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Evaluation of Nakagami and Birnbaum-Saunders probability distribution for wind speed and power estimation

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Abstract

This study compares three probability density functions (PDFs) for understanding and estimating wind power based on wind patterns: the commonly-used Weibull distribution, the relatively-new Birnbaum-Saunders distribution, and the Nakagami distribution. The wind profile of Jumla, Nepal was analyzed using data from 2004 to 2014. The Nakagami distribution performed similarly to the Weibull distribution in terms of understanding wind patterns. However, for estimating wind power, the Nakagami distribution was found to be more effective than the Weibull distribution in most cases. The Birnbaum-Saunders distribution was found to be the least effective of the three PDFs compared.

Keywords: *Birnbaum-Saunders distribution, Nakagami distribution, Weibull distribution*

1. Introduction

The rapid growth of the economy, society, and industry has led to an increasing demand for energy in our lives. Global primary energy consumption has increased by 2.4% per year and shows no signs of slowing down (Jarvis, Leedal, and Hewitt, 2012). A significant portion of this demand is met by fossil fuels. However, the rising price of fossil fuels, their limited availability, environmental concerns, and the need for a diverse energy mix have cast doubt on their future. In response to economic development, technological advancement, and climate change concerns, many countries are investing in renewable energy (Lin, Omoju, and Okonkwo, 2016). Studies have shown that renewable energy sources like solar PV and thermal, hydro, wind, and biomass-derived fuel provide wide-ranging socioeconomic benefits and reduce pollution (Aliyu, Modu, and Tan, 2018). Wind energy has become increasingly popular due to its modular and environmentally-friendly nature (Elhadidy and Shaahid, 2000). Additionally, large-scale wind power systems have a shorter lead time and can coexist with other land uses such as farming which reduces initial investment.

The feasibility of a wind energy conversion system depends largely on the amount of energy that can be harnessed, which is determined by factors such as wind characteristics, the interaction between the wind and the turbine (aerodynamic efficiency, mechanical stress on the structure, etc.), and its operating and maintenance strategies. The first step in this analysis is to understand the wind characteristics, which can be best done with long-term, high-resolution meteorological data. However, such data is not always available for all desired locations. In these cases, statistical analysis of limited wind data is used to predict wind energy. Furthermore, data such as the mean wind speed alone is not sufficient to accurately estimate wind power density because it does not provide a complete picture of the wind profile. Therefore, various probability density functions (PDFs) are used in wind power evaluation research (Ouarda et al., 2015; Pishgar-Komleh, Keyhani, and Sefeedpari, 2015).

The choice of a probability density function (PDF) is crucial for accurately representing wind characteristics. Some researchers suggest that the selection of a PDF should depend on its objective, such as representing the wind speed profile, estimating wind power density, or evaluating fatigue load (Morgan et al., 2011). As a result, various parametric, mixture, nonparametric, and hybrid models have been tested. Parametric models such as the Normal, Lognormal, Gamma, Generalized Gamma, Weibull, Inverse Weibull, Rayleigh, Generalized Rayleigh, Logistics, Log Logistics, Kappa, Wakeby, Birnbaum-Saunders, Burr, Beta, and Nakagami distributions are widely used (Aririguzo and Ekwe, 2019; Badawi et al., 2019; Carta et al. 2009; Morgan et al. 2011; Ouarda et al. 2015; Salim et al., 2019; Samal and Tripathy 2019; Wang et al., 2016; Xu et al., 2015).

There is a lack of detailed research on the suitability of PDFs for windy areas in Nepal. The author previously studied the applicability of the Weibull function in the Himalayan region (Parajuli, 2016, 2021). Recent studies have also examined the use of the two-parameter Weibull distribution and its parameter estimators in terrains of Nepal (Dhakal et al., 2020; Pandeya et al. 2022). The Weibull distribution is also the most widely used PDF (Burton et al., 2011; Garcia et al., 1998; Manwell et al., 2010). However, it has been shown in several studies that the Weibull distribution has limitations in certain wind profiles. Therefore, it is important to evaluate various PDFs at a given location to identify the best one. This research aims to evaluate two probability distribution functions: the Nakagami and Birnbaum-Saunders distributions. These distributions have been used in different fields and have only recently been introduced for wind power estimation. The performance of these PDFs will be compared to the Weibull distribution.

