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Infiltration-Induced Landslide: An Application of Richards Equation

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Abstract: *In this work, we incorporate the solution of Richards equation in infinite slope model to locate the potential landslide hazards area and prediction of landslide hazards induced by heavy rainfall, continuously precipitation and redistribution. Firstly, we modeled the Richards equation in cylindrical coordinate with axial symmetry and use Kirchhoff's transformation to linearize it. With Kirchhoff's transformation the nonlinear axi-symmetric model is transformed into the nonlinear parabolic equation. Because of its high non-linear properties, analytical solutions of Richards equation are rare and limited for particular cases with hardly reliable. To solve the equation numerically, different approximation techniques as FDM, FVM are used on the prescribed model. Since, the landslide hazards problems are accelerated by safety factor which are related to the forces that restrain the surface from failure and endow the surface to smash. Hence to evaluate the safety factor, we use the infinite slope model characterized with moisture content, pressure head in variably saturated (unsaturated) soils. The attachment of Richards equation along with the infinite slope model in the expression of safety factor which helps to explore the surface failure condition for different soils for their different physical characteristic including precipitation, infiltration, redistribution and moisture content.*

Keywords: Richards equation, Moisture content, Kirchhoff transformation, Landslide hazards, Safety factor

1 Introduction

Landslides occur especially in hilly area of the Himalaya region around the world. Depending on tectonic activity, varied topography, intense rainfall, and geological instability, the 2400 km long Himalaya mountain chain is highly susceptible to landslide. These mountainous terrains are the habitat to millions of living beings scattered in Nepal, Bhutan, northern Pakistan, northern India, and the other area of the world. Uneven landscape, changeable geological structures, weak and breakable rocks together with heavy and unusual concentrated precipitation during rainy seasons collectively cause rigorous landsliding problems and related phenomena in the Himalaya mountain region.

The main cause for severe and concentrated rainfall events which are common nowadays is climate change. The effect of heavy and uneven rainfall, infiltration-induced landslides are found everywhere in the Himalaya mountain in the world. Losses of natural and man made infrastructures, damaging of construction including roads, houses, and fatalities exposed describe that the landslides are dramatically increased nowadays. In variably saturated (unsaturated) soil, infiltration-induced landslide related to the infiltration process generally coincides to the ground surface where the shallow landslide may befall due to the continuous precipitations. Especially shallow landslide occurred within the vadose zone of unsaturated soil by sliding the surface of sloppy areas. Slope failure is the tectonic behavior of soil surface, which depends on the infiltration process in which fluctuation of matric potential in variably saturated (unsaturated) soil plays a vital role and triggers of landslide from rainfall.

The infiltration-induced landslides in the Himalaya make tremendous damage to lives, property, infrastructure and environment, particularly in the monsoon season. Therefore, it is necessary to discuss about the factors, issues and environmental effects to the infiltration-induced landslide in this region. The main causes of such kind of landslides are surface failure that depends on the nature and connectivity of slope instabilities. Now it is relevant to analyze the behavior of slope stability in variably saturated (unsaturated) soil. The concrete study of soil structure associated to slope model helps to calculate the factor of safety which are conventional to analyze the slope stability of sliding surface along the failure plane.

Particular weather conditions such as continuous and concentrated precipitation, rapid snow melting or uneven weather change are the main causes for slope instabilities. These phenomena are directly related with the pore water pressure of soil. Pore water pressure can may up and down in abnormal manner in infiltration and redistribution processes, i.e., during rainfall and after the rainfall. Thereby increasing the pore saturation, the corresponding pore pressure also increases. Continuously repeated these processes the effective stress of the soil may be deducted and it becomes the main cause of slope failure. Richards equation is a non-linear partial differential equation, which describes the soil water flow and can be applied for a quantitative assessment of the landslide hazards. The formulation of new model, connecting to Richards equation with infinite slope stability model and its numerical solution can be a powerful tool to capture the integrity of slope failure corresponding to the available weather information. The efficiency and accuracy of such formulation and procedure depends on the quality of the available soil parameters which are directly connected to weather data and physical feature of the landscape. Therefore, an acceptable mechanisms to landslide hazards assessment can accurately be adapted to the potential area considering to account.

In this work, we break a new ground for infiltration-induced landslides, flourishing on the importance of infiltration processes to predict landslide hazards. With the higher capability of adopting to disparate types of moisturity, axi-symmetric Kirchhoff transformed Richards equation [1] is purposed and incorporate it to infinite slope model [2] for the prediction of landslide hazards. Along with different hydraulic conductivity constitutive functions (verified with experimentally), which describes the behavior of water that can percolate in the soil pore and depends on inherent permeability, density, viscosity, degree of saturation, shearing stress and strength of soil. There are many expressions invoke for the hydraulic conductivity and the moisturity functions [3], with these, for accounting the stability of landslide, the hydrological slope stability model is corporate on the nonlinear Richards equation and formulate it for the ahead processes. Accordingly, the solution of this new formulation will appropriate to infiltration-induced landslide in variably saturated (unsaturated) soils. The stability and consistency of the proposed approaches will be incorporate by comparing the exact data interpolation.

