that can be

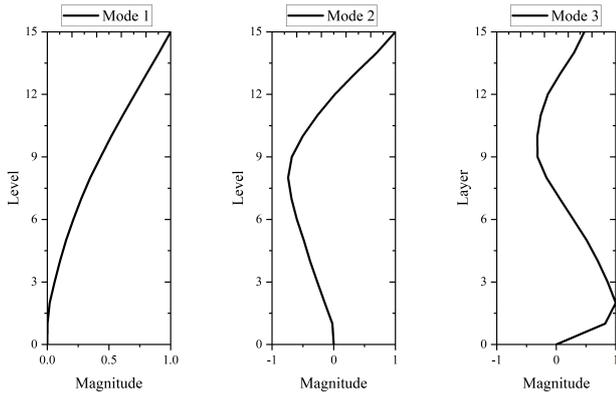


Figure 4. Mode shapes of uncontrolled primary structure

equipped with different properties such as elastic and viscous properties to simulate the spring and dashpot, respectively. The inerter element as been provided by Li et al. (2019) for simulating inerters in OpenSeespy. As described above, TVMD consists of dashpot and inerter in parallel which are connected in series with spring as depicted in Figure 2. To simulate this in Openseespy, a mid-node (k_i) is generated in between the two terminal nodes (l_i and r_i) of TVMD. The twoNodeLink element with properties of dashpot and an inerter element are connected in parallel between node l_i and k_i and a twoNodeLink element with properties of spring is connected in between node k_i and r_i .

In this study, only the linear behavior of TVMD is considered and hence the dashpot is not modeled with a power law constitutive behavior. The twoNodeLink elements are set to be infinitely rigid by assigning very large stiffness in the transverse and rotational DOFs compared to the axial stiffness of the spring in the local coordinate to ensure that the mid-node (k_i) moves co-linearly with the two terminal nodes of the TVMD. This method has been verified by Ji et al. (2020) by using the new element called InertiaTruss element which has the same properties as the inerter element used in this study.

3. Design and Analysis

3.1. Single-mode tuning design method

For optimal TVMD design, tuning its vibration frequency to align with a particular mode of the primary structure is advisable. This will help to minimize the dynamic response of the main structure. Ikago, Saito, and Inoue (2012) used the fixed-point method for the optimal design of TVMD, an extended version of the fixed-point method proposed by Hartog (1985) for the optimal design of TMDs. Ikago, Sugimura, et al. (2012) extended the fixed point method from an SDOF system to an MDOF system, a shear-type frame structure. According to Ikago, Sugimura, et al.

(2012), the distribution of mass in TVMDs is directly related to the stiffness of the structure when TVMDs are activated by inter-story drifts in shear-type buildings.

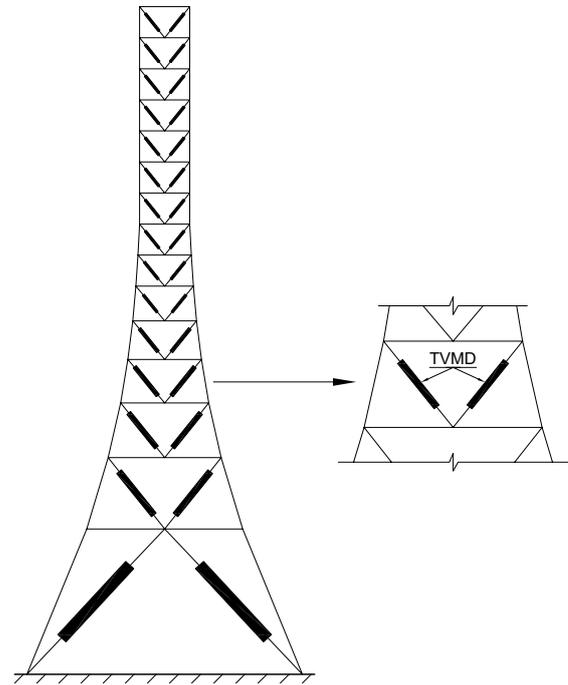


Figure 5. Telecommunication tower incorporated with TVMDs on all the layers

Ji et al. (2021) have proposed a design approach for flexure-type structures. The method uses a single tuning approach to a coupled wall system. The design primarily mobilizes TVMDs via vertical displacements between adjacent wall piers. Ji et al. assumed that the larger TVMD shall be assigned at a location of more significant displacement and hence, calculated the modal displacement demand of a TVMD for the target tuning mode based on the relative displacement between the two nodes connected by the TVMD in the corresponding mode shape vector. This study applies the same design approach as Ji et al.'s single-mode method to design the TVMDs for the telecommunication tower, which is a flexure-type structure.

