

# ECHOCARDIOGRAPHY IMAGE DENOISING USING FRACTAL WAVELET TRANSFORM

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## Abstract

One of the most important areas in image processing is medical image processing where the quality of the images has become an important issue. Most of the medical images are corrupted with the visual noise, and one of the such images is echocardiography image where this effect is more. So, this research aims to denoise the echocardiography image with fractal wavelet transform and to compare its performance with other wavelet based algorithm like hard thresholding, soft thresholding and wiener filter. Initially, the image is corrupted by the Gaussian noise with varying noise variances and is denoised using above mentioned different wavelet based denoising techniques. On comparison of the obtained results, it is observed that the fractal wavelet transform is well suited for highly degraded echocardiography images in terms of Mean Square Error (MSE) and Peak Signal To Noise Ratio (PSNR) than other wavelet based denoising methods. Further, the work could be enhanced to denoise the echocardiography image corrupted by other different types of noise. This research is limited to denoise the echocardiography image corrupted with Gaussian noise only.

**Keywords:** Image denoising, wavelet transform, thresholding, wiener filter, fractal wavelet transform

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## 1. Introduction

Nowadays, the digital imaging is being used widely. So, the quality of these images has become a very important issue. Generally, medical images are corrupted by noise during acquisition and transmission (B. Chinna Rao, et. al., 2010). Most of the medical images have visual noise and the image like echocardiography image, this effect is more. These noise not only degrade the image quality of echocardiograms but also make difficulty in clinical diagnosis based on echocardiograph. So, for achieving the best result in diagnosing disease, medical images must have good quality without noise and artifact. Many denoising algorithms were proposed previously to obtain good quality image but with the improvement of technologies that are

used in acquiring digital medical image, the noise has not been removed completely (M. Salarian, et. al., 2007) .

There are various wavelet-based methods, like hard thresholding, soft thresholding and wiener filter that have been used for the purpose of image enhancement of the echocardiography image, but there is still problem in the quality of image when the noise variance is significant. So, to overcome this issue, another wavelet based denoising method called the fractal wavelet transform is implemented in the present work to denoise the echocardiography image corrupted with Gaussian noise and is compared with basic wavelet image restoration technique based on thresholding, like Visushrink and Levelshrink thresholding methods where the wavelet coefficient of the image is compared to a given threshold that means if the coefficient is smaller than the threshold, then it is set to zero, otherwise it is kept or slightly reduced in

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magnitude. This method is also compared with wavelet based wiener filter. The methods that are used in this research work to denoise the echocardiography image are described below;

### 1.1. Wavelet Thresholding For Image Denoising

The Image denoising techniques using thresholding on wavelet domain attempt to remove the noise present in the signal while preserving most of the signal characteristics, regardless of its frequency content (Mohen Ghazel, 2006). The most common method for thresholding function are the hard-thresholding function and the soft-thresholding function (which is also known as the wavelet shrinkage function) given by equations i and ii respectively. The hard-thresholding function sets the wavelet coefficients to zero which are smaller than threshold value  $\lambda$  and chooses all wavelet coefficients that are greater than the given threshold  $\lambda$  (Kenta Nakayama, et. al., 2009).

$$fh(x) = \begin{cases} x, & |x| \geq \lambda \\ 0, & otherwise \end{cases} \quad (1)$$

Depending on the signal energy and the noise variance  $\sigma$ , the threshold  $\lambda$  is chosen. The soft-thresholding function follows different rule than the hard-thresholding function. It shrinks the wavelet coefficients by  $\lambda$  towards zero, so that this method is also called the wavelet shrinkage function (Byung-Jun Yoon et al., 2004).

$$fs(x) = \begin{cases} x - \lambda, & \text{if } x \geq \lambda \\ 0, & \text{if } |x| < \lambda \\ x + \lambda, & \text{if } x \leq -\lambda \end{cases} \quad (2)$$

### 1.2. Wiener Filter

The Wiener filter is also known as Least Mean Square filter which assumes the noise and power spectra of the object a priori (Saleem Zaroubi et al., 1995).The wiener filter is defined by the following expression (M. Salarian, et.al., 2007).

$$F(u, v) = \left[ \frac{H(u,v)^*}{|H(u,v)|^2 + [S_n(u,v)/S_f(u,v)]} \right] G(u, v) \quad (3)$$

Where  $G(u,v)$  and  $H(u,v)$  are degraded image and degradation function respectively. Similarly,  $S_n$  and  $S_f$  are the power spectra of noise and the original image.