The main purpose of this work is to provide the notions from weather information and a mathematical approach developed for the study of water flow in variably saturated (unsaturated) soil and infiltration-induced landslide hazards. In addition, for modeling of water flow, some intuitions are given how it is possible to derive water flow relations for unsaturated soil from fundamental equations at pore scale. This brief description enables to set an important concepts in order to describe the governing equation for the modeling of water flow in unsaturated soil [1]. Study of water flow in variably saturated (unsaturated) porous media required proper formulation of the governing equation with appropriate constitutive relationship [3]. Along with specific boundary and initial conditions are also introduced to complete the flow model. Generally, to describe water flow through unsaturated soil, Richards equation is used, which is based on semi–empirical equation derived by Buckingham [4] and Richards [1]. However, Richards equation has some limitations and drawbacks, it is still the most widely used equation for modeling of water flow through saturated as well as unsaturated soil. Therefore, Richards equation is a well known mathematical model and widely used for simulation of infiltration in many fields with a long history. Indeed, solving Richards equation may be very difficult due to the nonlinearities of constitutive relations especially with the hydraulic properties. To obtain the solution of Richards equation numerically and analytically, various techniques are employed. Analytical solutions are rare and limited for special cases only with appropriate substitutions. Numerous numerical solutions are developed and applied but still a great effort is required to reach the accuracy, robustness and cost effectiveness for the model solution. Solution of Richards equation is an essential part of discussion for the complex flow phenomenon in variably saturated (unsaturated) soil with its numerical challenges as well as in computational techniques. Therefore, a robust technique should be required for the solution. And here a qualitative analysis for its approximate solution is presented to sort Richards equation behaviors.

To solve unsaturated flow equation with Kirchhoff transformation Zhang et al. [27] introduced Finite Analytic Method (FAM). In this method, a set of algebraic equation are formed depending on the Kirchhoff transformed variable by applying a local analytical solution. This method established an accurate and efficient solution. The FAM method is applied by Zhang et al. [27] on mixed form of Richards equation. Also Zhang et al. [27] put forth it into the two–dimensional Richards equation. However, it is good achievement, the Finite Analytic Method is limited only for homogeneous porous media soil. Ji et al. [24] used the transient pressure head–based method and compared a steady state solution in between Kirchhoff transformation and time marching scheme. They used Gardner's constitutive relationship [28] and their result showed that the computational cost in unsaturated flow simulation can greatly be reduced and the use of Kirchhoff transformation has great implication for sub-surface water problems. But they only considered pseudo–heterogeneous layered porous media and limited it only to steady–state conditions. Although, Kirchhoff transformation method has some limitations, it seems to be very promising method for simulation of Richards equation in unsaturated porous media region. Following into this method, numerical errors will be reduced effectively because variations of Kirchhoff head are strictly lesser than pressure head in its integral nature. Indeed, when the Kirchhoff transformation is employed along with some specific constitutive relations to solve nonlinear Richards equation, a full linearization of the unsaturated Richards equation is required. This linearization has an ability of analytic and semi–analytic solutions. Furthermore, numerical methods for solving nonlinear Richards equation are pertaining to specific constitutive relationships. The popular and adopted constitutive relationships were developed by Gardener [28], van Genuchen [35], Brooks and Corey [34] and Haverkamp et al. [3]. Correspondingly, they gave the well measured relationship between hydraulic conductivity, pressure and moisture content. For example, Heejun Suk et al. [29] obtained an approximate result on Kirchhoff transformed Richards equation in variably saturated flow in non homogeneous layered soil. This method has followed the Gardener's relationships and is used to solve the linearized partial differential equation. Egidi et al. [30] used van Genuchten consecutive relationship for a numerical solution of Richards equation. A constitutive relationship developed experimentally by Haverkamp et al. [3] were used by Liu Fengnan et al. [25] to finite difference method (linearized) for the Richards equation under variable–flux boundary conditions. Among these, a most robust numerical method an explicit stabilized Runge–Kutta–Legendre super time–steeping scheme is developed [31] to solve mixed form of nonlinear Richards equation with no source and sink terms and with sink terms as evapotranspiration [32] numerically adopting the constitutive relationship developed by Haverkamp et al. [3]. The scheme is based on the natural phenomena taking into account the hydraulic conductivity as a function of water pressure head as a real situation with experimental data.

Finally, there are several processes and causes that contribute to landsliding in the Himalaya mountain. Tectonic activity, geological instability, varied topography, along with physical causes, human causes are the most important factor for the landslide hazards. In the foothill side of Himalaya, unusual, heavy and concentrated precipitation (rainfall) be regarded as the main triggering factor of landslides. That means, when monsoon started in this region most of landslides were occurred and make number of disasters. That is why numbers of people and other living things are suffering by a large and small scale landslide in the Himalaya region. For those consequences, it is necessary to understand the processes and causes for the infiltration-induced landslide in the Himalaya mountain region.

The paper is systematized as follows: we present the Kirchhoff transformed axi-symmetrical form of Richards equation in section 2. In section 3, we present the approximation procedure based on FDM with different time-stepping schemes. In section 4, we describe the slope failure models for landslide stability in unsaturated soils. In section 5, results of the simple numerical procedure implemented in python are presented. In section 6, we concluded it with the obtained results and the future development of this work.

2 Richards Equation in Cylindrical Coordinates with Axial Symmetry

The soil water flow conducting by Darcy's law along with mass conservation equation [1] can be written in axi-symmetrical form as

$$\frac{\partial \theta}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(rK(\psi)\frac{\partial \psi}{\partial r}\right) + \frac{\partial}{\partial z}\left(K(\psi)\frac{\partial \psi}{\partial z}\right) - \frac{\partial K(\psi)}{\partial z} + W - ET. \tag{1}$$

Where $\theta(\psi)$ is moisture content, $K(\psi)$ is the hydraulic conductivity, ψ is pressure head, W is the precipitation rate (recharge), ET is the evapotranspiration (rate of water loss) and transpiration of plants. To describe the water movement phenomena in variably saturated (unsaturated) soil, equation (1) can be used.

