Rational Type Contraction in b-Metric Spaces and Common Coupled Fixed Point Theorems

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Abstract: *The aim of this article is to construct and obtain some new results for common coupled fixed points in b-metric space for a pair of mappings satisfying rational type contraction. The present results extend, modify, and generalize the existing literature. The results are verified with the help of suitable examples.*

Keywords: b-Metric space, Couple fixed point, Common coupled fixed point, Rational type contraction

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1 Introduction

In 1922, Banach [5] first introduced the fixed point (FT) for contraction mapping in a metric space, which is also known as the Banach contraction mapping theorem (BCMT) or Banach fixed point theorem (BFPT). After that, many researchers have extended and generalized BFPT or BCMT for different types of mappings in various metric spaces.

The concept of b-metric space was introduced by Bakthin [4] and Czerwik [12, 13] as an extension of metric space by weakening the triangular inequality. As a result, many authors have generalized and improved the extension of fixed point theorems in b-metric space (for example, [1, 2, 3, 8, 9, 10, 11, 14, 15, 16, 17, 19, 22, 23, 24, 29, 30].

The concept of coupled fixed point was started by Guo and Lakshmikanthan [18] for partially ordered sets. Bhaskar and Lakshmikanthan [7] investigated the existence and uniqueness of coupled fixed point results in partially ordered metric space using the concept of mixed monotonic property. After that, various authors have studied the coupled fixed point and discussed its application (see, for instance, [6, 20, 21, 25, 27, 28].

The aim of this article is to construct and obtain new results on common coupled fixed points for a pair of mappings satisfying rational type contraction. The found results are extended and change various results from the existing literature of [26].

2 Basic Concept and Mathematical Preliminaries

In this section, we give some prominent definitions and mathematical preliminaries, which are needed for our main results.

Definition 2.1. [4, 12, 13] A mapping $\xi_b : \Sigma \times \Sigma \to R^+$, where Σ is a non empty set, is called a b-metric, if there exists a number σ such that for all $\kappa_1, \kappa_2, \kappa_3 \in \Sigma$

(*a*) $\xi_b(\kappa_1, \kappa_2) \geq 0$ and $\xi_b(\kappa_1, \kappa_2) = 0$ if and only if $\kappa_1 = \kappa_2$,

(*b*) $\xi_b(\kappa_1, \kappa_2) = \xi_b(\kappa_2, \kappa_1)$,

(*c*) $\xi_b(\kappa_1, \kappa_3) \leq \sigma[\xi_b(\kappa_1, \kappa_2) + \xi_b(\kappa_2, \kappa_3)]$.
Then the pair (Σ, ξ_b) is a b-metric space with parameter $\sigma \geq 1$. Clearly, every metric space is b-metric space with $\sigma = 1$, but the converse is not true in general. In fact, the class b-metric space is larger than the class of metric spaces.

Definition 2.2. [14] Let (Σ, ξ_b) be a b-metric space with $\sigma \geq 1$. Then a sequence $\{e_k\}$ in Σ is called
(i) a Cauchy sequence if for every $\delta > 0$, $\exists\, k_0 \in N$ such that $\xi_b(e_k, e_l) < \delta$ for all $k, l \geq k_0$.
(ii) convergent to $e \in \Sigma$ if for every $\delta > 0$, $\exists\, k_0 \in N$ such that $\xi_b(e_k, e) < \delta$ for all $k \geq k_0$.
(iii) The b-metric space(Σ, ξ_b) is complete if every Cauchy sequence in Σ converges in Σ.

Remark 2.1. In a b-metric space(Σ, ξ_b) the following assertions hold.
(1) A convergent sequence has a unique limit.
(2) Every convergent sequence is Cauchy.

Definition 2.3. [7] An element $(e, \lambda) \in \Sigma \times \Sigma$ is said to be a coupled fixed point of $\zeta : \Sigma \times \Sigma \to \Sigma$ if $e = \zeta(e, \lambda)$ and $\lambda = \zeta(\lambda, e)$.

