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Comparative Study on Wavelet-based Linear and Non-Linear Image Denoising Techniques

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Abstract— Denoising an image is still a challenging domain in the image processing research area. Denoising means removing noise from a corrupted image but the challenge is to retain different details of an image. The search for efficient image-denoising methods is still a valid challenge at the crossing of functional analysis and statistics. In spite of the sophistication of the recent methods, most algorithms have not yet attained a desirable level of applicability. All the algorithms show a high outstanding performance when the image model corresponds to the algorithm assumptions but it generally fails to create artifacts or change the main structures of the original image. De-noising of natural images corrupted by white Gaussian noise using wavelet techniques is very effective because of its ability to capture the energy of the signal in a few energy transform values or coefficients. This method performs well under a number of applications because wavelet transform has the compaction property of having only a small number of large coefficients where the remaining wavelet coefficients are very small. The choice of threshold in wavelet-based image denoising is very critical as well as image restoration and noise reduction are eminent problems in almost all image processing applications. Numerous image restoration methods have been developed, each of which has its own advantages and limitation. For the removal of salt and pepper noise on natural images, different wavelet techniques are tested on various grayscale and colour images. Here, linear and non-linear wavelet transform denoising techniques of images are studied and compared using thresholding techniques such as Weiner, soft, hard, semi-soft, LevelShrink, SUREShrink, VisuShrink and BayesShrink.

I. INTRODUCTION

Noise removal is the most common and important preprocessing step in image processing applications. The main objective of denoising is to recover the best estimate of the original image from its noisy version. The denoising of a raw image corrupted by Gaussian noise is an abiding problem in signal processing. Images are also frequently corrupted by impulse noise due to transmission errors, faulty memory locations or timing errors in analog-to-digital conversion. The impulse noise is salt and pepper noise and generally, non-linear filtering techniques are used in the removal of salt and pepper noise. The aim of an image-denoising algorithm is to recover a clean image from its noisy version by removing the noise and retaining the maximum possible image information.

Linear Filters. Linear filters such as the Wiener filter in the wavelet domain yield optimal results when the signal corruption can be modelled as a Gaussian process and the accuracy criterion is the mean square error “MSE”. [1] [2] [3] However, designing a filter based on this assumption frequently results in a filtered image that is more visually displeasing than the original noisy signal, even though the filtering operation successfully reduces the MSE.

Non-Linear Threshold Filtering. The most investigated domain in denoising using wavelet transform is the non-linear coefficient thresholding-based methods. In the wavelet hard thresholding technique [3], each coefficient after applying wavelet transform is compared with a threshold value. If the coefficient is smaller than the threshold, set it to zero, else it is preserved.

In order to overcome the demerits of hard thresholding, wavelet transforms using soft thresholding [1],[6]. An improvement in wavelet thresholding is soft thresholding. In this method firstly the k-level decomposition is performed then, and thresholding is applied to the noisy coefficient.

One of the most known algorithms in non–adaptive threshold is VisuShrink [1] [6] which depends only on the number of data points. VISUShrink is known to yield overly smoothed images because its threshold choice can be unwarrantedly large due to its dependence on the number of pixels in the image.

SUREShrink uses a hybrid of the universal threshold and the “SURE” (Stein’s Unbiased Risk Estimator) threshold and performs better than VISUShrink. BayesShrink [1] [7] [8] minimizes the Bayes’ Risk Estimator function assuming Generalized Gaussian prior and thus yielding data adaptive threshold. BayesShrink outperforms SUREShrink most of the time.

II. RELATED WORK

Literature on topic image denoising Techniques is much old and frequent papers also published. Image denoising was first studied by Nasser Nahi in early 1970s. Since then, numerous wavelet-based image denoising algorithms have appeared.

Asem Khmag, Abd Rahman Ramli, Shaiful Jahari Hashim, Syed Abdul Rahman Al-Haddad [1] summarized and reviewed some algorithms and techniques that used to improve the image and found compromise between noise reduction and preserving significant signal details.

Raghuram Rangarajan, Ramji Venkataramanan, Siddharth Shah [4] investigated many soft thresholding schemes viz. VisuShrink, SureShrink and BayesShrink for denoising images and they found that sub band adaptive thresholding performs better than a universal thresholding. Among these, BayesShrink gave the best results.

Divya Sharma [5] proposed different approaches of wavelet based image denoising methods and provided a comprehensive evaluation of those methods.