The Nakagami distribution is a two-parameter distribution that is related to the Gamma distribution. It has been widely used to model the attenuation of wireless signals that travel through multiple paths and to evaluate the impact of fading channels (Parsons 2001; Sanchez-Iborra, Cano, and Garcia-Haro 2013). It has also been used in fields such as medicine, hydrological science, and reliability theory (Datta, Gupta, and Agrawal 2014; S. Sarkar, Goel, and Mathur 2010; Shibayan Sarkar, Goel, and Mathur 2009; Zhou et al. 2015). Similarly, the Birnbaum-Saunders distribution is also a two-parameter distribution that is closely related to the skewed Normal distribution and is commonly used in reliability and fatigue life applications (Awad and Khanna 2015; Leiva et al. 2007). In addition, the applicability of the Birnbaum-Saunders distribution has been tested in a wide range of fields, including water quality, air pollution, economics, agriculture, engineering, and medicine (Gomes, Ferreira, and Leiva 2013; Leiva, Sanhueza, and Angulo 2009). Recently, a few researchers have also applied the Nakagami (Alavi, Mohammadi, and Mostafaeipour 2016; Aries, Boudia, and Ounis 2018; Gugliani 2020; Haq et al. 2021; Idriss et al. 2020) and Birnbaum-Saunders (Jia et al. 2020; Mahbudi, Jamalizadeh, and Farnoosh 2020; Mohammadi, Alavi, and McGowan 2017) distributions to wind applications. However, the performance of a probability distribution function should be evaluated across different terrain, altitude, and locations. Therefore, this research aims to compare the performance of the Nakagami and Birnbaum-Saunders distributions with the conventionally preferred Weibull distribution in the Himalayan region of Nepal.

2. Materials and Methods

2.1 Site location and data collection

There is limited wind data available for the Himalayan region of Nepal. This study used data from a single site in the region where wind data was available for a ten-year period. The site is located in the Chandannath Municipality of Jumla District in Nepal and is at an altitude of 2300 meters above sea level. Wind speed was measured at a height of 10 meters above the ground and the average daily wind speed was recorded. Data from 2004 to 2014 (excluding 2012) was used for analysis. The data availability for the site was 98.05%, and 0.2% of the data represented calm wind.

The earth's surface provides vertical shear for the wind. To accurately calculate wind energy, the measured wind speed must be adjusted for the height of the turbine hub. Researchers have suggested that a logarithmic relationship exists between altitude and velocity, which can be used to modify the wind speed. The following relationship is commonly used to adjust the wind speed for different altitudes and account for the vertical shear of the wind (Abbas et al. 2012).

$$v = u\left(\frac{z}{y}\right)^{a} \tag{1}$$

where, u is the wind speed at normalized height (m/s), y is the normalized height (m), and z is the turbine hub height (m). The exponent *a* is a shear parameter and depends on atmospheric stability and surface roughness. In neutral or stable conditions, *a* is approximately 0.143, which is often assumed to be constant in wind resource assessments (Pishgar-Komleh, Keyhani, and Sefeedpari, 2015).

