

Teaching Real Analysis in Teacher Preparation Program through Content Bridging Strategy*Deb Bahadur Chhetri¹***Abstract**

This study aims to investigate the impact of content bridging, based on the ARCS Model of Instructional Design, on student motivation in learning Real analysis within a teacher preparation program at the bachelor's level. The strategy aims to establish connections between Real analysis and secondary-level mathematics, emphasizing the relevance and applicability of the subject. The study employed a participatory action research approach, employing content bridging and observing the students' behaviour during the instruction to study the change. The results indicate remarkable improvements in student behaviour, engagement, and active participation. Students exhibited increased motivation, completed assigned work, posed thoughtful questions, and demonstrated enthusiasm during class. These findings highlight the effectiveness of the content bridging in enhancing student motivation and understanding of Real analysis within the context of teacher preparation programs.

Keywords: Instruction, Motivation, Real Analysis, Students

Article Information

Received: 2023-05-10 Reviewed: 2023-06-01 Revised: 2023-6-10 Accepted: 2023-06-24

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Cite this article as:

*Chhetri, D.B. (2023). Teaching real analysis in teacher preparation program content bridging strategy. *Janabhawana Research Journal, 2(1), 50-58**

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Introduction

Teaching advanced mathematics in a university teacher training course poses a significant challenge due to the inherent difficulty of the subject and the lack of student motivation regarding its practical applicability in their future professions. The perception that the content may not be relevant to their future careers adds a layer of complexity to the agenda of teaching advanced mathematics. Real analysis is one of the subjects which is included in teacher preparation programs at the university level, and it is considered a less motivating subject due to its nature and the feeling of relevance of its application in professional life.

Real analysis is a branch of mathematics that deals with the rigorous study of real numbers, sequences, limits, continuity, and calculus. It is considered a specific branch of mathematics that comes under advanced mathematics (Wasserman et al., 2017). Potential secondary mathematics educators are obliged to complete a real analysis course during their undergraduate studies in Nepal and worldwide. The fundamental properties of a subset of real numbers are studied under real analysis (Bartle & Sherbet, 2011). It is a course that nearly all mathematics majors and some mathematics education majors are required to take (Conference Board of the Mathematical Sciences, 2021). The course of real analysis has been taught at the university level either at the bachelor level or at the master level or both levels not only in pure mathematics programs but also in teacher preparation programs around the world.

The real analysis teaching in a teacher preparation program is one challenging job at the university level not only by the course nature but also by its rationale and relevancy as well as teaching strategies. The major challenge in teaching real analysis is motivating students to learn it with the reasons why we learn it for being teachers. To deal with this problem different teaching strategies can be found. Real analysis knowledge that has been taught at the undergraduate level has contributed more or less in preparing mathematics teachers for the secondary level with sound knowledge of mathematics. If we follow the model suggested by Wasserman et al. (2017) then valuable content knowledge and pedagogical content knowledge can be provided for the prospective teacher.

According to Alcock and Simpson (2016), key teaching methods for Real analysis include lectures, active learning, guided practice in constructing proofs, visualizing technology, scaffolding, real-world applications, reflection, and metacognition. Despite using various methods, students have shown a lack of motivation. The main issue seems to be their inability to comprehend the purpose and relevance of studying Real analysis concerning becoming teachers. This complex question has highlighted the need for establishing connections and logical reasoning within the content of Real analysis for secondary-level mathematics instruction. Therefore, a strategy focusing on content and logical connections between secondary-level mathematics and Real analysis has been chosen for implementation in teaching Real analysis.

For the past five years, there has been a consistent observation of students displaying a poor perception towards the practical application of mathematical knowledge in real analysis. Additionally, their behaviour towards real analysis at the bachelor's level of the teacher preparation program has shown a lack of motivation. This situation has inspired me to conduct a study on strategies to enhance student motivation

towards real analysis in the bachelor's level of the teacher preparation program. As a tutor of real analysis of the teacher preparation program of the Tribhuvan University of Nepal, I always face the students' queries which is why we learn it. And I feel that students always feel sad while teaching real analysis due to its application in their professional life. I share my problem with the university mathematics teacher. They also shared that type of feeling during teaching not only analysis but also modern algebra. From the discussion with another tutor, a common problem was found that teaching Real analysis in a teacher preparation program at the university level is indeed a challenging job, not only due to the nature of the course itself but also a lack of students' motivation due to its rationale and relevance and teaching strategies. This entire problem is guided by the students' unmotivated state towards real analysis.