Dharmpal D Doye, Sachin D Ruikar [9] proposed a technique which is computationally faster and gives better results. They also proposed a new threshold function which is better as compare to other threshold function.

Rajesh Kumar Rai, Trimbak R. Sontakke [13] found that sub band adaptive thresholding performs better than a universal thresholding and also noted that although SureShrink performed worse than BayesShrink, it adapts well to sharp discontinuities in the signal. Among these, SURE shrink gave the best results.

Sonali Singh and Sulochana Wadhwani [19] obtained results by different methods of wavelet thresholding Visu shrink, Sure shrink, Bayes shrink and concluded that Bayes shrink produces better restoration results in terms of PSNR and visual effects suppressing the Gaussian noise.

III. IMAGE DENOISING TECHNIQUES

A. NOISE MODEL

Gaussian Noise: Noise having Gaussian-like distribution is very often encountered in acquired data. This means that each pixel in the noisy image is the sum of the true pixel value and a random Gaussian distributed noise value. [14] This type of noise has a Gaussian distribution, which has a bell-shaped probability distribution function given by,

$$F(g) = \frac{1}{\sqrt{2\pi\sigma*\sigma}} e^{\frac{-(g-m)^2}{2\sigma^2}} \tag{1}$$

Where g represents the gray level, m is the mean or average of the function and σ is the standard deviation of the noise as shown in fig 1.

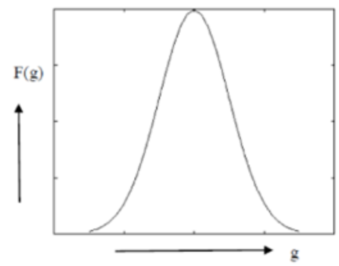


Fig. 1. Gaussian distribution

Salt and Pepper Noise: Salt and Pepper noise is a kind of impulse noise and is also referred to as intensity spikes. The salt and pepper noise gives a “salt and pepper” like appearance to the image and the affected (or corrupted) pixels are given minimum and maximum values alternatively and leave the unaffected pixels unchanged. For an 8-bit image, the minimum value i.e. pepper noise is set as 0 and the salt noise which has a maximum value is set as 255. Salt and pepper noise is occurred due to defective pixels in the camera sensors, timing errors in the digitization error or faulty memory locations. [10] [12].

B. DISCRETE WAVELET TRANSFORM

A wavelet is a wave-like oscillation with an amplitude that starts out at zero, increases, and then decreases back to zero. The finite scale multi-resolution representation of a discrete function can be called a discrete wavelet transform. DWT is a fast linear operation on a data vector, whose length is an integer power of 2. An image can be decomposed into a sequence of different spatial resolution image using DWT. In the case of an image, an N-level decomposition can be performed resulting in 3N+1 different frequency bands (sub-bands) namely, LL, LH, HL and HH as shown in fig 2.

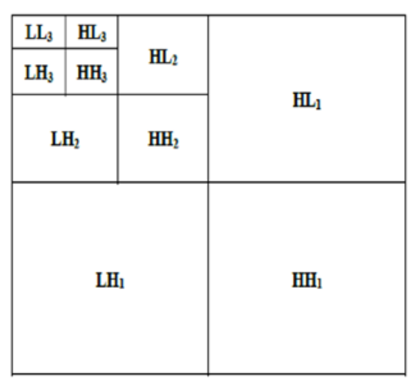


Fig. 2. DWT with 3-Level decomposition [15]

The sub-bands HHk, HLk, LHk are called the details coefficients, where k = 1, 2, …, j; k is the decomposition level and j denotes the largest or coarsest scale in decomposition and LLk is the approximation coefficient which is a low-resolution component. The next level of wavelet transform is applied to the low-frequency sub band image LL only. The Gaussian noise will nearly be averaged out in low-frequency wavelet coefficients.

Therefore, only the wavelet coefficients in the high-frequency levels need to be the threshold. As a final step in the denoising algorithm, the inverse discrete wavelet transform is applied to build back the modified image from its coefficients.

C. THRESHOLDING METHODS

Thresholding methods use a threshold and determine the clean wavelet coefficients based on this threshold.

1) Hard Thresholding Method

The hard-thresholding function chooses all wavelet coefficients whose magnitude are greater than the selected threshold value λ to remain as they are and the others with magnitudes smaller than λ are set to zero [16]. Mathematically it is

$$f_h(x) = \{x,\ |x| \geq \lambda\ 0,\ otherwise \tag{2}$$

The threshold λ is chosen according to the signal energy and the noise variance (σ^2) as shown in fig 3.

