Application of Fuzzy Logic Through Bellmen-Zadeh Maximin Method

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Abstract: *Fuzzy Logic is a multiple valued logic, from which the truth values of variables might be any real numbers between 0 and 1. It has wide application in different fields of real life. We might face different challenges in our daily life to select the best of the best for the concrete result. There are various criteria in the literature for ranking alternatives in the realm of decision-making under uncertainty in a crisp environment. In this paper, we address the challenges seen in making a decision for selecting the best of the best as studied by Bellmen-Zadeh maximin method. It is used to address such challenges using fuzzy real numbers.*

Keywords: Fuzzy logic, Membership function, Fuzzy number, Decision problem, Alternatives

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1 Introduction

Fuzzy logic is a multiple-valued logic form, in which the truth values of variables might be any real number between 0 and 1 [16]. It is a strategy for dealing with linguistic variables and describing modifiers such as vary, fairly, and not, among others. It aids common sense reasoning with imprecise and ambiguous propositions in natural language and acts as a foundation for decision analysis and control action.

Lots of the applications of fuzzy logic can be found in various sectors of the real world. Fuzzy logic has been studied and investigated with many results in economic fields. Stojic [14] presented a model for evaluating the level of economic development of countries and regions using fuzzy logic. In 2016, to obtain optimal solution, Kripa and Govindarajan [10] proposed various strategies for solving fuzzy sequencing problems with trapezoidal fuzzy numbers. In 2017, Sahoo [13] proposed a solution procedure to solve the fuzzy job sequencing problem, in which processing time is represented as a trapezoidal fuzzy number, and Yager's Ranking Index method is used to convert the fuzzy processing time into crisp ones, in which the optimal job solution (order) and ideal time for each machine are determined. Leelavathy and Kowsalya [8] suggested a new fuzzy arithmetic operation and ranking algorithm in 2019 to find an ideal sequence for fuzzy sequencing problem using trapezoidal fuzzy number. Making a decision is a problem-solving process that leads to a certain action and is a choice between numerous methods for achieving a goal [1]. In business, finance, management, economics, social and political science, engineering and computer science, biology and medical science, decision-making plays a vital role. Due to elements such as insufficient and inaccurate information, subjectivity, and language, which tend to be present in real-life circumstances to varying degrees, it is a tough procedure. These characteristics suggest that a decision-making process occurs in a fuzzy environment. According to Bellman–Zadeh [1], decision making is defined as the intersection of goals and constraints expressed by fuzzy sets. A decision process in a fuzzy environment is one in which the goals and constraints are both fuzzy characters. This means that the goal and constraints define classes of alternatives whose boundaries are not clearly defined. John and Sunny [7] conducted a research in 2011 on decision making in a fuzzy environment employing preference relations and comparative uncertainty, in which the probabilities of natural states are unknown a priori. There are various criteria in the literature for ranking alternatives in the realm of decision-making under uncertainty in a crisp environment. Jianping and et al. [9], in 2021 designed a method that is connected to the green supplier selection(GSS) and conducted some comparative analysis to illustrate the designed method's superiority. The best alternative can be chosen based on the final score calculated by combining the weights of various sets of criteria and alternatives. In 2021, Hussain and et al. [6] suggested a consistency ratio to measure the dependability of the fuzzy technique's best-worst analysis results and conducted two case studies to validate the practicality and consistency and results revealed that the proposed model can be used to determine the best green supplier. Also Chen and Ye [4] constructed a multi-attribute decision-making method based on the single-valued neutrosophic Dombi weighted geometric average in a single-valued neutrosophic number environment, and used an example of an investment selection problem to show how the method may be used and is viable. Fuzzy logic and fuzzy set theory are not only used in applicable fields, also used in pure and abstract mathematics. Paudel and Pahari [11] used the concept of fuzzy logic and fuzzy set to study the topological properties of metric space in 2021. Similarly, in 2022, Paudel and et al. [12] used the fuzzy real numbers to study the generalized form of p-bounded variation of fuzzy real numbers. In this paper, we discuss how we can select the best person from a set of persons of the same environment using the Bellman-Zadeh method in decision making.

The paper is structured as follows: in section 2, we review some background material on fuzzy set theory, triangular fuzzy numbers, decision making as a concept, and Bellman-Zadeh decision-making process. Through numerical examples, the Bellman-Zadeh decision-making method will be explained in Section 3.1 and 3.2. The paper is concluded in Section 4.