The study was completed based on the following research questions.

1. How can content bridging strategy be applied for the instruction of Real analysis?
2. Does content bridging strategy improve students' motivation towards learning Real analysis at the bachelor level?

Theoretical Framework

The following theories were utilized to conduct the study. The first two theories were utilized for framing the teaching strategy so that the strategy can motivate the students to learn mathematical knowledge of real Analysis.

ARCS Model of Instructional Design

The ARCS stands for attention, relevance, confidence, and satisfaction which is a motivational designed framework developed by John M. Keller. This Model is a framework for instructional design. It involves attention, relevance, confidence, and satisfaction (Keller, 1987). Attention pertains to capturing and maintaining student attention, as well as centralizing their focus towards the relevant subject. Relevance entails ensuring that the instruction is connected to present and future career prospects, highlighting the enjoyment derived from learning itself and addressing students' psychological needs by emphasizing the process rather than just the result. Confidence refers indicates fostering the belief among students that they can achieve a certain level of success if they exert effort. Finally, satisfaction indicates the students in feeling a sense of pride and accomplishment in their achievements and granting them some level of control over the learning process. The relevance strategy among different strategies is applicable for motivating students in learning mathematical knowledge of real analysis. Connecting the students' prior knowledge, understanding the students' interests related to the instruction, and presenting the intrinsic value of learning content which is useful to the future carrier are remarkable techniques for motivation based on the relevant element of the ARCS model (Yarborough & Fedesco,2020).

When teaching real analysis, greater emphasis is placed on the relevance component compared to the other elements of the ARCS model. This is because students recognize the increasing importance of this subject in their future careers in teaching. The content connection and motivation strategies were prepared based on this model.

Motivating Behavior of University Students

Several studies explore motivation and learning achievement and make a common understanding that motivating learning improves learning achievement. University students' motivation improves their cognitive learning outcomes (Kenneth et al, 2022). The students' self-belief influences the students' success in the transition to university, additionally motivating behaviour improving engagement manner and doing work (Susan et al.,2019). Stimulated behaviours such as active participation, getting the reward, and prizes indicate student motivation in teaching (Deci & Ryan, 2020). Additionally, students' daily behaviour such as submitting assignments timely, rising queries on the teaching topic, and goal-oriented activities indicates the students' motivation towards the subject taught (Simpson & Balsam, 2016).

From the above discussion, it can be concluded that the students' behaviour that demonstrates active participation, timely completion of assigned work, posing queries during instruction and deriving enjoyment from learning all indicates the students' motivation and interest towards a particular subject.

Content bridging Strategy

Content bridging strategy in teaching refers to the techniques used to establish connections between the content of mathematics and its related fields. These strategies aim to show students how mathematical concepts and principles are applicable and interconnected with other disciplines, real-world scenarios, or different areas of mathematics itself. By using connecting strategies, educators can enhance students' understanding, engagement, and appreciation for the subject. It is not a well-established strategy but it was considered to teach Real analysis aim to increase the student's motivation towards learning Real analysis at the bachelor of teacher preparation program at the university level.

To develop content bridging, two main approaches were incorporated: connecting the content of Real analysis with the content of mathematics taught at the secondary level and justifying that the related content is helpful for further study and is an expert in the subject area. The first strategy establishes a bridge between advanced mathematical concepts and the curriculum covered in secondary-level mathematics by creating connections between these two areas, to deepen students' understanding of mathematical principles mentioned in Real analysis and their application for teaching mathematics at the secondary level. The second strategy aims to convince the students; Real analysis knowledge is not only about being a teacher but also about being a good mathematics educator, writer, etc.

Method

Dhawalagiri Multiple Campus, Baglung of Tribhuvan University is my working place and I have been teaching real analysis for 10 years. There were nearly five to ten students in each batch. Students of each batch raise questions on the relevance of real analysis and are important for being a secondary school teacher. Therefore, I selected this subject and this place to conduct the study. There were 6 students in the academic year 2020.