### 1.3. Fractal Wavelet Transform

The Fractal Wavelet Transform is an effective approach to avoid blocking artifacts in the fractal approximation. This method involves a scaling and copying of wavelet coefficient in higher level subtrees to lower subtrees. The essence of this method is to predict the fractal code of a noiseless image from its noisy observation and from this predicted fractal code, we can generate a denoised estimate of original image (Mohen Ghazel, 2006).

## 2. Literature Review

In 2010, B.chinna Rao, et. al., presented a new denoising technique which combines the wavelet transform and fractal transform with recursive gaussian diffusion for the images corrupted with additive white gaussian noise and found that the proposed method gives better performance when compared to curvelet-based denoising and fractal based denoising.

In 2006, Mohen Ghazel, George H.Freeman, and Edward R Vrscay made the comparative study on wavelet thresholding methods and fractal based image denoising method and found that the fractal based image denoising method are quite competitive with standard wavelet thresholding methods for image denoising .

In 2007, M. Salarian, et. al., made the comparative study on the algorithms based on wavelet such as wiener filter, hard thresholding, soft thresholding and found that the wiener filter is superior to other methods in echocardiography image.

In 2012, Rashmikant A Madaliya, et. al., proposed a mix wavelet–fractal denoising method and found that the results obtained by this method are comparable to some of the most efficient known denoising methods like hard and soft thresholding method.

In 2009, Kenta Nakayama, et. al., proposed the new speckle reduction algorithm for clinical echocardiogram. The proposed method employs wavelet shrinkage to reduce the noise on an ultrasonic signal and found that the method can

remove specific frequency components and provides superior performance on speckle noise reduction compared to that of existing speckle reduction method.

In 2003, K.U. Barthel, et. al., proposed a fractal denoising scheme operating in a non sampled overcomplete wavelet decomposition and denoising results are significantly improved compared to a subsampled wavelet decomposition

In 2010, S.Suhaila, et. al., proposed a denoising method which utilizes the additive noise estimated from the smooth region in the degraded image and found that the proposed method outperforms other denoising method like thresholding techniques, wiener filter.

In 2002, Lakhwinder Kaur, et. al., proposed the adaptive wavelet threshold technique to denoise the natural image corrupted by Gaussian noise and found that the proposed method is better than other thresholding techniques when compared in terms of PSNR.

In 2009, S.Sudha, et. al., proposed wavelet thresholding based on weighted variance to reduce the speckle noise in ultrasound images and found that the results obtained by the proposed method demonstrate the higher performance for speckle reduction when compared with the results achieved from the other speckle noise reduction techniques.

### 3. Research Methodology

#### 3.1. Generic Model

For the effective reduction of visual noise present in the echocardiography image, different wavelet based denoising techniques like Visushrink, Levelshrink, Wiener filter and Fractal wavelet are used and found that the Fractal wavelet image denoising technique significantly reduces the noise and errors in the noisy images than other denoising techniques.

The Fig.1 shows the generic model of proposed method for denoising the echocardiography image. Initially, the original echocardiography image is

corrupted with Gaussian Noise to create the noisy image. Then, the discrete wavelet transform on grey scale image of the noisy image is performed to construct the wavelet decomposition tree and the Fractal wavelet scheme is applied on the image where the fractal code is estimated and the fractally denoised estimate of the image is reconstructed. Finally, the inverse wavelet transform of the image lead to the output of denoised image.