The function $\theta(\psi)$ and $K(\psi)$ are relative to the work of Van Genuchten [35] and Haverkamp et.al. [3] models. These models are widely used in scientific computations due to their formulation by smooth function. Commonly used version of these models can be written as

$$K(\psi) = K_s \left[1 - |\alpha\psi|^n\right]^{-\frac{m}{2}} \left[1 - \left(\frac{|\alpha\psi|^n}{1 + |\alpha\psi|^n}\right)^m\right]^2 \tag{2}$$

$$\theta(\psi) = \theta_r + \frac{(\theta_s - \theta_r)}{(1 + |\alpha\psi|^n)^m} \tag{3}$$

$$K(\psi) = K_s \frac{A}{\alpha + |\psi|^\gamma} \tag{4}$$

$$\theta(\psi) = \theta_r + \frac{\alpha(\theta_s - \theta_r)}{\alpha + |\psi|^\beta} \tag{5}$$

Here, equations (2), (3) represent the Van Genuchten and (4), (5) represent the Havercamp models. Here, α is the reciprocal value of ψ, i.e., the air entry point [35], θ_s and θ_r are the saturated water content and residual water content respectively. K_s is the permeability of saturated soil. A, β, γ, n, m are empirical parameters depending on the soil, also m and n have the relation

$$m = 1 - \frac{1}{n} \tag{6}$$

Parameters involving in constitutive relations and used in soil types are adopted from Edi et al. [30].

2.1 Linearization techniques

We apply Kirchhoff integral transformation in equation (1), that transformed the equation in non-linear parabolic from. For the transforming process, we assume $h = \psi - z$ and computing as

$$\phi(h) = \int_0^h \bar{K}(\kappa) d\kappa. \tag{7}$$

Since $K(h) > 0$ from (5), the function $\phi(h) >>$ with $\bar{K}(h) = K(\psi)$. Taking derivative of equation (7) with respect to r

$$\frac{\partial\phi}{\partial r} = \frac{\partial\phi}{\partial h}\frac{\partial h}{\partial r} = \bar{K}(h)\frac{\partial(\psi - z)}{\partial r} = \bar{K}(h)\frac{\partial\psi}{\partial r} = K(\psi)\frac{\partial\psi}{\partial r}. \tag{8}$$

Again taking derivative of (8) with respect to r,

$$\frac{\partial^2\phi}{\partial r^2} = \frac{\partial}{\partial r}(K(\psi)\frac{\partial\psi}{\partial r}) \tag{9}$$

and differentiating equation (7) with respect to z

$$\frac{\partial\phi}{\partial z} = \frac{\partial\phi}{\partial h}\frac{\partial h}{\partial z} = \bar{K}(h)\frac{\partial(\psi - z)}{\partial z} = K(\psi)\left(\frac{\partial\psi}{\partial z} - 1\right) = K(\psi)\frac{\partial\psi}{\partial z} - K(\psi) \tag{10}$$

Again differentiating of equation (10) with respect to z

$$\frac{\partial^2\phi}{\partial z^2} = \frac{\partial\left(K(\psi)\frac{\partial\psi}{\partial z}\right)}{\partial z} - \frac{\partial}{\partial z}\left(K(\psi)\right). \tag{11}$$

Using equations (8)-11, the Richards equation (1), transformed as

$$\frac{\partial \theta}{\partial t} = \frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\left(\frac{\partial \phi}{\partial r}\right) + \frac{\partial^2 \phi}{\partial z^2} - ET + W \tag{12}$$

with $\bar{\theta}(\phi) = \theta(h)$. Since (12) is a parabolic PDE, to obtain the solution of the prescribed model (12) in cylindrical coordinate system, we set the initial and boundary values assuming the potential landslide area as an annulus. Thus, we set the initial and boundary conditions as

$$\begin{cases} \phi(r, z, 0) = \phi_0(r, z),\ R_{in} \le r \le R_{out}\ , 0 \le z \le Z_{top} \\ \dfrac{\partial \phi}{\partial z} = \bar{q}(t),\ r > 0\ z = 0,\ t > 0 \\ \phi(r, Z_{bot}, t) = \bar{\beta}(t),\ r > 0,\ t > 0 \\ \dfrac{\partial \phi}{\partial r} = 0,\ r = R_{out}\ , z > 0,\ t > 0 \\ \phi(R_{in}, Z, t) = \bar{\beta}_1(t),\ t > 0 \end{cases} \tag{13}$$

Now the nonlinear equation (1) is transfered to a nonlinear parabolic (heat like) equation (12) with this transformation technique. Also, the above transformation scheme has a property to preserve the uniqueness of the solution for the new problem.

3 Approximation Schemes

Now we have the transformed equation in two state variables θ and ϕ. To solve (12) numerically with the given initial and boundary conditions (13), a distinguish state variable is feasible. For this, we assumed θ and ϕ as continuous functions of ϕ and θ simultaneously, and constructing a chain rule into these variables as

$$\frac{\partial \theta}{\partial t} = \frac{\partial \theta}{\partial \phi}\frac{\partial \phi}{\partial t} = \left(\frac{1}{\frac{\partial \phi}{\partial \theta}}\right)\frac{\partial \phi}{\partial t}, \quad \frac{\partial \phi}{\partial \theta} = \frac{\partial \phi}{\partial h}\frac{\partial h}{\partial \theta}. \tag{14}$$

Taking derivative with respect to h of (5) and (4), we get

$$\frac{\partial \theta}{\partial h} = \beta |h|^{\beta-1}\alpha(\theta_s - \theta_r)(\alpha + |h|^{\beta})^{-2} \qquad \frac{\partial \phi}{\partial h} = \bar{K}(h) = K(\psi). \tag{15}$$