2.2 Probability distribution

2.2.1 Weibull distribution

The PDF of Weibull distribution is obtained by following function (Ahmed, 2013; Weibull, 1951):

$$f(v;k,c) = \left(\frac{k}{c}\right)\left(\frac{v}{c}\right)^{k-1} \exp\left[-\left(\frac{v}{c}\right)^{k}\right] \tag{2}$$

where, *k* and *c* are shape and scale parameter respectively. They can be estimated as (Azad, Rasul, and Yusaf 2014)

$$k = \left(\frac{0.9874}{\frac{\sigma}{\bar{v}}}\right)^{1.0983} \tag{3}$$

and

$$c = \frac{\bar{v}}{\Gamma(1+1/k)} \tag{4}$$

where, Γ is the gamma function and is given by

$$\Gamma(x) = \int_0^\infty e^{-t} t^{x-1} dx \tag{5}$$

2.2.2 Nakagami distribution

The PDF of Nakagami distribution is obtained by (Alavi, Mohammadi, and Mostafaeipour 2016; Nakagami 1960)

$$f(v;m,\Omega) = \frac{2m^m}{\Gamma(m)\Omega^m} v^{2m-1} \exp\left[-\frac{m}{\Omega}v^2\right] \tag{6}$$

where m and Ω are shape and scale parameters and are calculated using (Nakagami 1960)

$$m = \frac{\overline{v^2}^2}{\overline{v^2 - \overline{v^2}}^2} \tag{7}$$

and

$$\Omega = \overline{v^2} \tag{8}$$

2.2.3 Birnbaum-Saunders distribution

The PDF of Birnbaum-Saunders distribution is obtained by following function (Z. W. Birnbaum and Saunders 1969; Coleman et al. 1996; Ng, Kundu, and Balakrishnan 2003):

$$f(v;\alpha,\beta) = \frac{1}{2\sqrt{2\pi}\alpha\beta}\left[\left(\frac{\beta}{v}\right)^{1/2} + \left(\frac{\beta}{v}\right)^{3/2}\right]\exp\left[-\frac{1}{2\alpha^2}\left(\frac{v}{\beta} + \frac{\beta}{v} - 2\right)\right] \tag{9}$$

where, α and β are shape and scale parameters and are calculated using a relation (Z. Birnbaum and Saunders 1969)

$$\alpha = \left(\frac{s}{\beta} + \frac{\beta}{r} - 2\right)^{1/2} \tag{10}$$

and

$$\beta^2 - \beta[2r + K(\beta)] + r[s + K(\beta)] = 0 \tag{11}$$

where, s and r are arithmetic and harmonic mean of v and K(β) is defined as

$$K(\beta) = \left[\frac{1}{n}\sum_{i=1}^{n}(\beta + v)^{-1}\right]^{-1} \tag{12}$$

The equation for β is non-linear and is to be solved iteratively.

2.3 Performance evaluation of PDFs

In this study, we evaluate the performance of wind profile fitting by using two metrics: Root Mean Square Error (RMSE) and coefficient of determination (R^2) related to PP plot. We also consider the error in power estimation, as the ultimate goal of using PDFs is to estimate wind power density. The error in power estimation is a key factor in determining the performance of PDFs.

2.3.1 Root mean square error

Root Mean Square Error (RMSE) is a commonly used measure of the difference between observed and predicted values. It represents the square root of the second moment of the residual error. RMSE can be calculated using (Jung et al. 2017)

$$RMSE = \left[\frac{1}{n} \sum_{i=1}^{n} (F_{i,est} - F_{i,obs})^2 \right]^{1/2} \tag{13}$$

where, n is number of observations, $F_{i,est}$ is estimated Cumulative Density Function (CDF) and $F_{i,obs}$ is observed CDF of ith data.

2.3.2 Coefficient of determination (R^2) related to PP plot

TheR^2 value is a measure of how well a dependent variable can be predicted from independent variables. In curve fitting, a high R^2 value (close to 1) indicates a high degree of accuracy in predicting the dependent variable. In the context of wind applications, we can use the R^2 value to assess the accuracy of predicting the Cumulative Density Function (CDF) from velocity by treating velocity as the independent variable and CDF as the dependent variable (Azad, Rasul, and Yusaf 2014; Chang 2010). Then,

$$R^2 = \frac{\sum (F_{i,est} - \overline{F_{i,est}})^2}{\sum (F_{i,est} - \overline{F_{i,est}})^2 + \sum (F_{i,est} - F_{i,obs})^2} \tag{14}$$

where, $F_{i,est}$ is mean value of estimated CDF. All other symbols have same meaning as described above.