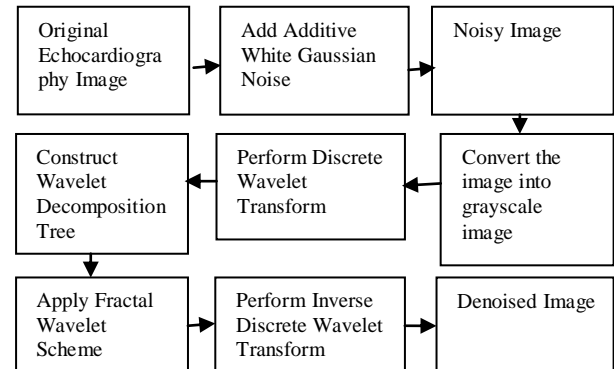


Fig.1 Generic Model for Echocardiography Image Denoising Using Fractal Wavelet Transform.

#### 3.2. Proposed Algorithm

Step(1) Input image  $x(t)$ .

Step(2) Add Gaussian noise  $n(t)$ .

$$y(t) = x(t) + n(t)$$

Where  $y(t)$  is the noisy image.

Step(3) Convert noisy image  $y(t)$  into grayscale image using following relation.

$$G(x) = 0.3R + 0.59G + 0.11B$$

Where R,G,B are red, green and blue component values of a pixel.

Step(4) Perform Haar Wavelet transform.

$$y(t) \xrightarrow{\text{Wavelet Transform}} w(t)$$

Step(5) Organized the wavelet coefficients  $A_{kij}^\lambda$ ,  $\lambda \in \{h, v, d\}$  of an image in a pyramid structure known as the wavelet decomposition tree constructed through a recursive four-subband splitting, starting with the original image.

Step(6) Consider the fixed set of parent and child level values  $(k_1, k_2)$ , where  $k_1 < k_2$ .

Step(7) For each uncoded child subtree,  $A_{k_2 i, j}^\lambda$ ,  $i, j = 1, 2, \dots, 2^{k_2}$ , find the parent subtree  $A_{k_1 i', j'}^\lambda$  and the corresponding scaling coefficient  $\alpha^*$  given by

equation (4);

$$\alpha^* = \frac{E[XY]}{E[X^2]} \quad (4)$$

So that the so-called ‘‘collage distance’’ given by equation (5) is minimized.

$$\Delta_{i,j,i',j'}^\lambda = E \left[ A_{k_2,i,j}^\lambda - \alpha_{i,j,i',j'} A_{k_1,i',j'}^\lambda \right] \quad (5)$$

Where,

$$E[XY] = \frac{1}{n} \sum_{m=1}^n x_m y_m \text{ and } E[X^2] = \frac{1}{n} \sum_{m=1}^n x_m^2 \quad (6)$$

$x$  and  $y$  are wavelet coefficients considered as random samples drawn from the random variables  $X$  and  $Y$  representing the wavelet coefficient distributions of a parent subtree  $D$  and its corresponding child subtree  $R$ , respectively.

Step(8) From this predicted code, reconstruct a fractally denoised estimate of the original image.

Step(9) Compute the inverse discrete wavelet transform.

### 3.3. Performance Parameters

The above mentioned methods are evaluated using the quality measure Peak Signal to Noise Ratio and Mean Squared Error. Both methods have been used widely due to their easy statistical computations.

#### 3.3.1. Mean square error (MSE)

MSE is calculated using following formula given below;

$$MSE = \frac{1}{MXN} \sum \sum [x(i,j) - x'(i,j)]^2 \quad (7)$$

Where ‘ $x$ ’ is the original image and  $x'$  is the denoised image.  $MXN$  is the size of image.

#### 3.3.2 Peak Signal to Noise (PSNR)

PSNR is calculated using following formula given below;

$$PSNR = 10 \log_{10} (255)^2 / MSE \text{ (db)} \quad (8)$$

## 4. Result and Discussion

Different experiments are carried over different echocardiography images of size 512X512 that are acquired from library of echo-web.