Definition 2.4. [25] An element $(e, \lambda) \in \Sigma \times \Sigma$ is called a common coupled fixed point of $\zeta_1, \zeta_2 : \Sigma \times \Sigma \to \Sigma$ if
$e = \zeta_1(e, \lambda) = \zeta_2(e, \lambda)$ and $\lambda = \zeta_1(\lambda, e) = \zeta_2(\lambda, e)$.

Example 2.1. Let $X = \mathbb{R}$ and $\zeta_1, \zeta_2 : \Sigma \times \Sigma \to \Sigma$ defined as $\zeta_1(e, \lambda) = e\lambda$ and $\zeta_2(e, \lambda) = e + (\lambda - e)^2$ for all $e, \lambda \in \Sigma$. Then $(0, 0)$ and $(1, 1)$ are common coupled fixed points of ζ_1 and ζ_2.

Lemma 2.1. *[24] Suppose that (Σ, ξ_b) is a b-metric space with coefficient $\sigma \geq 1$ and $e : \Sigma \to \Sigma$ is a mapping and $\{e_k\}$ is a sequence in Σ induced by $e_{k+1} = \zeta_1 e_k$ such that $\xi_b(e_k, e_{k+1}) \leq \alpha\, \xi_b(e_{k-1}, e_k)$, for all $k \in N$, where $\alpha \in [0, 1)$ is a constant. Then $\{e_k\}$ is a Cauchy sequence.*

3 Main Results

In this section, we present main results and deliver theirs proofs.

Theorem 3.1. *Let (Σ, ξ_b) be complete b-metric space with $\sigma \geq 1$ and $\zeta_1, \zeta_2 : \Sigma \times \Sigma \to \Sigma$ be any two mappings satisfying the condition*

$$\xi_b(\zeta_1(e, \lambda), \zeta_2(\theta, v) \leq i\xi_b(e, \theta) + j \left[\frac{\xi_b(e, \zeta_1(e, \lambda))\xi_b(e, \zeta_2(\theta, v)) + \xi_b(\theta, \zeta_2(\theta, v))\xi_b(\theta, \zeta_1(e, \lambda))}{\xi_b(e, \zeta_2(\theta, v)) + \xi_b(\theta, \zeta_1(e, \lambda))} \right] \tag{3.1}$$

$\forall e, \lambda, \theta, v \in \Sigma$, where $i, j \geq 0, \sigma(i + j) < 1$ and $\xi_b(e, \zeta_2(\theta, v) + \xi_b(\theta, \zeta_1(e, \lambda)) \neq 0$.
Then ζ_1, ζ_2 have unique common coupled fixed point in Σ.

Proof. Suppose that, $(e_0, \lambda_0) \in \Sigma$ and define the sequence $\{e_{2k}\}$ and $\{\lambda_{2k}\}$ in Σ such that
$e_{2k+1} = \zeta_1(e_{2k}, \lambda_{2k}), \lambda_{2k+1} = \zeta_1(\lambda_{2k}, e_{2k})$
and
$e_{2k+2} = \zeta_1(e_{2k+1}, \lambda_{2k+1}), \lambda_{2k+1} = \zeta_2(\lambda_{2k+1}, e_{2k+1})$.
Then from (3.1), we have

$$\begin{aligned}
\xi_b(e_{2k}, e_{2k+1}) &= \xi_b[\zeta_1(e_{2k-1}, \lambda_{2k-1}), \zeta_2(e_{2k}, \lambda_{2k})] \\
&\leq i\xi_b(e_{2k-1}, e_{2k}) + j \left[\frac{xi_b(e_{2k-1}, e_{2k})\xi_b(e_{2k-1}, e_{2k+1}) + \xi_b(e_{2k}, e_{2k+1})\xi_b(e_{2k}, (e_{2k})}{\xi_b(e_{2k-1}, (e_{2k+1} + \xi_b(e_{2k}, (e_{2k})} \right] \\
&\leq (i + j)\xi_b(e_{2k-1}, e_{2k}).
\end{aligned}$$

This implies that
$\xi_b(e_{2k}, e_{2k+1}) \leq r\xi_b(e_{2k-1}, e_{2k})$, where $r = (i + j) < \frac{1}{\sigma}$.
By Lemma 2.1 $\{e_{2k}\}$ and $\{\lambda_{2k}\}$ are Cauchy sequence in Σ. Since Σ is a complete b-metric space, there exist $e, \lambda \in \Sigma$ such that $e_{2k} \to e$ and $\lambda_{2k} \to \lambda$, *i.e.*, $lim_{k\to\infty}\, e_{2k} = e$, $lim_{k\to\infty}\, \lambda_{2k} = \lambda$.
Now we will show that $e = \zeta_1(e, \lambda)$ and $\lambda = \zeta_1(\lambda, e)$, *i.e.*, we will show that (e, λ) is a common fixed point in Σ.