2.3.3 Power prediction error

The wind power density is a measure of the amount of power that can be generated from the wind in a specific area. It is calculated as the average power available from the wind across all wind speeds. This value is crucial for the analysis of wind turbines and wind farms. To get a rough estimate of the power that can be generated, the wind power density is multiplied by the rotor area and the efficiency of the turbine. However, to get a more accurate estimate, it is necessary to consider factors such as the cut-in speed, rated speed, cut-off speed, and efficiency profile of the turbine. If high-resolution wind speed data is available, it is possible to more accurately calculate the wind power density by using this data. The wind power density can be calculated using

$$P_{obs} = \frac{\sum \frac{1}{2} \rho v^3}{n} \tag{15}$$

where, ρ is the air density at the location, which is typically a function of the temperature and atmospheric pressure at that location. The power density can also be estimated using the probability density function as

$$P_{est} = \sum \frac{1}{2} \rho v^3 f(v) \tag{16}$$

where f(v) is probability density function selected to estimate the wind power density. The percentage error in estimation of wind power density is evaluated using

$$P_{Error} = \frac{\left|P_{obs} - P_{est}\right|}{P_{obs}} \times 100\% \tag{17}$$

3. Results and Discussions

To facilitate the analysis, the wind speed data was divided into five groups, each containing data from two consecutive years. Table 1 shows the average wind speed and standard deviation for each group and the overall data. The data shows that the average wind speed is decreasing over time, a trend which was previously discussed by the author (Parajuli 2016). The overall average wind speed is 5.98 m/s with a standard deviation of 2.15 m/s. The table also includes the skewness and kurtosis for each group. The skewness for all groups is positive indicating that the distribution has a longer right tail. Similarly, the kurtosis for all groups is greater than 3 indicating a leptokurtic distribution with thicker tails compared to a normal distribution. In 2004 and 2005, the kurtosis of the wind speed data is particularly high. The relationship between kurtosis and estimation error will be examined later in the study. The three different distributions used in this research are suitable for fitting data with a positive skew and leptokurtic distribution.

Table 1: Wind Speed Characteristics

Particulars	04-05	06-07	08-09	10-11	13-14	Overall
Mean Speed	6.94	6.62	5.78	5.26	5.22	5.98
Standard Deviation	2.35	2.05	2.07	1.79	1.83	2.15
Skewness	0.67	0.24	0.16	0.09	0.17	0.45
Kurtosis	5.97	3.19	3.39	3.09	3.22	4.42

In this study, the accuracy of three different probability density functions in curve-fitting and power estimation will be evaluated. The shape and scale factors of these distribution functions are calculated and are listed in Table 2. It was observed that there is an inverse correlation between the Weibull shape factor and the Birnbaum-Saunders shape factor while there is a positive correlation between the Weibull scale factor and the Birnbaum-Saunders/Nakagami scale factor. However, no direct relationship was found between the Weibull shape factor and the Nakagami shape factor.

Table 2: Shape and Scale Parameters

PDF	Particulars	04-05	06-07	08-09	10-11	13-14	Overall
Weibull	Shape (k)	3.23	3.58	3.05	3.22	3.12	3.03
	Scale (c)	7.75	7.35	6.46	5.87	5.84	6.69
Nakagami	Shape (m)	1.97	2.79	2.20	2.46	2.28	1.94
	Scale (Ω)	53.74	48.02	37.64	30.85	30.64	40.32

BS	Shape (α)	0.36	0.32	0.39	0.37	0.39	0.42
	Scale (β)	6.59	6.35	5.41	4.96	4.91	5.50