### 4.1 Experiments

Several experiments are carried out to verify the performance of proposed method. For each experiments performance parameters are measured and compared with previous existing methods.

#### Experiment 1

This experiment is carried on the echocardiography image shown in Fig.2(a). Initially, the image is corrupted by Gaussian noise with variance 22 as shown in Fig.2(b), and FW scheme is implemented to encode the noisy image. The wavelet based technique used here is haar wavelet with 4-levels of decomposition.

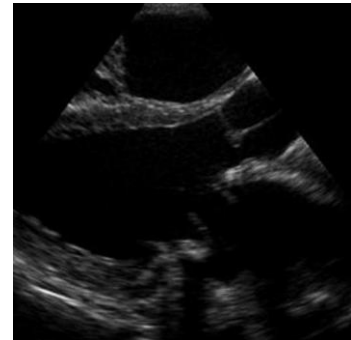


Fig.2(a) Original Image



Fig.2(b) Noisy Image

The results illustrated in Fig.2(c) and Fig.2(d) show that the best result reflected by the MSE and the PSNR is obtained when using Fractal Wavelet (FW) scheme at  $(k_1, k_2)=(6,7)$ . So, throughout this research work, the FW scheme at  $(k_1, k_2)=(6,7)$  is used to denoise the image.

### Experiment 2

This experiment is carried on the echocardiography image shown in Fig.3 and for comparison purpose, the image is corrupted by Gaussian noise with variance 18 as shown in Fig.3(b) and denoised using various denoising techniques.



Fig.2(c) FW scheme  $(k_1, k_2)=(5,6)$   
MSE=5, PSNR=34



Fig.2(d) FW scheme  $(k_1, k_2)=(6,7)$   
MSE=20, PSNR=22

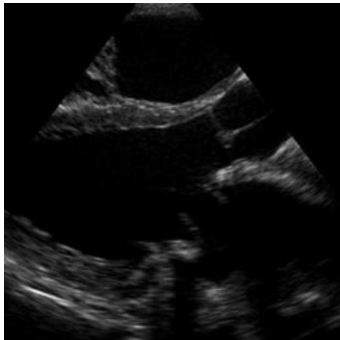


Fig.3(a) Original Image



Fig.3(b) Noisy Image



Fig.3(c) Denoising by Visushrink hard thresholding  
MSE =19, PSNR=23

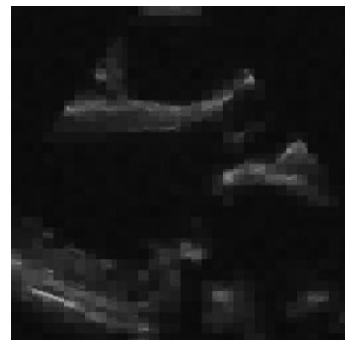


Fig.3(d) Denoising by Visushrink soft thresholding  
MSE=20, PSNR=22



Fig.3(e) Denoising by Levelshrink hard thresholding  
MSE=14, PSNR=25

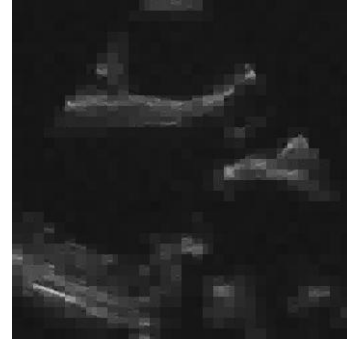


Fig.3(f) Denoising by Levelshrink soft thresholding  
MSE=19, PSNR=22



Fig.3(g) Denoising by Wiener filter  
MSE=10, PSNR=28



Fig.3(h) Denoising by Fractal Wavelet  
MSE=4, PSNR=36

The results shown in Fig. 3(c) to Fig. 3(h) are the denoised images initially corrupted by the Gaussian noise with variance 18. The various wavelet based denoising techniques have been implemented to get the denoised image. On the basis of PSNR and MSE, the result shows that the level dependent thresholding algorithm i.e Levelshrink which adopts different thresholds for different levels of wavelet tree improved the performance of the original wavelet thresholding method, Visushrink which adopts the universal threshold to be used uniformly throughout the wavelet decomposition tree of the noisy image.