Adopting equations (14) and (15), the equation (12) becomes

$$c(\phi)\frac{\partial \phi}{\partial t} = \frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial z^2}, \tag{16}$$

in which the coefficient (functional) c prescribed in ϕ through h is defined as

$$c(\phi(h)) = \frac{\alpha\beta(\theta_s - \theta_r)|h|^{\beta-1}}{\bar{K}(h)(\alpha + |h|^{\beta})^2}. \tag{17}$$

3.1 Discretization

We perform finite difference scheme for discretization to equation 16. We build an uniform grid of two dimension in (r, z) for model equation in the axi-symmetric cylindrical geometry by partially dividing the radial length $[R_n, R_{out}]$ into M_r partial intervals of width $\Delta r = \frac{R_{out} - R_{in}}{M_r}$ and the height $[0, Z_{top}]$ into M_z subintervals of width $\Delta z = \frac{Z_{top}}{M_z}$. We set up a grid (r_i, z_j, t_n) with $r_i = i\Delta r, i = 0, 1, 2, \ldots, M_r$, $z_j = j\Delta z$, $j = 0, 1, 2, \ldots, M_z$, and $t_n = n\Delta t$, $n = 1, 2, \ldots, N$. Let $\phi^n_{i,j}$ denote $\phi(r_i, z_j.t_n)$. Now the PDE (16) can be simulated using finite difference approximation forward in time and central in space (FTCS) as

$$\left.\frac{\partial \phi}{\partial t}\right|_{(r_l, z_j, t_n)} \approx \frac{\phi^{n+1}_{i,j} - \phi^n_{i,j}}{\Delta t}, \quad \left.\frac{\partial \phi}{\partial r}\right|_{(r_l, z_j, t_n)} \approx \frac{\phi^n_{i+1,j} - \phi^n_{i-1,j}}{2\Delta r}.$$

$$\left.\frac{\partial^2\phi}{\partial r^2}\right|_{(r_l,z_j,t_n)} \approx \frac{\phi^n_{i-1,j} - 2\phi^n_{i,j} + \phi^n_{i+1,j}}{\Delta r^2}, \quad \left.\frac{\partial^2\phi}{\partial z^2}\right|_{(r_l,z_j,t_n)} \approx \frac{\phi^n_{i,j-1} - 2\phi^n_{i,j} + \phi^n_{i,j+1}}{\Delta z^2} \tag{18}$$

expressing the given derivative terms using a weighted average as $\left(\frac{\partial\phi}{\partial r}, \frac{\partial^2\phi}{\partial r^2}, \frac{\partial^2\phi}{\partial z^2}\right)$ at the two time levels, t_n and t_{n+1}. Adopting Crank-Nicolson (CN) scheme, equation (16) is discretized as

$$\begin{aligned}\frac{\phi^{n+1}_{i,j} - \phi^n_{i,j}}{\Delta t} &= \frac{1}{2c^n_{i,j}(\Delta r)^2}\left[\phi^{n+1}_{i-1,j} - 2\phi^{n+1}_{i,j} + \phi^{n+1}_{i+1,j} + \phi^n_{i-1,j} - 2\phi^n_{i,j} + \phi^n_{i+1,j}\right] \\ &+ \frac{1}{2c^n_{i,j}(\Delta z)^2}\left[\phi^{n+1}_{i,j-1} - 2\phi^{n+1}_{i,j} + \phi^{n+1}_{i,j+1} + \phi^n_{i,j+1} - 2\phi^n_{i,j} + \phi^n_{i,j-1}\right] \\ &+ \frac{1}{4r_i c^n_{i,j}(\Delta r)}\left[\phi^{n+1}_{i+1,j} - \phi^{n+1}_{i-1,j} + \phi^n_{i+1,j} - \phi^n_{i-1,j}\right] + W^n_{i,j} - ET^n_{i,j}\end{aligned} \tag{19}$$

Collecting the unknown terms in the left hand side:

$$\begin{aligned}&-\left(F_r - \frac{F}{r_i}\right)\phi^{n+1}_{i-1,j} + (1 + 2F_z + 2F_r)\phi^{n+1}_{i,j} - \left(F_r + \frac{F}{r_i}\right)\phi^{n+1}_{i+1,j} - F_z(\phi^{n+1}_{i,j-1} + \phi^{n+1}_{i,j+1}) \\ &= \left(F_r - \frac{F}{r_i}\right)\phi^n_{i-1,j} + (1 - 2F_z - 2F_r)\phi^n_{i,j} + \left(F_r + \frac{F}{r_i}\right)\phi^n_{i+1,j} - F_z(\phi^n_{i,j+1} + \phi^n_{i,j-1}) + W^n_{i,j} - ET^n_{i,j},\end{aligned} \tag{20}$$

where $F_r = \frac{\Delta t}{2c^n_{i,j}(\Delta r)^2}$, $F_z = \frac{\Delta t}{2c^n_{i,j}(\Delta z)^2}$, $F = \frac{\Delta t}{4c^n_{i,j}(\Delta r)}$.
We coupled equation (20) at the new time level $n+1$. That coupled equation performs a system of linear equation as $Mk = D$, where M is the augmented matrix, k is the unknown vector, D is the right side resources. Now, we have to solve the system of equations.

To solve system of equation $Mk = D$, where the index of unknown vector k must be one. Instead of two, a single index number is required to each unknown in the mesh. From a mesh point with indices (i, j), a mapping $q(i, j) = w(i, j)$ is performed to the corresponding unknown q in the system of equation.

$$q = w(i, j) = j(W_r + 1) + i \text{ for } i = 0, 1, 2, \ldots, W_r,\ j = 0, 1, 2, \ldots, W_z.$$

Performing in this manner, the points along radial direction are numbered starting with $z = 0$ and each mesh line are filled one by one at a time. Correspondingly we get,

$$q = w(i, j) = i(W_z + 1) + j \text{ for } i = 0, 1, 2, \ldots, W_r,\ j = 0, 1, 2, \ldots, W_z.$$

with $r = 0$ and each mesh line is filled one at a time. Regarding to this processes, the discretized equation (20) is transformed to the coefficient matrix.