Now cosider

$$\begin{aligned}\xi_b(e,\zeta_1(e,\lambda)) &\leq \sigma[\xi_b(e,e_{2k})+\xi_b(e_{2k},\zeta_1(e,\lambda))]\\ &\leq \sigma[\zeta_2(e_{2k-1},\lambda_{2k-1},\zeta_1(e,\lambda]+\sigma\xi_b(e,e_{2k})\\ &\leq \sigma i\xi_b(e_{2k-1},e)+\sigma\left[j\frac{\xi_b(e,\zeta_1(e,\lambda)).\xi_b(e,\zeta_2(e_{2k-1},\lambda_{2k-1}))+\xi_b(e_{2k-1},\zeta_2(e_{2k-1},\lambda_{2k-1}))}{\xi_b(e,\zeta_2(e_{2k-1},\lambda_{2k-1}))+\xi_b(e_{2k-1},\zeta_1(e,\lambda))}\right]+\sigma\xi_b(e,e_{2k})\\ &= i\sigma\xi_b(e_{2k-1},e)+\sigma\left[j\frac{\xi_b(e,\zeta_1(e,\lambda).\xi_b(e,e_{2k}+\xi_b(e_{2k-1},e_{2k}.\xi_b(e_{2k-1},\zeta_1(e,\lambda)}{\xi_b(e,e_{2k})+\xi_b(e_{2k_1},\zeta_1(e,\lambda)}\right]+\sigma\xi_b(e,e_{2k}.\end{aligned}$$

Taking limit as $k\to\infty$, we have $\xi_b(e,\zeta_1(e,\lambda)=0\Rightarrow\zeta_1(e,\lambda)=e$. Similarly, we can prove that $\lambda=\zeta_1(\lambda,e)$. Now it follow similarly that $e=\zeta_2(e,\lambda)$ and $\lambda=\zeta_2(\lambda,e)$. Therefore, (e,λ) is a common coupled fixed point in Σ.
Now to prove the uniqueness of common coupled fixed point in $\Sigma\times\Sigma$.
Let (e^*,λ^*) be another common coupled fixed point of ζ_1,ζ_2. Then from (1) we get

$$\begin{aligned}\xi_b(e,e^*) &= \xi_b(\zeta_1(e,\lambda),\zeta_2(e^*,\lambda^*))\\ &\leq i\xi_b(e,e^*)+j\left[\frac{\xi_b(e,\zeta_1(e,\lambda)).\xi_b(e,\zeta_2(e^*,\lambda^*)+\xi_b(e^*,\lambda_2)),\xi_b(e^*,\zeta_1(e,\lambda))}{\xi_b(e,\zeta_2(e^*,\lambda^*)),\xi_b(e^*,\zeta_1(e,\lambda))}\right].\\ &= i\xi_b(e,e^*)+j\left[\frac{\xi_b(e,e)\xi_b(e,e^*)+\xi_b(e^*,e^*),\xi_b(e^*,e)}{\xi_b(e,e^*)+\xi_b(e^*,e)}\right].\end{aligned}$$

This implies that $(1-i)\xi_b(e,e^*)\leq 0$. Similarly, one can easily prove that $(1-i)\xi_b(\lambda,\lambda^*)\leq 0$. Now adding both equation, we get $(1-i)[\xi_b(e,e^*)+\xi_b(\lambda,\lambda^*)]\leq 0$, Implying that $\xi_b(e,e^*)+\xi_b(\lambda,\lambda^*)=0$. Therefore, $e=e^*$ and $\lambda=\lambda^*$.
Thus, (e,λ) is a unique common coupled fixed point of ζ_1 and ζ_2 of Σ. □