Similarly, the result also shows that the image denoising by Wiener filter have better performance than above mentioned thresholding techniques and the proposed method i.e Fractal Wavelet is still

better than wiener filter on the basis of PSNR and MSE.

### Experiment 3

This experiment is carried on the echocardiography image shown in Fig.4(a) and for comparison purpose, the image is corrupted by Gaussian noise with variance 22 as shown in Fig.4(b) and denoised using different denoising techniques. The result shown in Fig. 4(c) to Fig. 4(h) shows that though the noise variance is high, the proposed method is still better than other wavelet based denoising techniques in terms of PSNR. Also the result shows that the performance of wiener filter is better than the thresholding techniques as in Experiment 2 despite of higher noise variance.



Fig.4(a) Original Image

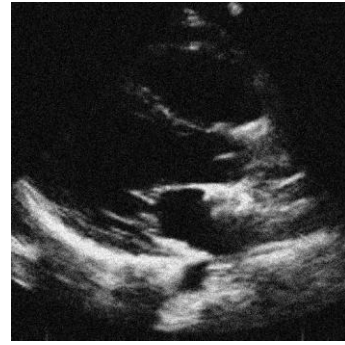


Fig.4(b) Noisy Image



Fig.4(c) Denoising by Visushrink hard thresholding  
MSE=23, PSNR=21

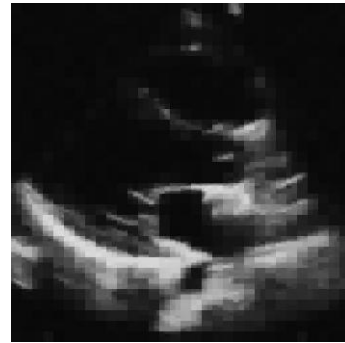


Fig.4(d) Denoising by Visushrink soft thresholding,  
MSE=25, PSNR=20



Fig. 4 (e) Denoising by Levelshrink hard thresholding  
MSE=17, PSNR= 23



Fig.4(f) Denoising by Levelshrink soft thresholding  
MSE=24, PSNR=21

#### 4.2 Comparative Analysis of Result

Following table shows the quantitative measurement of evaluation metrics. The performance of different denoising schemes Visushrink hard thresholding, Visushrink soft thresholding, Levelshrink hard thresholding, Levelshrink soft thresholding, Wiener

filter and Fractal Wavelet Transform is compared in following table and presented the comparative study of various wavelet based denoising techniques for Echocardiography images in terms of PSNR and MSE. All the wavelet-based techniques used here is haar wavelet with 4-levels of decomposition.