The TVMDs are added on all the layers of the primary structure in parallel with the truss members as depicted in Figure 5. The V-shaped braces of the tower contributes in controlling the vertical deflection and this is advantageous for TVMD as well. In the configuration shown in Figure 5, the TVMDs act as a couple providing vibration control in both positive and negative lateral direction while the vertical deflection is controlled by the V-shapes bracing. In order to design the TVMDs, authors use the single-mode tuning design method, which involves the procedures described below.

Table 2. Optimal design parameters of TVMDs.

Storey	Tuned to 1 st Mode (T1M)			Tuned to 2 nd Mode (T1M)			Tuned to 3 rd Mode (T3M)		
	m_r (ton)	k_b (kN/mm)	c_d (kNs/mm)	m_r (ton)	k_b (kN/mm)	c_d (kNs/mm)	m_r (ton)	k_b (kN/mm)	c_d (kNs/mm)
1	38.21	11.63	0.85	1.57	4.61	0.10	24.75	146.22	1.65
2	75.80	23.08	1.68	8.66	25.43	0.53	5.37	31.71	0.36
3	83.36	25.38	1.85	7.88	23.15	0.48	4.24	25.06	0.28
4	89.49	27.24	1.98	7.52	22.09	0.46	4.87	28.80	0.32
5	100.73	30.66	2.23	6.47	18.99	0.40	5.54	32.72	0.37
6	118.30	36.01	2.62	6.65	19.54	0.41	6.46	38.18	0.43
7	126.07	38.38	2.80	4.88	14.34	0.30	6.33	37.43	0.42
8	144.05	43.85	3.19	2.58	7.58	0.16	6.17	36.46	0.41
9	166.12	50.57	3.68	4.53	13.32	0.28	4.25	25.11	0.28
10	172.65	52.56	3.83	12.20	35.83	0.75	0.34	1.99	0.02
11	185.73	56.54	4.12	16.39	48.13	1.01	1.57	9.27	0.10
12	194.31	59.15	4.31	19.56	57.43	1.20	3.20	18.90	0.21
13	196.56	59.83	4.36	22.86	67.14	1.40	5.96	35.20	0.40
14	200.03	60.89	4.44	24.60	72.24	1.51	7.03	41.56	0.47
15	191.25	58.22	4.24	21.56	63.32	1.32	5.40	31.93	0.36
Sum	2082.66	633.99	46.18	167.91	493.14	10.31	91.48	540.54	6.08

I. Calculate the modal properties of the primary structure and select the target tuning mode.

After obtaining the modal properties from the FE model of the primary structure, the target tuning mode is selected based on the modal participation mass factors of the modes, which were 41.4%, 15.5%, and 41.2% for mode 1, 2, and 3, respectively. The target percentage of the total mass participation for determining the controlled modes of the telecommunication tower should be greater than 85% based on study by Palermo et al. (2015). The summation of these three modes is 98.33%, greater than 85%. The modal displacement demand vector of the TVMD (ϕ_{di}) of the i^{th} target mode is defined as the relative displacement between the two nodes connected by the TVMD in the corresponding mode shape vector only in X-direction. This is based on the assumption that the relative vertical displacement of the terminal nodes connected by TVMD is negligible since the V-shaped bracing contributes in controlling the vertical displacements of the nodes. The k^{th} component of $\{\phi_{di}\}$ is calculated using Equation (1). The modal displacement demand vector of the telecommunication tower are depicted in Figure 6.

$$\phi_{di,k} = |\phi_{i,kr} - \phi_{i,kl}| \quad (1)$$

Here,

- $\phi_{i,kl}$ = the displacement components of Node K_l in ϕ_i
- $\phi_{i,kr}$ = the displacement components of Node K_r in ϕ_i
- ϕ_i = the i^{th} mode shape vector of the primary structure