Example 3.1. Let $\Sigma=\{0,1\}$. Consider a b-metric $\xi:\Sigma\times\Sigma\to R$ defined as $\xi_b(e,\lambda)=|e-\lambda|^3$, for all $e,\lambda\in\Sigma$. Then (Σ,ξ_b) is a b-metric space with parameter $\sigma=4$. Define $\zeta_1,\zeta_2:\Sigma\times\Sigma\to\Sigma$ as follows

$$\zeta_1(e,\lambda)=\begin{cases}0 & for\quad e=\lambda=1\\ \frac{3}{4}, & for\quad 0\ otherwise\end{cases}=\zeta_2(e,\lambda)$$

for all $e,\lambda\in\Sigma$. It can be easily verified that the maps ζ_1 and ζ_2 satisfying all the conditions of Theorem 3.1 and $(\frac{3}{4},\frac{3}{4})$ are its unique common coupled fixed point in Σ. From Theorem 3.1 yields the following corollary by taking $\zeta_1=\zeta_2=\zeta$.

Corollary 3.1. *Let (Σ,ξ_b) be complete b-metric space with $\sigma\geq 1$ and $\zeta:\Sigma\times\Sigma\to\Sigma$ be any two mappings satisfying the condition*

$$\xi_b(\zeta(e,\lambda),\zeta(\theta,v)\leq i\xi_b(e,\theta)+j\left[\frac{\xi_b(e,\zeta(e,\lambda))\xi_b(e,\zeta(\theta,v))+\xi_b(\theta,\zeta(\theta,v)).\xi_b(\theta,\zeta(e,\lambda))}{\xi_b(e,\zeta(\theta,v))+\xi_b(\theta,\zeta(e,\lambda))}\right]$$

$\forall e,\lambda,\theta,v\in\Sigma$, where $i,j\geq 0,\sigma(i+j)<1$ and $\xi_b(e,\zeta(\theta,v)+\xi_b(\theta,\zeta(e,\lambda))\neq 0$.
Then ζ has unique coupled fixed point in Σ.

Theorem 3.2. *Let (Σ,ξ_b) be complete b-metric space with $\sigma\geq 1$ and $\zeta_1,\zeta_2:\Sigma\times\Sigma\to\Sigma$ be any two mappings satisfying the condition*

$$\begin{aligned}\xi_b(\zeta_1(e,\lambda),\zeta_2(\theta,v)\leq p\xi_b(e,\theta)&+q\left[\frac{\xi_b(\theta,\zeta_2(\theta,v))\{1+\xi_b(e,\zeta_1(e,\lambda))\}}{1+\xi_b(e,\theta)}\right]\\ &+r\left[\frac{\xi_b(\theta,\zeta_2(\theta,v))+\xi_b(\theta,\zeta_1(e,\lambda))}{1+\xi_b(\theta,\zeta_2(\theta,v)).\xi_b(\theta,\zeta_1(e,\lambda))}\right]\end{aligned}$$

$\forall e, \lambda, \theta, v \in \Sigma$, *where* $p, q, r \geq 0$ *and* $\sigma(p+q+r) < 1$. *Then* ζ_1, ζ_2 *have unique common coupled fixed point in* Σ.

Proof. Suppose that, $(e_0, \lambda_0) \in \Sigma$ and define the sequence $\{e_{2k}\}$ and $\{\lambda_{2k}\}$ in Σ such that
$e_{2k+1} = \zeta_1(e_{2k}, \lambda_{2k}), \lambda_{2k+1} = \zeta_1(\lambda_{2k}, e_{2k})$ and $e_{2k+2} = \zeta_1(e_{2k+1}, \lambda_{2k+1}), \lambda_{2k+1} = \zeta_2(\lambda_{2k+1}, e_{2k+1})$.
Then from (2), we have