The study was completed under the approach of participatory action research (PAR). The study was completed in four states.

Steps I Understand the Problem

The study was conducted to achieve the goal of finding out how to motivate students to learn real analysis at the undergraduate level. The background of the study rests on the question of the students: Why do we learn real analysis? What it takes us to be a good secondary school teacher.

Undergraduate-level mathematics students in the pre-teacher program expect each course to provide knowledge that will make it easier for us to teach mathematics at the secondary level. However, the real analysis course does not directly correspond to the expectations and leads to the fact that the students are not motivated for this topic. Therefore, I believe that the content of real analysis teaching needs to be linked to secondary mathematics teaching.

Step II Designing the Strategy

Content bridging strategies were prepared based on the ARCS model of instruction. The strategy includes how each topic of real analysis can be applied to teach secondary-level mathematics and how logic mentioned in real analysis can be applied to secondary-level mathematics. Certain examples were prepared which show the relation and justification of the content of real analysis relates to the teaching of real analysis. Additionally, two reasons were made to convince the students to learn real analysis. The first one was connectivity in content and the second was being an expert in mathematics for being a book writer, curriculum developer and studying the further level. However, the focus was on justifying the content by connecting the content with pedagogical practices. Only the first three chapters of real analysis, which includes real numbers, open and closed sets, and real sequences were considered for applying the prepared strategy. The lesson plan was prepared including the content bridging strategy more than the usual method of instruction.

Step III Applying the Strategy

The strategy was applied in the academic year of 2077 for those students who were studying real analysis regularly. Only 6 students were studying in this batch by taking major mathematics in the teacher preparation program. My position was as assigned a teacher to teach real analysis of this batch. The lesson plan and content bridging were applied during the instruction. Only 54 classes were used for the intervention of this strategy. First their chapter on real analysis: Real numbers, Open and closed sets and Real sequences were taught by using a content bridging strategy. During the strategy content was delivered by using the discussion method and each topic related to teaching practices. For example, the Field axiom of the real number system related to the four basic operations and justified the logic of the operations performed in school mathematics concerning the axioms mentioned in the context of real analysis.

Step IV Observing the Change

Based on the theories explaining the motivating behaviour, the behaviour change of the students and the way of the student's learning activities were observed. The details are explained in the results section.

Validity

The common method of ensuring validity in participatory action research is presence and

elaboration. These methods term as being there and thick description (Bhattarai, 2015). Ensuring validity in participatory action research involves researchers actively engaging with participants in the field, where their activities play a significant role (Wallerstein & Durban, 2006; Minker, 2004). Hence the researcher used these techniques. Additionally, a detailed explanation of the process of research researcher used the thick description method as explained by Patterotto (2006).

Findings

During the instruction, the students' behaviour was observed. Mainly students' activeness in learning, frequency of submitting assigned work, enthusiastic manner in class, and positive or negative response type queries about the subject matter were observed. Students were found to be active during the instruction. A good level of concentrated behaviour was observed. Students were trying to search for new connected logic in taught logic and theorem and its application in secondary-level instructional procedures. This evidence proves the good level of concentration. Some scenarios of the classroom was as below.

Scenario 1: Application of Commutative Law

When I was teaching commutative law, " If $a, b \in R$ then $a + b = b + a$ and $a \cdot b = b \cdot a \forall a, b \in R$ " In this axiom students A said "Oh! This idea is used in our daily mathematics practices at the school level, but it is unknown why. But is a law on real numbers so we do it in school without reasoning" other students support the view. This scenario clearly states the students' motivation status.

Scenario 2: Application of Cancellation law

During the teaching of properties of real numbers, students were themselves linking the law, and how we are practicing at the school level. When we were discussing If $a, b, c \in R$ then $a + c = b + c$ then $a = b \forall a, b, c \in R$, Student B said "we accept really when $x+3=y+3$ then $x=y$, this operation is guided by this law on the real number". This scenario also motivates students on the teaching topic.