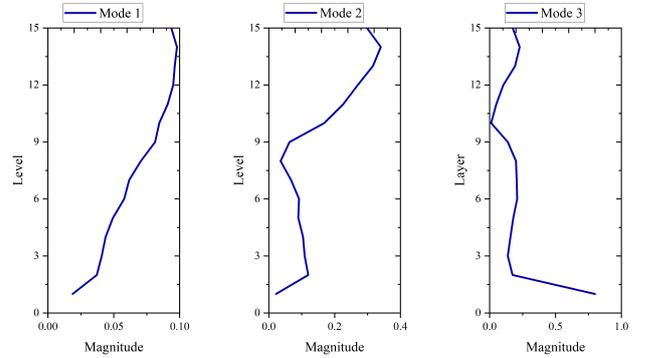


Figure 6. Modal displacement demand of TVMDs

II. Determine the apparent masses of TVMDs.

For the given mass ratio of μ_i , the apparent mass vector of TVMDs $\{m_{r1}, m_{r2}, \dots, m_{rn}\}^T$ is calculated by assuming it to be proportional to the modal displacement demand vector of TVMDs (ϕ_{di}) and using Equations (2).

$$\mu_i = \frac{\{\phi_{di}\}^T [M_r] \{\phi_{di}\}}{\{\phi_i\}^T [M_p] \{\phi_i\}} \quad (2)$$

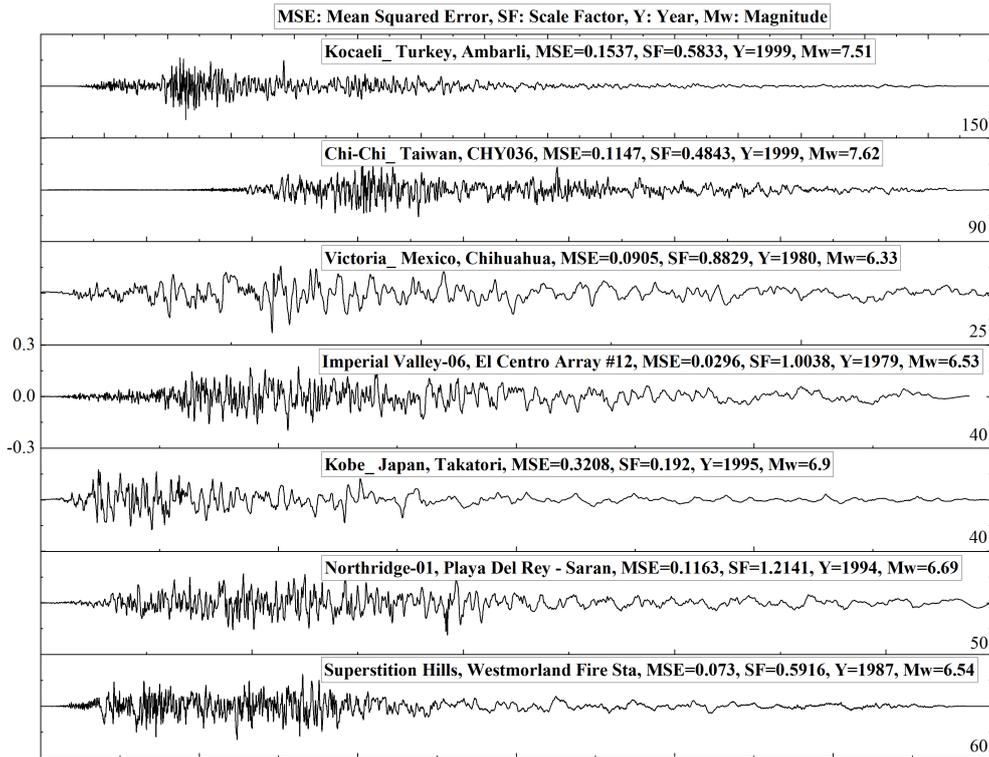


Figure 7. Acceleration time history of selected ground motions

get spectrum for record selection is the design based earthquake (DBE) response spectrum specified in the Chinese code for seismic design of electric installations GB 50260-2013. The criteria for choosing the ground motions include records with magnitudes higher than 6 and an average shear wave velocity that corresponds to site class IV without restrictions on fault type and fault distance. Time history records of selected ground motion records are shown in Figure 7 while the matched response spectrum, and the mean response spectrum are shown in Figure 8.