Considering $M_{g,h}$ be the corresponding value of element (g, h) in the augmented matrix M, where g and h are the unknown in the system of equation. Providing $M_{g,h} = 1$ for $g = l$ correspondingly to the all known boundary values. Again let g be $w(i, j)$, i.e., the single index corresponding to the mesh point (i, j). Now the positions of interior mesh with along boundary mesh can located as

$$\begin{aligned}M_{w(i,j),w(i,j)} &= M_{g,g} = 1 + (F_Z + F_r), \\ M_{g,w(i-1,j)} &= M_{g,g-1} = -F_r, \\ M_{g,w(i+1,j)} &= M_{g,g+1} = -F_r \\ M_{g,w(i,j-1)} &= M_{g,g-(W_z+1)} = -F_z \\ M_{g,w(i,j+1)} &= M_{g,g+(W_z+1)} = -F_z.\end{aligned}$$

Numbers containing in the vector D_g, where g numbers the equations with the given boundary values represents the coefficients of right hand side of the system of equation involving the boundary values.

Using above algorithm, the value of $\phi_{i,j}^{n+1}$ in time level $n+1$ is updated from the value of $\phi_{i,j}^{n}$. After that we employ the equation (17) which can be approximated as

$$h_{i,j}^{n+1} = h_{i,j}^{n} + \frac{\phi_{i,j}^{n+1} - \phi_{i,j}^{n}}{\bar{K}(h_{i,j}^{n})}. \tag{21}$$

To accelerate the algorithm to the next time level $\phi_{i,j}^{n+2}$. it is essential to upgrade the function $c(\phi_{i,j}^{n+1})$ which requires computing the intermediate variable $h_{i,j}^{n+1}$. we can not get the new approximation without computing $h_{i,j}^{n+1}$.

4 Landslide Stability and Instability for Unsaturated Soils

We assume the wide implementation of Richards equation for a significant assessment of landslide hazards. For modeling of infiltration-induced landslide in variably saturated (unsaturated) soils, the instability/stability analyses connected to the axi-symmetrical Kirchhoff transformed Richards equation with the variation of moisture content relative to variation of water pressure head and vice-versa is adapted. The characteristic of infiltration-induced landslide in unsaturated soil on the ground surface is seemingly closed to the physical features of shallow landslide that occurs during the period of intense rainfall. This study governs the landslide hazards problem involving infinite slope model for safety factor. Safety factor is the ratio of two forces in which the one prevents the slope from falling and the other adheres the slope to be collapsed. The numerical value of safety factor represent the landslide hazards index, depending on it prediction can be made for the possibility of landslide hazards to the given topography. Generally, index greater than one defines stable condition and less than one defines unstable condition. Different types of slope model are developed to calculate the safety factor, the following infinite slope model is used here for its easiest and reliable calculation [30].

$$FS = \frac{C}{zY_t \sin\beta \cos\beta} + \frac{\tan\phi}{\tan\beta} - \frac{hY_w(\theta)\tan\phi}{zY_t \sin\beta\cos\beta}, \tag{22}$$

where C is the effective cohesion, θ is the moisturity, Y_t and Y_w are the unit weight of the soil and water respectively, ϕ is internal frictional angle, z is soil thickness, β is the slope of the inclined surface. The combination of the infinite slope model (22) with Richards equation, explore the landslide hazard calculation directly from weather data. In particular, the values of θ obtained from solution of Richards equation are used to get information about safety factor in the given infinite slope model. In collaborating the weather data with this moisturity θ, we can easily obtained the potential landslides area.

5 Results and Discussion

5.1 Simulation setup

To demonstrate the approximate solution of the prescribed model, the approximating process carried out in the above section is written in python and executed. We define a specific infiltration experiment in unsaturated soil and observe the corresponding outcomes. At first the setup consisted on a landslide area having dimension 70cm × 100cm. The physical constituent of the landslide area is sandy soil. The corresponding soil parameters and consecutive relationship between $\theta(\psi)$ and $K(\psi)$ are interpolated from the work of Haverkamp et al. [3] and van Genuchten [35].

$$\begin{aligned} K(\psi) &= K_s \frac{A}{\alpha + |\psi|^{\gamma}} = \frac{34 \times 1.175 \times 10^6}{1.175 \times 10^6 + |\psi|^{4.74}} \\ \theta(\psi) &= \theta_r + \frac{\alpha(\theta_s - \theta_r)}{\alpha + |\psi|^{\beta}} = 0.075 + \frac{1.611 \times 10^6 \times (0.287 - 0.075)}{1.611 \times 10^6 + |\psi|^{3.96}} \end{aligned} \tag{23}$$

The simulation process is started with a flux of $\theta = 0.1\ cm^3/cm^3$ maintaining water pressure head $\psi = -61.5$ cm at the bottom of the annulus $z = Z_{bot}$ as lower boundary. At $z = Z_{top}$, which coincides to the soil surface, a constant flux $q(t) = 13.69$ cm/hr for $t < 0.7hr$ and zero normal flux for $t > 0.7$ hr maintained as upper boundary. For curved surface of the annulus we set zero flux as an boundary. These phenomena are also observed by using different parameters of soil connecting to the soil moisture content $\theta(\psi)$ and the hydraulic conductivity $K(\psi)$ from the work of van Genuchten [35].