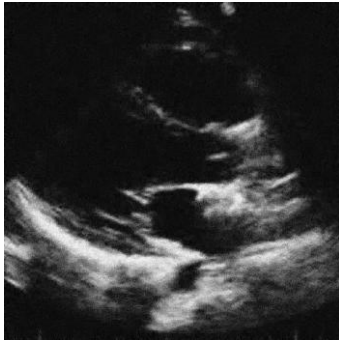


Fig. 4(g) Denoising by Wiener filter  
MSE=12, PSNR=27

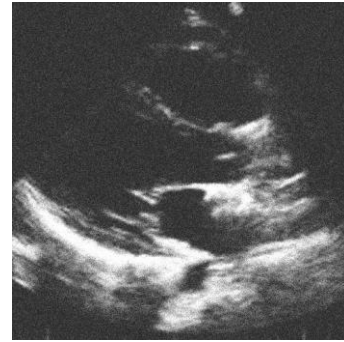


Fig.4(h) Denoising by Fractal Wavelet  
MSE=5, PSNR=34

Table.1 Evaluation result

Variance	Method	Test Image: Fig.3(a)		Test Image :Fig.4(a)	
		MSE	PSNR	MSE	PSNR
18	Visushrink hard	19	23	19	23
	Visushrink soft thresholding	20	22	21	22
	Levelshrink hard	14	25	14	25
	Levelshrink soft	19	22	20	22
	Wiener filter	10	28	9	29
	Fractal Wavelet Transform	4	36	4	36
20	Visushrink hard	21	22	22	21
	Visushrink soft thresholding	22	21	24	21
	Levelshrink hard	16	24	16	24
	Levelshrink soft	21	22	23	21
	Wiener filter	11	17	11	28
	Fractal Wavelet Transform	4	35	5	35
22	Visushrink hard	22	21	23	21
	Visushrink soft thresholding	24	21	25	20
	Levelshrink hard	18	23	17	23
	Levelshrink soft	23	21	24	21
	Wiener filter	12	27	12	27
	Fractal Wavelet Transform	7	34	5	34
24	Visushrink hard	25	20	26	20
	Visushrink soft thresholding	26	20	28	19
	Levelshrink hard	19	22	19	22
	Levelshrink soft	25	20	27	20
	Wiener filter	13	26	13	26
	Fractal Wavelet Transform	5	34	5	34
26	Visushrink hard	27	20	27	19
	Visushrink soft thresholding	28	19	30	19
	Levelshrink hard	21	22	21	22
	Levelshrink soft	27	19	28	19
	Wiener filter	14	25	14	25
	Fractal Wavelet Transform	6	33	6	33



Table.1 illustrates the result obtained by six different wavelet based denoising techniques when applied to test image shown in Fig.3(a) and Fig.4(a) with noise variance  $\sigma = 18, 20, 22, 24, 26$ . The Table.1 shows that the Levelshrink which is the adaptive wavelet thresholding method is better than the Visushrink which adopts the universal threshold

to be used uniformly throughout the wavelet decomposition tree of the noisy image in terms of PSNR . Similarly the table shows that though the wiener filter is better than thresholding techniques, its performance seems to be poor when compared with proposed method.

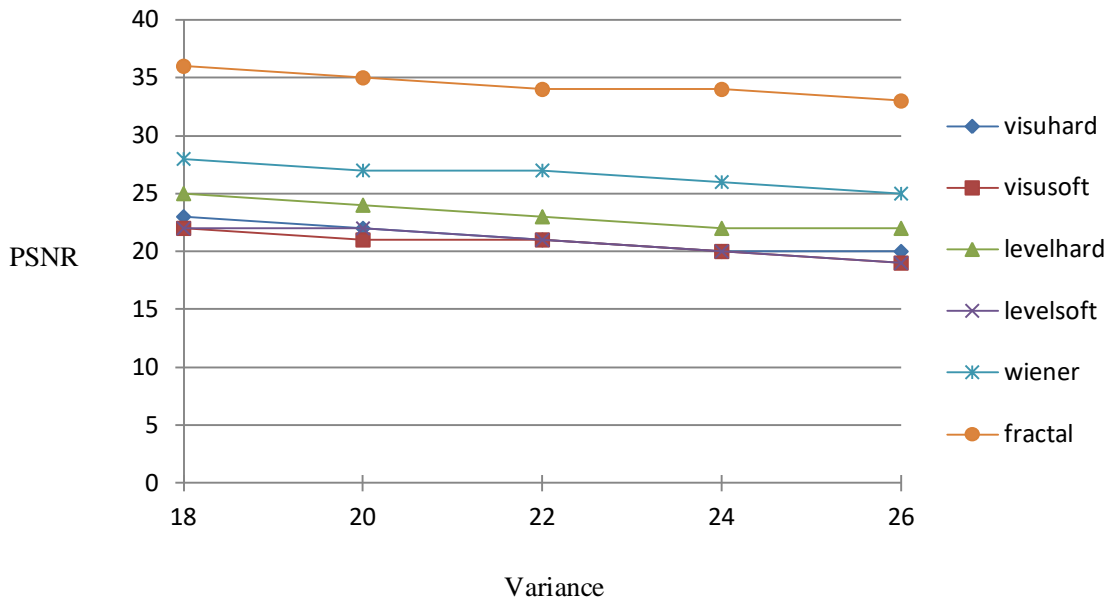


Fig.5 PSNR Comparison of Test Image shown in Fig.3(a) .