In this study, the ground motion of seven different time histories are scaled to the design response spectrum obtained from the code for analysis and linear time-history analysis is used. The three engineering demand parameters (EDPs): mean nodal displacements, acceleration, and stress, with and without TVMDs at each level are compared. It is worth noting that, due to the transmission tower's large base area, the response at the base differs significantly from the upper parts. The displacement control is negligible for the T2M system, but significant for the T1M system, with a reduction of up to 16%. The use of TVMDs is found to be effective for T3M system, with a reduced displacement value of up to 23%, as shown in Figure 9a. Similarly, acceleration of the top level is also studied, which is decreased by 9%, 6%, and 20% for T1M, T2M, and T3M systems, respectively. In the case of stress, after applying TVMDs,

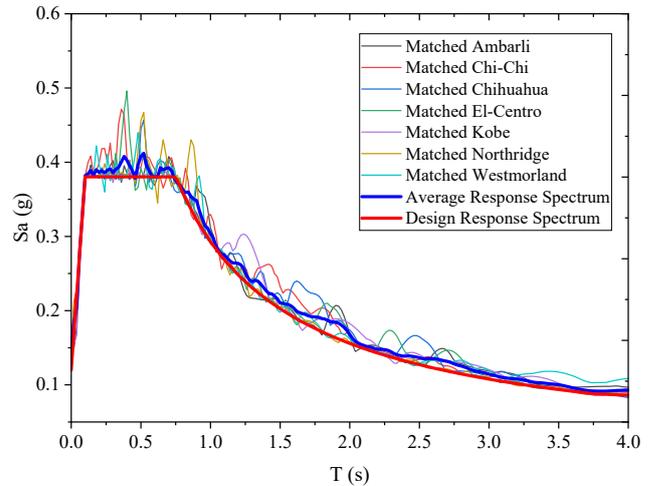


Figure 8. Response spectra of selected ground motions

the stress reduction at the 6th layer is 22.24% in the T3M system. The stress reduction at the same layer for the T1M system is 14.18%. The most negligible reduction of stress is for the T2M system (Figure 9c).

Figure 10 shows the time history graph regarding acceleration and displacement to the tower's top level for all four systems. The T3M system dominates the effective reduction in displacement and acceleration. Some significant ac-