The axial direction are partitioned with uniform spatial step size $\Delta z = 2$ cm and the radial direction with step size of $\Delta z = 2$ cm to get the approximate solution from Crank-Nicolson scheme. For the observation the simulation was conducting for 0.75 hr.

To analyze the infiltration-induced landslide, we have interpolate the moisturity variation obtained from the above experiment to the hydrological model to obtain the stability analysis. We use the experimental data obtained from [30] to the equation (22).

5.2 Experimental results and discussion

The 2+1 dimensional model equation (12) was employed to approximate the vertical water infiltration into an unsaturated homogeneous soil for simulation and to analyze the landslide hazards, hydrological model was used. Simulation operation were carried out by designing a vertical annulus completely filled by sandy soil (proposed by Haverkamp et al. [3] and van Genuchten [35]). Numerical computation were computed

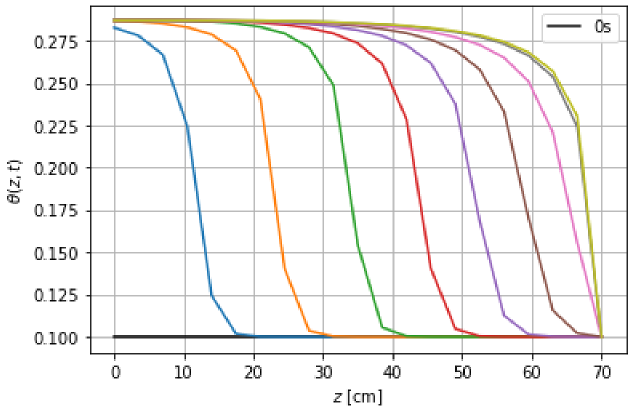

Figure 1: Moisture content profile (one dimension case).

by different finite difference schemes as RKL, FTCS, BTCS, and CN and tested the results obtained. Since the model is highly non-linear and has no analytical solution, then for the stability and consistency of two dimensional model, we compiled the obtained solution with the numerical solution obtained from above (RKL) finite difference scheme in one dimension as the reference solution as mentioned in [31]. The nature of flow phenomena in the infiltration process are depict in Figure 1. The moisture content profile in sandy soil for one dimension, put forth further development in two dimension as discussed in [31].

In this simulation process, consecutive relations are adopted from Haverkamp et al. [3]

We set the Richards equation in cylindrical coordinates with axial symmetry in an variably saturated (unsaturated) soil water flow into a foothill domain of soil (sandy). In this movement, we maintained the domain as specified an annulus, $r_{inner} \leq r \leq r_{outer}$ with r_{inner} strictly greater than zero. We have arranged the dimension of the annulus as $70cm \times 100cm$.

We set up the experiment regarding the soil water flow in vertically downward direction taking z axis as downward positive with a constant water pressure head $\psi = -61.5cm$ at left side of the annulus as a fixed boundary (Dirichlet) assuming with $r = R_{in}$. On the right side $r = R_{out}$ of the annulus, we set a constant flux as a boundary and at the bottom $z = Z_{bottom}$ a constant water pressure head $\psi = -61.5$ cm as lower boundary . At the ground level (soil surface), i.e., at $z = 0$, a constant flux $q(t) = 13.69$ cm/hr for $t < 0.7hr$ and zero normal flux condition for $t > 0.7$hr is adapted. Grid structure of solution domain was meshed