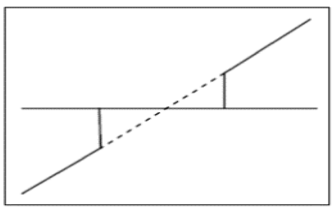


Fig. 3. Hard-thresholding [1] [4]

2) Soft Thresholding Method

The soft-thresholding function has a somewhat different rule from the hard-thresholding function. It shrinks the wavelet coefficients by λ towards zero, which is the reason why it is also called the wavelet shrinkage function. Soft thresholding is where the coefficients with greater than the threshold are shrunk towards zero after comparing them to a threshold value. [16].

$$f_s(x) = \{sign(x)(|x| - \lambda),\ for\ x\ \geq\ \lambda\ 0, \tag{3}$$

The soft-thresholding rule is chosen over hard-thresholding, for the soft-thresholding method yields more visually pleasant images over hard thresholding as shown in fig 4.

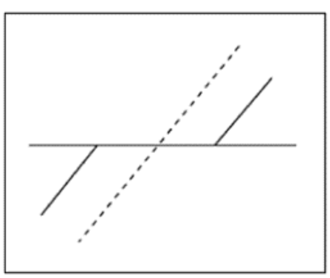


Fig. 4. Soft-thresholding [1] [4]

3) Semi- Soft Thresholding Method

By choosing appropriate thresholds, semi-soft shrinkage offers advantages over both hard shrinkage and soft shrinkage. [16]

$$f_{ss}(x) = \{0,\ \ |x| \leq \lambda_1\ sign(x)\left(\frac{\lambda_2|x|-\lambda_1}{\lambda_2-\lambda_1}\right)\ x,\ \ |x| > \lambda_2\ ,\ \ \lambda_2 < |x| \leq \lambda_1 \tag{4}$$

4) Universal Thresholding Method

The most original threshold is Universal Threshold which was originally proposed to use by Donoho and Johnstone is formulated as [9]:

$$\lambda_{univ} = \sigma * \sqrt{2Ln(M)} \tag{5}$$

Where M is the signal size and σ^2 is noise variance estimated from the sub-band HH1.

5) Weiner Filter

Wiener filters are a class of optimum linear filters which involve linear estimation of a desired signal sequence from another related sequence. It is not an adaptive filter. The Wiener filter may also be used for smoothing. [21] The wiener filter is formulated as:

$$F(u,v) = \left[\frac{H(u,v)^*}{|H(u,v)|^2 + \left[\frac{S_n(u,v)}{S_f(u,v)}\right]}\right] G(u,v) \tag{6}$$

Where G (u, v) and H (u, v) are degraded image and degradation function respectively. S_n and S_f are the power spectra of noise and original image (before adding of noise).

6) VisuShrink Filter

Visushrink is thresholding by applying the Universal threshold proposed by Donoho and Johnstone. For denoising images, Visushrink is found to yield an overly smoothed estimate. It is formulated as

$$T = \sigma * \sqrt{2Log(M)} \tag{7}$$

Where σ^2 is the noise variance present in the signal and M represents the signal size or the number of samples [16].

7) SureShrink Filter

An adaptive threshold, called SureShrink, was developed by Donoho and Johnstone, which is named after Stein's unbiased risk estimation (SURE). SureShrink is the combination of the universal threshold and the SURE threshold [17]. Sure Shrink is an adaptive thresholding method where the wavelet coefficients are treated in a level-by-level fashion [18]. Sure Shrink is used for suppression of additive noise in wavelet domain where a threshold **'T'** SURE is employed for denoising. [19]

The threshold parameter **'T'** SURE is expressed as

$$T_j = argmin\left(SURE_j(t,y)\right) \tag{8}$$

Where the Stein's Unbiased Risk (SURE) is minimized as follows:

$$SURE_j(t,y) = \sigma^2 - \frac{1}{N_j}\left(2\sigma^2 \cdot \#\{i: |y_i| \leq t\} + \sum_{i=1}^{N_j} \quad min(|y_i|,t)^2\right) \tag{9}$$

where N is the number of samples; J is the number of channels; N_j is the number of samples in the channel j; σ^2 is noise variance and y is the coefficient of the sub band.