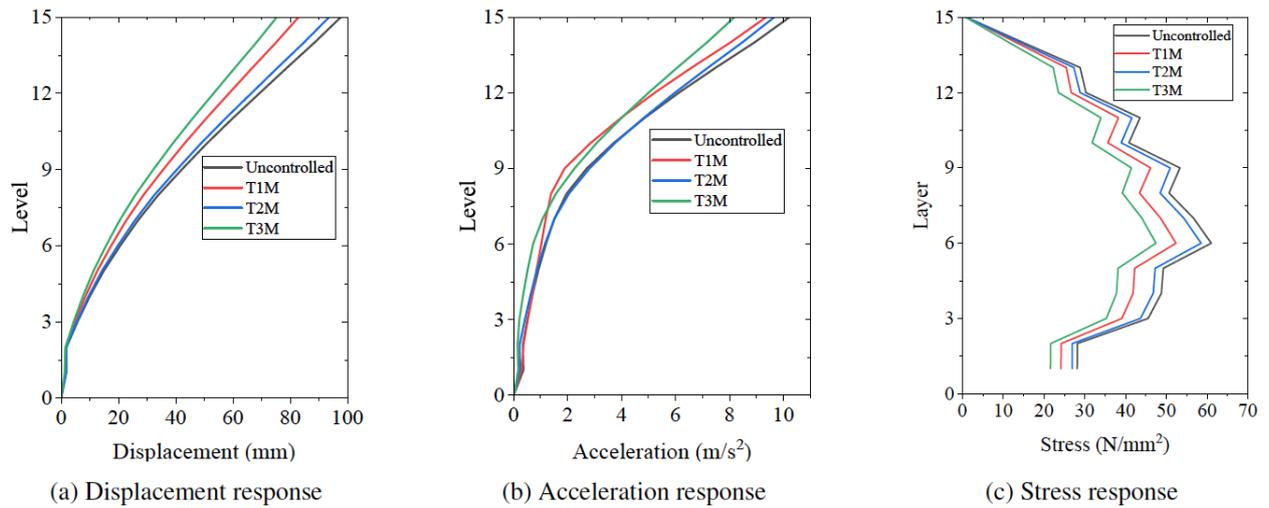


Figure 9. Mean peak responses of Telecommunication tower

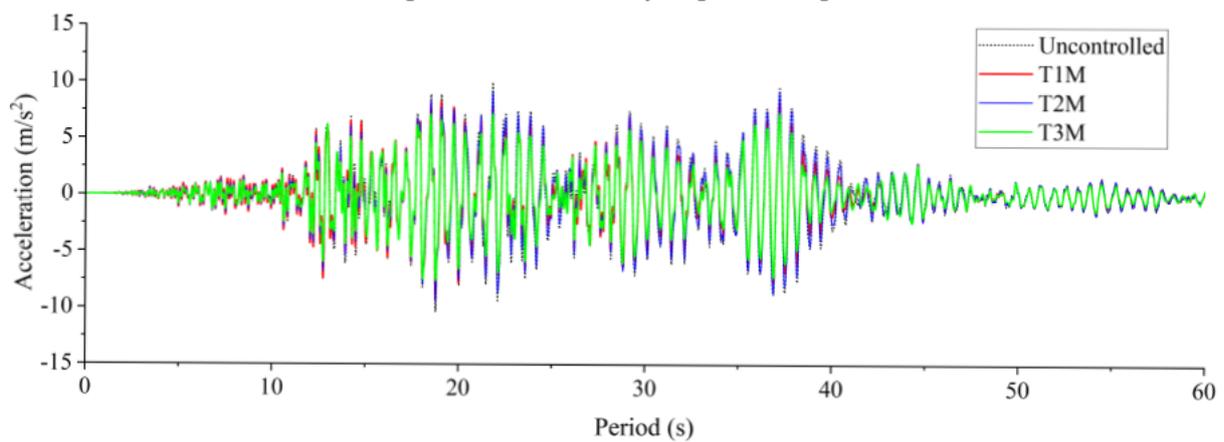
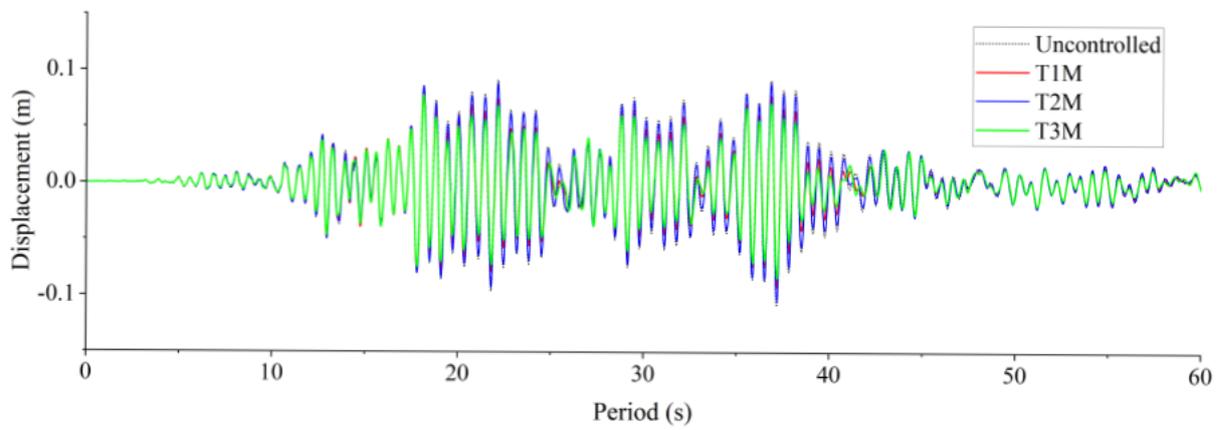
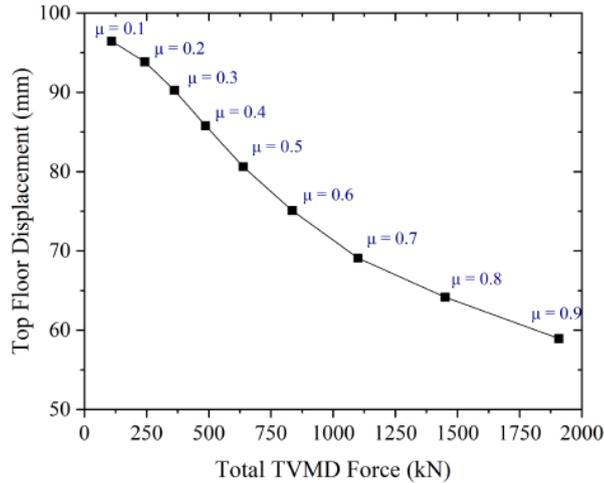
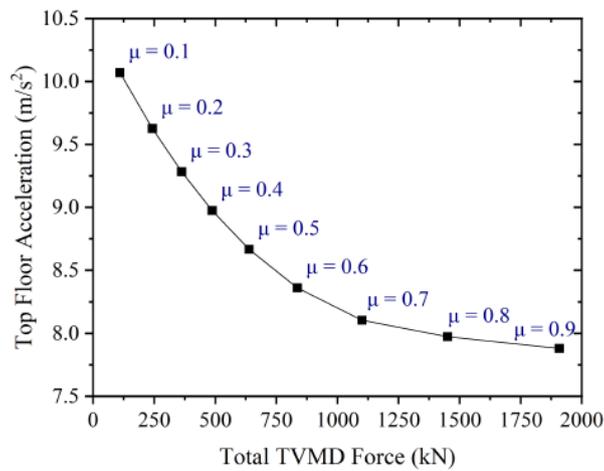