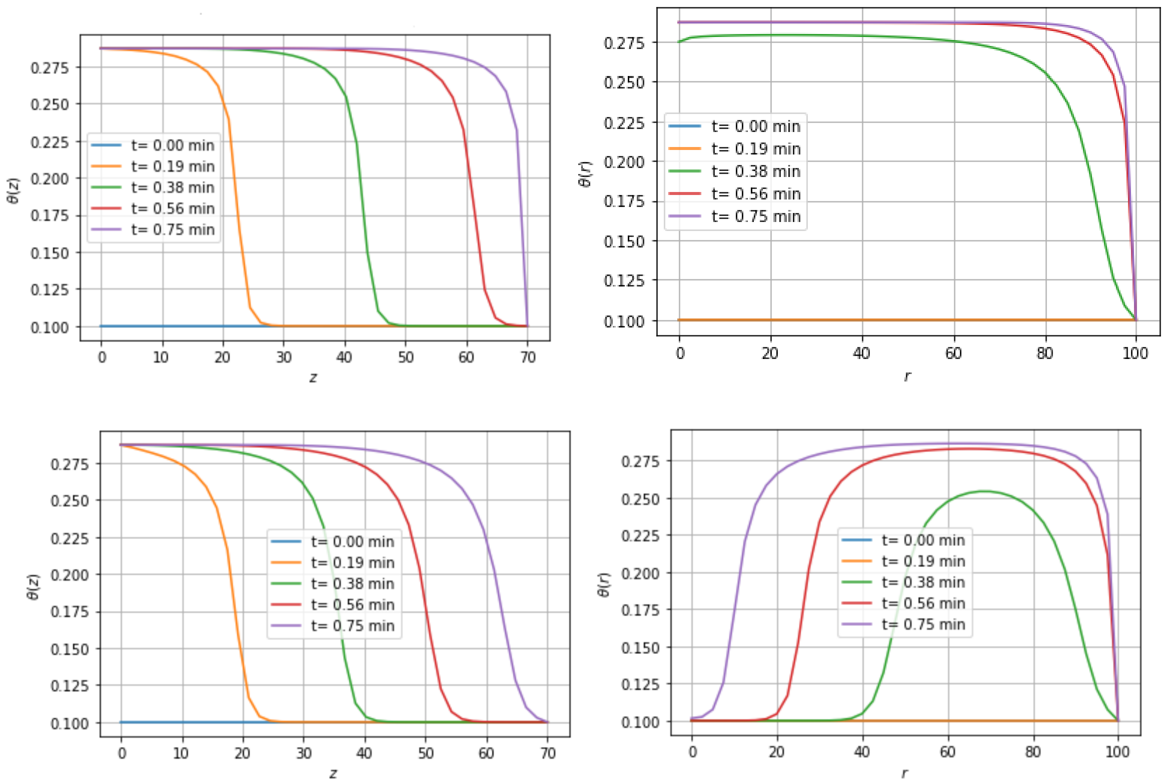

Figure 2: Soil moisture profile $\theta(z)$ at $r = 50cm$ to longitudinal direction (top left), soil moisture profile $\theta(z)$ at $z = 35cm$ to radial direction(top right), soil moisture profile $\theta(z)$ at $r = 25cm$ to longitudinal direction (bottom left), soil moisture profile $\theta(z)$ at $z = 35cm$ to radial direction (bottom right).

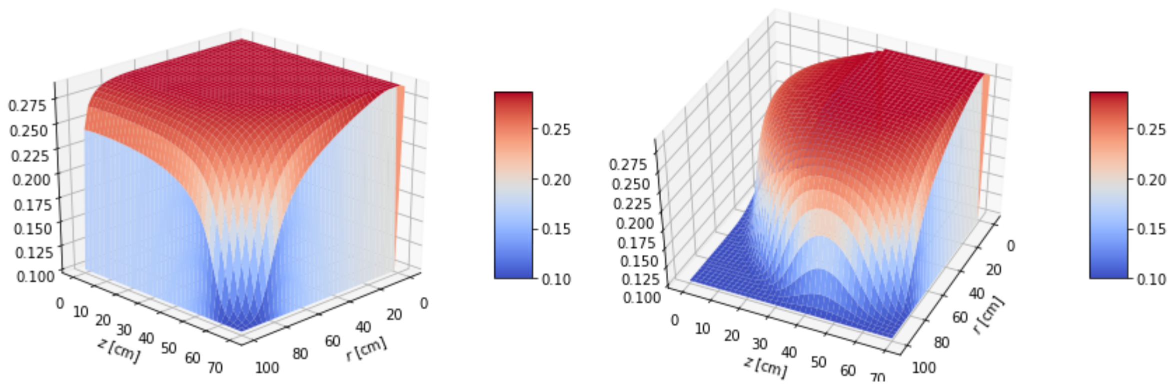

Figure 3: Moisture content $\theta(r, z)$ at $t = 0.75sec$ for $r = R_{out}$ (left), Moisture content $\theta(r, z)$ at $t = 0.75secs$ for $r = R_{out}/2$ (right).

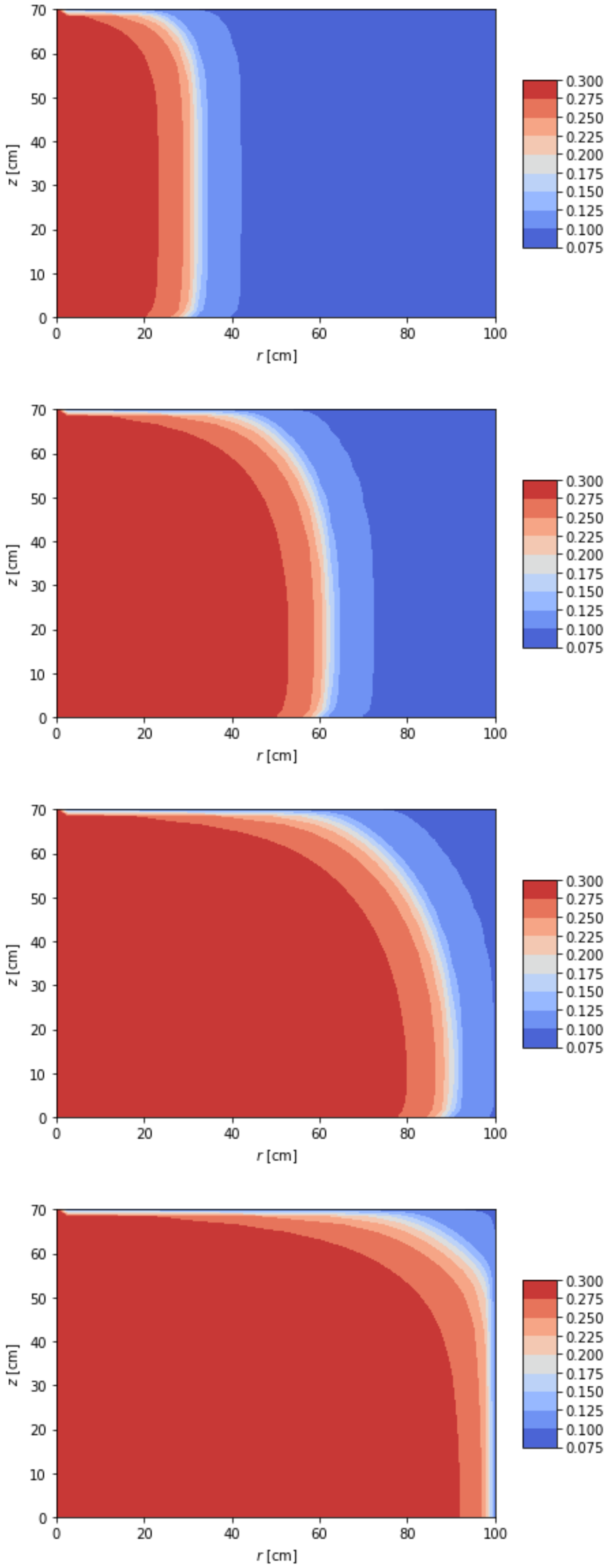

Figure 4: Moisture content $\theta(r, z)$ at $t = 0.1875sec$ for $r = R_{out}$ (top left), moisture content $\theta(r, z)$ at $t = 0.375sec$ for $r = R_{out}$ (top right), moisture content $\theta(r, z)$ at $t = 0.5625sec$ for $r = R_{out}$ (bottom left), Moisture content $\theta(r, z)$ at $t = 0.75sec$ for $r = R_{out}$ (bottom right).