$$\begin{aligned}
\xi_b(e_{2k}, e_{2k+1}) &= \xi_b[\zeta_1(e_{2k-1}, \lambda_{2k-1}), \zeta_2(e_{2k}, \lambda_{2k})] \\
&\leq p\xi_b(e_{2k-1}, e_{2k}) + q\left[\frac{\xi_b(e_{2k}, \zeta_2\xi_b(e_{2k}, \lambda))1 + \xi_b(e_{2k-1}, e\zeta_1(e_{2k-1}, \lambda_{2k-1}))}{1 + \xi_b(e_{2k-1}, e_{2k})}\right] \\
&\quad + r\left[\frac{\xi_b(e_{2k}, \zeta_2(e_{2k}, \lambda_{2k})) + \xi_b(e_{2k}, \zeta_1(e_{2k-1}, \lambda_{2k-1}))}{1 + \xi_b(e_{2k}, \lambda_{2k})).\xi_b(e_{2k}, \zeta_1(e_{2k-1}, \lambda_{2k-1}))}\right] \\
&= p\,\xi_b(e_{2k-1}, e_{2k}) + q\left[\frac{\xi_b(e_{2k}, e_{2k-1})1 + \xi_b(e_{2k-1}, (e_{2k-1}, e_{2k}))}{1 + \xi_b(e_{2k-1}, e_{2k})}\right] \\
&\quad + r\left[\frac{\xi_b(e_{2k}, e_{2k+1}) + \xi_b(e_{2k}, (e_{2k})}{1 + \xi_b(e_{2k}, e_{2k+1}).\xi_b(e_{2k}, (e_{2k})}\right]
\end{aligned}$$

This implies that
$\xi_b(e_{2k}, e_{2k+1}) \leq h\,\xi_b(e_{2k-1}, e_{2k})$, where $r = \frac{p}{1-q-r} < \frac{1}{\sigma}$. By Lemma 2.1, $\{e_{2k}\}$ and $\{\lambda_{2k}\}$ are Cauchy sequences in Σ. Since Σ is a complete b-metric space, there exist $e, \lambda \in \Sigma$ such that $e_{2k} \to e$ and $\lambda_{2k} \to \lambda$. *i.e.*, $lim_{k\to\infty}\, e_{2k} = e$, $lim_{k\to\infty}\, \lambda_{2k} = \lambda$.
Now we will show that $e = \zeta_1(e, \lambda)$ and $\lambda = \zeta_1(\lambda, e)$, *i.e.*, we will show that (e, λ) is a common fixed point in Σ.
Now cosider

$$\begin{aligned}
\xi_b(e, \zeta_1(e, \lambda)) &\leq \sigma[\xi_b(e, e_{2k}) + \xi_b(e_{2k}, \zeta_1(e, \lambda))] \\
&\leq \sigma[\zeta_2(e_{2k-1}, \lambda_{2k-1}, \zeta_1(e, \lambda] + \sigma\xi_b(e, e_{2k}) \\
&\leq \sigma p\xi_b(e_{2k-1}, e) + \sigma\left[q\frac{\xi_b(e_{2k-1}, \zeta_2(e_{2k-1}, \lambda_{2k-1}))1 + \xi_b(e, \zeta_1(e, \lambda))}{1 + \xi_b(e_{2k-1}, e))}\right] \\
&\quad + \sigma\left[r\frac{\xi_b(e_{2k-1}, \zeta_2(e_{2k-1}, \lambda_{2k-1})) + \xi_b(e_{2k-1}, \zeta_1(e, \lambda))}{1 + \xi_b(e_{2k-1}, \zeta_2(e_{2k-1}, \lambda_{2k-1})).\xi_b(e_{2k-1}, \xi_b(e, \lambda))}\right] + \sigma\,\xi_b(e, e_{2k}) \\
&= \sigma p\xi_b(e_{2k-1}, e) + \sigma\left[q\frac{\xi_b(e_{2k-1}, e_{2k})1 + \xi_b(e, e_{2k}, \zeta_1(e, \lambda)}{1 + \xi_b(e_{2k_1}, e)}\right] \\
&\quad + \sigma\left[r\frac{\xi_b(e_{2k-1}, e_{2k}) + \xi_b(e_{2k-1}, \zeta_1(e, \lambda))}{1 + \xi_b(e_{2k-1}, e_{2k}).\xi_b(e_{2k-1}, \zeta_1(e, \lambda))}\right] + \sigma\,\xi_b(e, e_{2k})
\end{aligned}$$