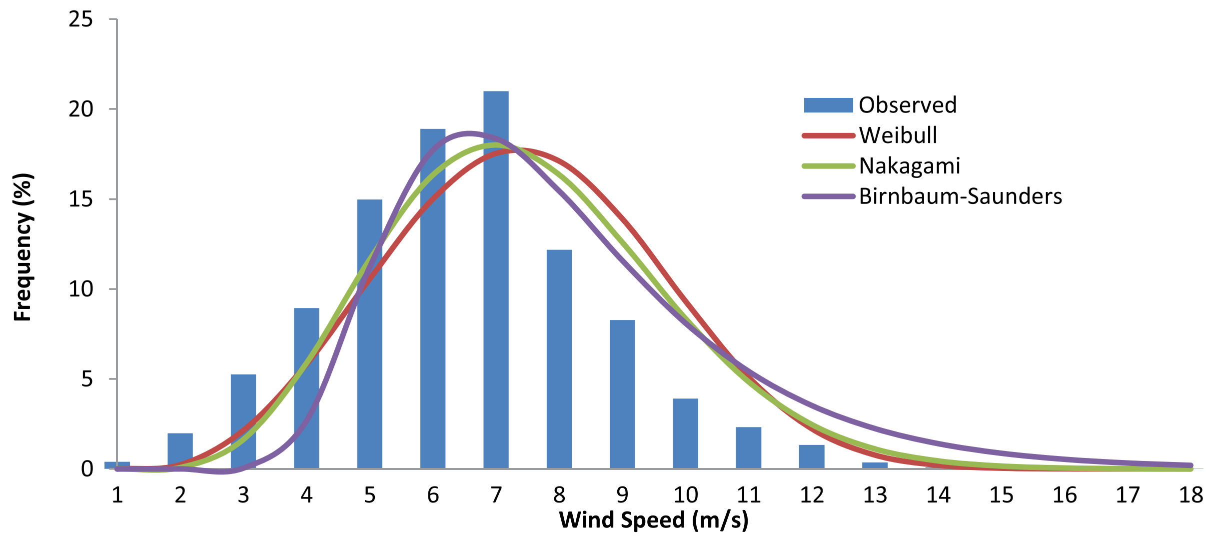

Figure 1: Observed Histogram and Fits of Various PDFs

Table 3: Wind Power Density

PDF	04-05	06-07	08-09	10-11	13-14	Overall
Observed	281.04	229.93	164.26	120.31	120.10	184.03
Weibull	276.70	229.01	164.52	120.46	120.03	182.37
Nakagami	283.18	229.85	164.14	120.07	119.96	184.97
BS	341.78	290.65	223.14	167.62	168.59	245.43

The PDFs of Weibull, Nakagami, and Birnbaum-Saunders for the overall dataset were plotted and compared with the observed probability distribution. This process is known as curve fitting. Figure 1 shows the histogram of the actual distribution and the three PDFs. It can be seen that the Birnbaum-Saunders distribution largely underpredicts small wind speeds and overpredicts large wind speeds, although it more accurately defines the peak of the distribution. Similarly, all PDFs overpredict the intermediate wind speeds. The wind power density of the observed data and the data predicted by the three distribution functions are presented in Table 3. The wind power density is observed to be decreasing, which is also reflected in the trend of decreasing average wind speeds. However, the reduction of wind power density is steeper than the wind speed due to the dependence of wind power density on the third power of wind speed. The main goal of estimating powers and curve fitting in this article is to evaluate the effectiveness of the PDFs. The root mean square error (RMSE) and coefficient of determination (R^2) are used to evaluate the accuracy of the curve fit, while the percentage power error estimates the error in power prediction. A summary of these errors is presented in Table 4, which also includes the best PDF as determined by different error estimators.