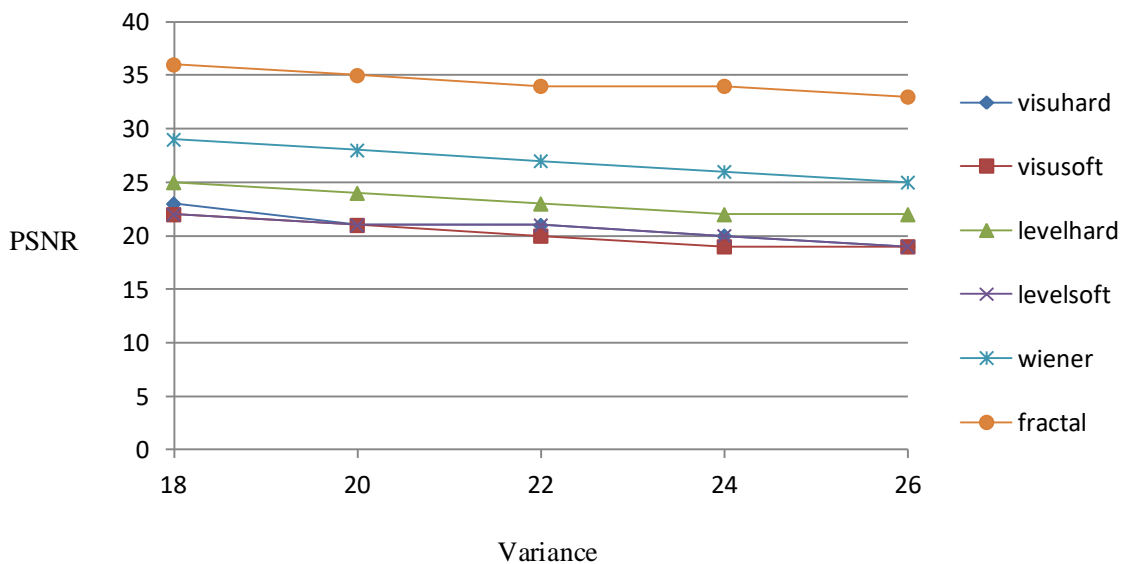


Fig.6 PSNR Comparison of Test Image shown in Fig.4(a)

## 5. Conclusion and Recommendations

In this research, various wavelet based denoising methods like wavelet thresholding, wiener filter and fractal wavelet transform are used to denoise the several echocardiography images with different noise variance and their performance based on PSNR and MSE is compared. From the results obtained above, it can be concluded that the fractal wavelet method is optimal compared to both thresholding and wiener filter. It produces the maximum PSNR for the output image compared to the other methods considered. The results also show that the image obtained from the adaptive thresholding technique i.e Levelshrink produces better PSNR than the Visushrink thresholding technique as it adopts different thresholds for different levels of wavelet tree. The experimental results also showed the performance of the wiener filter is better when compared with thresholding techniques but its performance becomes poor when compared with proposed method in terms of PSNR. Therefore among all, the proposed method performs well in terms of PSNR and MSE. Also the result shows that the Visushrink is the least effective method among the methods compared.

However, there are also some limitations with this method. The Fractal Wavelet denoising scheme is computationally more expensive. However, this issue could be overcome with a more efficient coding of the method as there are another Fractal Wavelet schemes also. So we can denoise the echocardiography image with them and compare with the proposed method in future. Similarly, this research is limited to denoise the echocardiography corrupted by other different types of noise.

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