Since $\{e_{2k}\}$ and $\{\lambda_{2k}\}$ are convergent. So, by taking limit as $k \to \infty$, we get $\xi_b(e, \zeta_1(e, \lambda) = 0 \Rightarrow \zeta_1(e, \lambda) = e$. Similarly, we can prove that $\lambda = \zeta_1(\lambda, e)$. Now it follows similarly that $e = \zeta_2(e, \lambda)$ and $\lambda = \zeta_2(\lambda, e)$. Therefore, (e, λ) is a common coupled fixed point of ζ_1 and ζ_2 in Σ.
Now to prove the uniqueness of common coupled fixed point in $\Sigma \times \Sigma$.
Let (e^*, λ^*) be another common coupled fixed point of ζ_1, ζ_2. Then from (2) we get

$$\begin{aligned}
\xi_b(e, e^*) &= \xi_b(\zeta_1(e, \lambda), \zeta_2(e^*, \lambda^*)) \\
&\leq p\xi_b(e, e^*) + q\left[\frac{\xi_b(e^*, \zeta_2(e^*, \lambda^*))\{1 + \xi_b(e, \zeta_1(e, \lambda))\}}{1 + \xi_b(e, e^*)}\right] + r\left[\frac{\xi_b(e^*, \zeta_2(e^*, \lambda^*)) + \xi_b(e^*, \zeta_1(e, \lambda))}{1 + \xi_b(e^*, \zeta_2(e^*, \lambda^*)).\xi_b(e^*, \zeta_1(e, \lambda))}\right] \\
&= p\xi_b(e, e^*) + q\left[\frac{\xi_b(e^*, e^*)\{1 + \xi_b(e, e^*) + \xi_b(e, e)\}}{1 + \xi_b(e, e^*)}\right] + r\left[\frac{\xi_b(e^*, e^*) + \xi_b(e^*, e)}{1 + \xi_b(e^*, e^*).\xi_b(e^*, e)}\right] \\
&= (p + r)\xi_b(e, e^*).
\end{aligned}$$

Thus, $(1-p-r)\xi_b(e,e^*)=0$. Similarly, $(1-p-r)\xi_b(\lambda,\lambda^*)=0$.
Adding both equation$(1-p-r)[\xi_b(e,e^*)+\xi_b(\lambda,\lambda^*)]=0 \Rightarrow \xi_b(e,e^*)+\xi_b(\lambda,\lambda^*)=0$. Therefore, $e=e^*$ and $\lambda=\lambda^*$. Thus, (e,λ) is a unique common coupled fixed point of ζ_1 and ζ_2 of Σ. □

Corollary 3.2. *Let (Σ,ξ_b) be a complete b-metric space with $\sigma \geq 1$ and $\zeta:\Sigma\times\Sigma\to\Sigma$ be the mapping satisfying the condition*

$$\xi_b(\zeta(e,\lambda),\zeta(\theta,v) \leq p\xi_b(e,\theta)+q\left[\frac{\xi_b(\theta,\zeta(\theta,v))\{1+\xi_b(e,\zeta(e,\lambda))\}}{1+\xi_b(e,\theta)}\right]$$
$$+r\left[\frac{\xi_b(\theta,\zeta(\theta,v))+\xi_b(\theta,\zeta(e,\lambda))}{1+\xi_b(\theta,\zeta(\theta,v)).\xi_b(\theta,\zeta(e,\lambda))}\right]$$

$\forall e,\lambda,\theta,v\in\Sigma$, where $p,q,r\geq 0$ and $\sigma(p+q+r)<1$. Then ζ has unique coupled fixed point in Σ.

Proof. From Theorem 3.2, we take $\zeta_1=\zeta_2=\zeta$ and get the above corollary. □

4 Conclusion

The theory of fixed points is a very amusing technique for contractive mapping. A number of authors have defined contractive type mapping in numerous directions. In this work, we found two main results, which are given in Theorem 3.1 and Theorem 3.2. with Corollaries 3.1 and 3.2, respectively. The present Theorem 3.1 and Theorem 3.2 extend, modify, and generalize the results of the Theorems 2.2 and 2.3 of the literature [26].

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