Table 4: Error in Velocity and Power Estimation

Year	PDF	RMSE	R^2	Power Error%
2004-2005	Weibull	1.207	0.9990	1.54%
	Nakagami	1.618	0.9969	0.76%
	BS	1.478	0.9919	21.61%
	Best	***Wei***	***Wei***	***Nak***
2006-2007	Weibull	1.095	0.9993	0.40%
	Nakagami	0.823	0.9997	0.04%
	BS	1.248	0.9914	26.41%
	Best	***Nak***	***Nak***	***Nak***
2008-2009	Weibull	1.511	0.9989	0.16%
	Nakagami	1.504	0.9989	0.07%
	BS	1.877	0.9923	35.85%
	Best	***Nak***	***Nak***	***Nak***
2010-2011	Weibull	1.707	0.9989	0.12%
	Nakagami	2.144	0.9980	0.20%
	BS	2.725	0.9896	39.32%
	Best	***Wei***	***Wei***	***Wei***
2013-2014	Weibull	1.196	0.9994	0.05%
	Nakagami	1.492	0.9989	0.11%
	BS	1.987	0.9911	40.38%
	Best	***Wei***	***Wei***	***Wei***
Overall	Weibull	3.009	0.9579	0.90%
	Nakagami	2.547	0.9631	0.51%
	BS	2.912	0.9313	33.37%
	Best	***Nak***	***Nak***	***Nak***

According to the error analysis, the Weibull and Nakagami distributions provide the best fit and are the most accurate for power prediction in all data groups. The Weibull distribution performs better in curve fitting for the 2004-2005, 2010-2011, and 2013-14 data groups, while the Nakagami distribution performs better in the remaining groups. However, the Birnbaum-Saunders distribution has significantly higher error in power estimation, making its usefulness in wind applications questionable. The Nakagami distribution was the best model for wind power density estimation in four out of six groups.

The findings of Alavi, Mohammadi, and Mostafaeipour (2016) and Gugliani (2020) suggest that the Nakagami distribution is similar to the Weibull distribution in different terrains. Although Haq et al. (2021) do not explicitly mention this in their conclusion, their data suggests that the Nakagami and Weibull distributions have similar performance. In contrast, Jia et al. (2020) found that the power density estimator error of the Birnbaum-Saunders distribution is much worse compared to the Weibull distribution. However, Mohammadi, Alavi, and McGowan (2017) concluded that the Birnbaum-Saunders distribution is superior to the Weibull distribution based on R^2, RMSE, Akaike Information Criterion (AIC), and Bayesian Infromation Criterion (BIC) criteria, but did not evaluate the error in the power estimator. Our study and the findings of Jia et al.

(2020) show that the inferior performance of the Birnbaum-Saunders distribution is due to power estimation error, not probability distribution curve fitting error.

Further analysis showed that the error in estimation using these three PDFs is positively correlated with the kurtosis of the wind speed. Additionally, the Nakagami distribution was found to be superior to the Weibull distribution in terms of accuracy when the kurtosis and skewness of the data were larger. However, the difference in error between these two distributions was very small in this study. Since the Weibull distribution is more widely used and accepted, and the Nakagami distribution has shown promising results for highly skewed and leptokurtic distributions, it would be beneficial to further test the application of the Nakagami distribution in various regions before it is widely accepted as an alternative to the Weibull distribution.

4. Conclusions

In this study, the effectiveness of the Weibull, Nakagami, and Birnbaum-Saunders probability distribution functions were compared for analyzing wind patterns and estimating wind power at the high-altitude site of Jumla, Nepal. The wind speed data was analyzed and the shape and scale factors of the distribution functions were estimated. The power density was evaluated and the errors in curve fitting and power estimation were calculated. It was found that both the Weibull and Nakagami distributions performed better than the Birnbaum-Saunders distribution. The estimated errors for the Weibull and Nakagami distributions were similar, suggesting that the Nakagami distribution could serve as an alternative to the Weibull distribution for this site. However, further analysis of multiple sites is needed to determine the wider applicability of the Nakagami distribution in wind science.

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Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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