8) BayesShrink Filter

BayesShrink is an adaptive data-driven threshold driven in a Bayesian framework, and we assume generalized Gaussian distribution (GGD) for the wavelet coefficients in each detail sub band and try to find the threshold T, which minimizes the Bayesian Risk. [21] The goal of this method is to minimize the Bayesian risk, and hence its name, BayesShrink [9]. The Bayes threshold, $\mathbf{T_B}$, is defined as

$$T_B = \frac{\sigma^2}{\sigma_s} \qquad (10)$$

Where σ^2 is the noise variance and σ_s is the signal variance without noise. The noise variance σ^2 is estimated from the sub band HH by the median estimator. Using this threshold, the wavelet coefficients are threshold at each band [9] [20].

IV. EVALUATION

The performance of each algorithm is compared by computing MSE and PSNR, besides the visual interpretation. MSE and PSNR are the two parameters used in this paper for comparison of denoising techniques. Mean square error (MSE) is calculated as:

$$MSE = \frac{1}{M \times N} \sum_{i=1}^{M} \sum_{j=1}^{N} [x(i,j) - x'(i,j)]^2 \qquad (11)$$

Where x is the original image and x' is the denoised image

PSNR is the peak signal to noise ratio. PSNR is the most commonly used parameter to measure the quality of reconstruction image with respect to the original image. A higher PSNR would normally indicate that the reconstruction is of higher quality. PSNR is usually expressed in terms of the logarithmic decibel scale (dB). PSNR of reconstructed image is formulated as:

$$PSNR = 10 log_{10}\left(\frac{255^2}{MSE}\right) \qquad (12)$$

A. PROPOSED ALGORITHM

1. Input image $x_{i,j}$ corrupted with visual noise.
2. Compute the Discrete Wavelet Transform

$$F(t) = \sum_k \sum_j \alpha_{j,k} \beta_{j,k} \qquad (13)$$

Where $\alpha_{j,k}$ and $\beta_{j,k}$ are the transform coefficients and basic functions respectively.

3. Apply different wavelet schemes.
4. Estimate the wavelet coefficients of an image produced by different wavelet schemes.
5. Compute the inverse discrete wavelet transform.

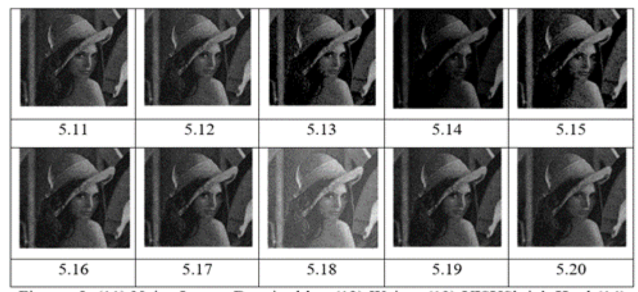
V. RESULT AND DISCUSSION

We have taken two test images i.e., Lena (512 x 512) and House (256 x 256) as input image for simulation of results. Different test images i.e. lena.jpeg, cameraman.jpeg, peppers.jpeg and mandrill.jpeg, of size 256 x 256 are taken for Gaussian noise application and airplane.png, barbara.png, and boat.png (512 x 512) images for Salt & Pepper Noise application as shown in fig 5.

A. *Test images for Gaussian Noise Application:*

Figure 5.0: (a) Lena.jpeg (b) Cameraman.jpeg (c) Peppers.jpeg (d) Mandrill.jpeg

B. *Visual Results:*



Figures 5: (11) Noisy Image, Denoised by: (12) Weiner (13) VISUShrink Hard (14) VISUShrink Soft (15) VISUShrink semi-soft (16) Level Shrink Hard (17) Level Shrink Soft (18) Level Shrink semi-soft (19) SureShrink (20) Bayes Shrink