Figure 10. Time history response of top level due to El-centro ground motion



(a) Top level lateral displacement



(b) Top level lateral acceleration

Figure 11. Seismic responses versus total TVMD force at various mass ratios

celeration reduction is observed in the T1M system but not as effective as in the T3M system. Similar results can be observed in the case of displacement time history, where a significant reduction in time history is observed in the T3M system. The T1M system's time history shows a less effective reduction pattern than the T3M system. These results can be easily understood as the modal participation mass ratio of the 3rd mode is similar to the 1st mode.

Based on other recent research (Ji et al., 2020, 2021), it has been found that the widely accepted notion of the first mode being the most crucial is not entirely accurate. In fact, it is highly recommended to consider the significance of higher modes as well since, as observed in this tower structure, they can possess greater contributing masses compared to the initial mode. In fact, it is highly recommended to con-

sider the significance of higher modes as well since, as observed in this tower structure, they can possess greater contributing masses compared to the first mode. In this study, particular attention was given up-to the third mode, given its substantial modal mass participation ratio of 41.188%, indicating a significance comparable to that of the first mode. Because, the sum of 1st to 3rd modes covered 96.79% of total modal mass participation ratio, the other modes higher than 3rd mode was not significant for the analysis.

The above analysis results are calculated based on the apparent mass ratio of 0.7. The apparent mass ratio of the TVMDs is optimized concerning the force versus EDPs. The economic cost of TVMDs is proportional to their force capacity (Ikago, Sugimura, et al., 2012) and the increase in the apparent mass ratio results in increase in force of

TVMD. Here, an additional analysis is conducted where the TVMDs are tuned to the 3rd mode where the mass ratio is taken as a variable ranging from 0.1 to 0.9 with increment of 0.1.

For each mass ratio, the TVMD parameters are determined using single-tuning design procedure described above. The responses of T3M system is obtained along with the TVMD forces from the time history analysis. The relation of the mean values of the maximum top level lateral displacement and acceleration versus the total TVMD force are shown in Figure 11. The top level lateral displacement is approximately inversely proportional to the total TVMD force whereas the top level lateral acceleration decreases rapidly with an increase in the total TVMD force upto 1100 kN. After this point, the top level lateral acceleration seems to be quite stable at about 8 m/s² for further increase in TVMD forces. Similar results have been observed in the study for TVMD coupled wall system (Ji et al., 2020). Therefore, a reasonable apparent mass ratio should be chosen for TVMD, that would be determined through a consideration of the required seismic control performance, economic costs and other engineering issues.

4. Conclusion

This study proposes an innovative approach to enhance the seismic performance of telecommunication towers through the integration of Tuned Vibration Mass Dampers (TVMDs). A single-mode tuning design methodology is developed to optimize the design of the TVMDs in the system. Time history analysis is conducted to demonstrate the effectiveness of the proposed system in controlling displacement, acceleration, and stress. Based on the results of this investigation, the following conclusions are drawn.

1. When using a single-mode tuning design method to tune the TVMDs to 1st mode, the displacement and the acceleration of the top level of the tower are reduced by 16% and 9%, respectively. Additionally, the maximum stress of the members decreased by 15%.
2. Implementing a single-mode tuning design methodology to tune the integrated TVMDs to 3rd mode has effectively reduced the displacement and acceleration of the tower's top level by 23% and 20%, respectively. Furthermore, the maximum stress experienced by the members is reduced by 23%.
3. When the integrated TVMDs are tuned to the 2nd mode, control effects have little to no effect.
4. The effects of controlling the system with TVMDs tuned to the 3rd mode are significant. This suggests that higher modes also have significant contributions, and the modal participating mass ratios of these higher modes should be carefully studied.

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