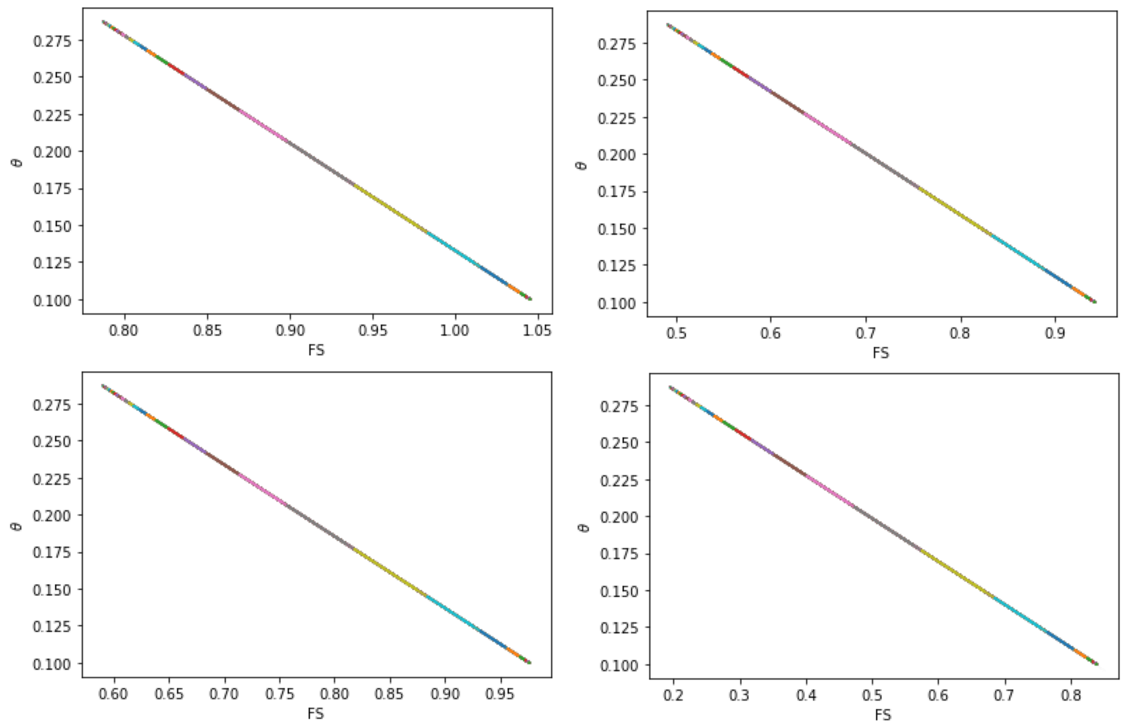

Figure 5: Factor of safety for various moisture profile.

with $\Delta z = 2cm$ on axial direction and a $\Delta r = 2.5cm$ on radial direction and the total time elapsed in simulation process was calculated $1hr$. Figure 2 (top) shows the observation obtained from the longitudinal and radial procedure of water contain profiles at time $0hr$ and next time domain accordingly. Figure 2 depicts that at time t is about $0.5hr$, the residual water coincide to water contain profile at the bottom of the annulus. Surface plots related to moisture content are shown in Figure 3 and the relative contours plots are in Figure 4.

We observed the flow phenomena and carried out observation in the variation of water content in the region we have prescribed. We have interconnected the value of θ to the infinite slope model to evaluate the safety factor. We have measured the slope angle and the thickness of the unsaturated soil as $25°$ and $10m$. We use the parameter of effective cohesiveness is $3.50kPa$, the frictional angle is $28.8°$, the soil unit weight is $19.75kN/m^3$. Figure 5 depicts the computed results of safety factor for different moisture profile observed in the experiment. The factor of safety obtained by the above model demonstrates the possibilities of surface failure and operation of landslide hazards. From Figure 5, we conclude that the situation of surface failure may increase when the level of ground water is increased by heavy rainfall meanwhile the pressure head (moisture content) energy increases and the level of unsaturated region decreases and the possibility of landslide increases.

6 Conclusion

In this work, we considered axi-symmetric Kirchhoff transformed Richards equation. We solved this equation numerically using forward in time and central in space, backward in time and central in space and Crank-Nicolson method, which are based on finite difference schemes. Because of more realistic and consistent behavior, we have used the solution obtained from Crank-Nicolson method in infinite slope stability model for the analysis of landslide hazards. We have used hydrological infinite slope stability model (22) to calculate the factor of safety. We obtained values of moisture content θ the most necessary quantity for evaluating safety factor in the model (22) and interconnected these values to model (22). The other parameters used in the model were obtained from available weather data. The result, we got after interconnecting the values of θ to the infinite slope equation shows that the prediction of landslide is possible for the available parameters depending on the constituent of the soil and nature of the land. We conclude that with the help of Richards equation we can evaluate the landslide hazards directly from the weather data also. This is the wide application of Richards equation from the point of geological view. This work can be extended to achieve accurate prediction of massive landslides to unsaturated heterogeneous soils with abruptly changing wetness conditions.

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Data Availability The data used for supporting the findings of this study are included within the article.

Conflicts of Interest The author declares that there is no conflict of interest.

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