C. *Test images for Salt & Pepper Noise Application:*

Figure 6.0: (a) airplane.png (b) barbara.png (c) boat.png

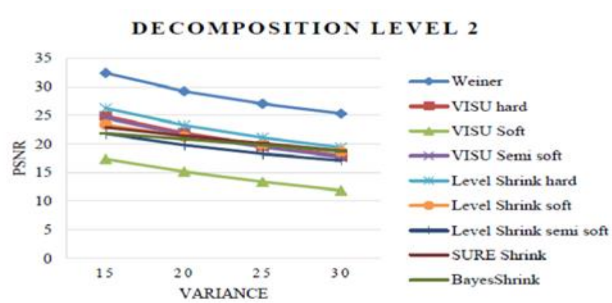


Fig. 5. PSNR vs VARIANCE for Lena image

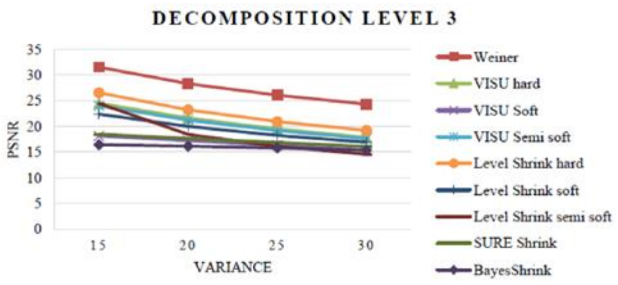


Fig. 6. PSNR vs VARIANCE for Cameraman image

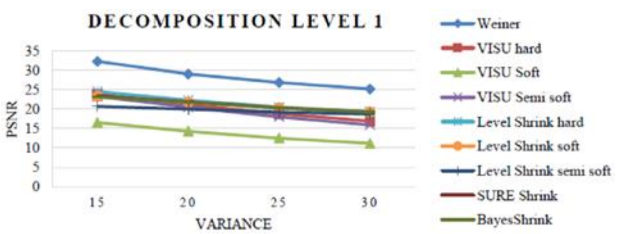


Fig. 7. PSNR vs VARIANCE for Mandrill image

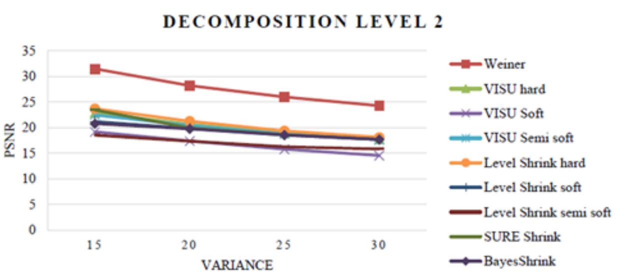


Fig. 8. PSNR vs VARIANCE for Peppers image

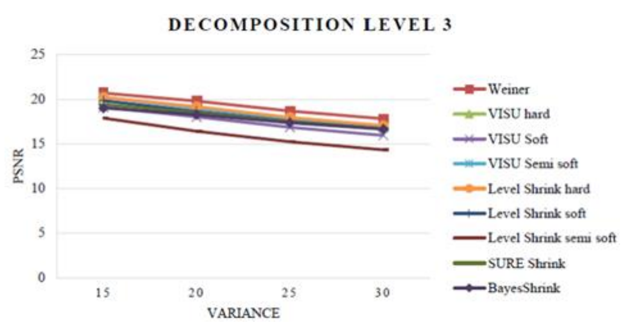


Fig. 9. PSNR vs VARIANCE for Airplane image

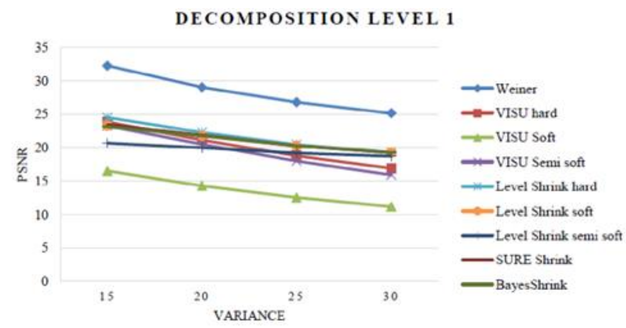


Fig. 10. PSNR vs VARIANCE for Barbara image

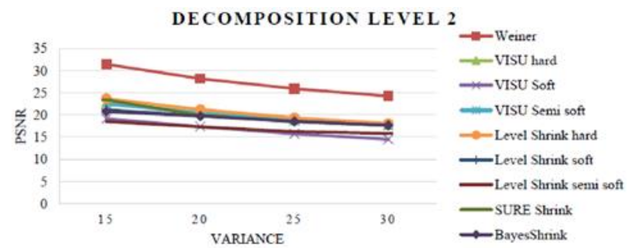


Fig. 11. PSNR vs VARIANCE for Boat image

VI. CONCLUSION

From the results obtained above, it can be concluded that the Wiener filter method is optimal compared to other thresholding methods. It produces the maximum PSNR for the output image compared to the other methods. Although Weiner has better PSNR value, it cannot denoise the salt & pepper properly. SUREShrink, BayesShrink and VISUShrink are poor in terms of PSNR values but work well for denoising Salt & Pepper Noise with pleasing visual effects.

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