Scenario 3: Convergent sequence

When we are discussing on convergent of a sequence, student A gives an example, " $\left\{\frac{1}{n}\right\}, n \in N$ this is a sequence which progressively goes to 0 and terminates at 0 when n tends to infinity" which was an interesting scenario during teaching where students create an example of the definition of example. This observation shows students are engaging to create examples and logic when teaching real analysis by bridging content with schools' mathematics.

Students were assigned multiple-choice type questions and created examples based on the taught topic. All the students completed the assigned work. Additionally, they tried to give the logic in each right answer to multiple questions based on why it is current, and others were not. Comparatively, these activities were observed as more active and interesting rather than the previous batch's manner of submitting the assigned work. Therefore, the content bridging was found to help motivate the student's engagement in learning Real analysis.

Satisfaction of the students was observed based on their manner of presentation in the classroom

and posing questions. In the previous batch, students were observed asking questions regarding the benefits of studying Real analysis for becoming a teacher and how it contributes to the instruction of secondary-level mathematics. This question was a result of being unsatisfied and not motivated by Real analysis. However, these types of unsatisfied and unmotivated questions were not observed during the intervention period. Thus, the motivation level can be increased by using the content bridging for teaching Real analysis in the teacher preparation program. Students were found to be engaged in seeking the logic behind the example, questions and theorem-proving style which shows an enthusiastic manner of participation. Additionally, students were engaging in creating examples and non-example of definitions and proof of the content of this subject's courses.

All the observed evidence indicates that content bridging is beneficial for increasing students' participation and motivation in Real analysis. The strategy is helpful to convince the students about why we study the mathematical knowledge of real analysis for being secondary-level mathematics teachers.

Discussion

The use of content bridging promotes the motivation of students towards learning real analysis at the bachelor level of a teacher preparation program. Students participating in teaching learning activities during teaching real analysis with smiling facts and active mode of participation indicates their motivating mode of learning as explained by Yarborough and Fedesco (2020). Susan et al. (2019) claim that the students' good engagement with their active and enthusiastic behaviour during learning indicates their motivated behaviour in the classroom. These types of behaviours were observed during the instruction of Real analysis. When students engage in critical thinking, analyze concepts, and ask probing questions, they are more likely to be motivated and achieve a deeper level of understanding (Fredricks et al., 2004). This claim was observed during teaching Real analysis. Students tried to create new logic and examples about teaching topics. The students' questions during the teaching of real analysis were found to be rational.

Questions asked by the students related to the future teaching profession. Questions were related to how we can use mathematical knowledge of real analysis to justify the mathematical rules in mathematical operations. A student said, "What is the connection and use of associative property?" During teaching students raise questions and prepare answers themselves through discussion which reflects the relevance of mathematical knowledge of real analysis for mathematics teaching at the secondary-level. This observation is supported by the claim that students' queries in the class and their participation in discussion reflect their motivation.

These observed behaviours were matching with the self-determination theory by Deci and Ryan (2000), when students feel a sense of autonomy, competence, and relatedness in the learning environment, their intrinsic motivation is enhanced. Rational inquiries demonstrate students' autonomy by taking ownership of their learning and seeking further understanding. Moreover, research by Zimmerman and Kitsantas (2002) on self-regulated learning emphasizes the role of metacognition in student motivation. Rational inquiries indicate that students are employing metacognitive strategies by monitoring their understanding, recognizing gaps in knowledge, and actively seeking solutions. This reflective and proactive approach reflects their motivation to learn and succeed. As mentioned, behaviors were observed during the

teaching real analysis. Therefore, content bridging stands as motivation supporting techniques for teaching Real analysis in the teacher preparation program.

Conclusion

The technique of content bridging involves connecting the content and logic of a subject, specifically secondary-level mathematics, during instruction. This study has found that content and course bridging can assist both learners and teachers in effectively delivering new mathematical content related to real analysis. By establishing connections between different mathematical concepts and courses, students are motivated to engage with and learn real analysis. Studying real analysis at the bachelor's level also enhances logical reasoning skills and encourages active student participation in the learning process. The bridging of courses and contents in mathematics is a valuable technique that increases student motivation and addresses the question of why real analysis is important. The implementation of content bridging in mathematics education supports the integration of real analysis into the curriculum, promotes student motivation, and facilitates the resolution of queries related to the significance of studying real analysis.

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