2 Preliminaries and Notations

Before proceeding with the main results, we now give some definitions and preliminaries that are used in our work.

Let U be universe of discourse and let $X \subseteq U$. A *fuzzy set* [15] in U is defined as the collection of order pairs (x, μ_X) where, $\mu_X : U \to [0,1]$ and $x \in U$, so that the fuzzy set X is defined as

$$X = \{(x, \mu_X(x)) : x \in U \text{ and } \mu_X : U \to [0,1]\}$$

Here, $\mu_X(x)$ is the degree of the element x and μ_X and is called *membership function*. Here, $\mu_X(x) = 1$ means x is fully included in the set X and $\mu_X(x) = 0$ means x in not included in X.

The maximum value of the membership function is known as the *height of the fuzzy set* and fuzzy set having height one is a *normal fuzzy set*.

A fuzzy set X in U is called *convex* [5] if for all $x_1, x_2 \in X$ and $\lambda \in [0,1]$

$$\mu_X(\lambda x_1 + (1-\lambda)x_2) \geq \text{minimum}[\mu_X(x_1), \mu_X(x_2)]$$

A fuzzy set X is a *fuzzy number* [2] if it is convex, normalized and membership function is piecewise continuous.

Let a, b, c be three real numbers with $a \leq b \leq c$. Then a number of the form $X = (a, b, c)$ is a *triangular fuzzy number* [3] with the membership function $\mu_X(x)$ defined by

$$\mu_X(x) = \begin{cases} \frac{x-a}{b-a}, & \text{if } x \in [a,b], \\ \frac{c-x}{c-b}, & \text{if } x \in (b,c], \\ 0, & \text{otherwise.} \end{cases}$$

In the above definition, we have $\mu_X(b) = 1$, while b need not to be the mid-point of a and c.

The *intersection* $X \cap Y$ of two fuzzy sets X and Y is defined as the largest fuzzy set contained in both X and Y and its membership function is defined by

$$\mu_{X \cap Y(x)} = min\{\mu_X(x), \mu_Y(x)\} \text{ for } x \in U.$$

It is also written as

$$\mu_{X \cap Y}(x) = \mu_X(x) \wedge \mu_Y(x)$$

.

And, the *union* $X \cup Y$ is defined as the smallest fuzzy set containing both sets X and Y and its membership function is defined by

$$\mu_{X\cup Y(x)} = max\{\mu_X(x), \mu_Y(x)\} \text{ for } x \in U.$$

We also write

$$\mu_{X\cup Y(x)} = \mu_X(x) \vee \mu_Y(x).$$

Decision-making is the process of making an important selection. In other words, the process of determining a course of action, an option, or a set from individual performances relative to the potential ions is called decision-making. So, in order to arrive at a solution to a particular problem, decision making entails choosing a course of action from two or more feasible alternatives. A decision process in a fuzzy environment is one in which the goals and constraints are both fuzzy in character. In view of the work studied in [6, 7, 9], the main components of the decision-making process in the traditional approach to decision making are:

i. a set of alternatives.

ii. a set of constraints on the choosing of distinct alternatives.

iii. a performance function that correlates the benefit or result of each alternative's selection with that alternative.

A decision-maker is an individual, group of individuals, or organization that has the necessary opportunity to choose between different options. *A decision problem* is a situation in which the decision-maker has to make a decision. *Alternatives* are an alternative choice open to the decision-maker that is under the control of the decision maker. In 1970, Bellman–Zadeh [1], defined decision-making as the intersection of goals and constraints expressed by fuzzy sets. In a simple decision-making problem, if a fuzzy set G represents the goal and a fuzzy set C represents the constraints, with membership functions $\mu_G(x)$ and $\mu_C(x)$ respectively, then the decision set D is $G \cap C$ and its membership function is

$$\mu_D(x) = \mu_G(x) \wedge \mu_C(x).$$

More generally, suppose $G_1, G_2, G_3, \cdots, G_n$ be n goals and $C_1, C_2, C_3, \cdots, C_m$ be m constraints. Then the result decision is the intersection of given goals and constraints, i. e.,

$$D = G_1 \cap G_2 \cap G_3 \cap \cdots \cap G_n \cap C_1 \cap C_2 \cap C_3 \cap \cdots \cap C_m.$$

And, the corresponding membership function is

$$\mu_D = \mu_{G_1} \wedge \mu_{G_2} \wedge \mu_{G_3} \wedge \cdots \wedge \mu_{G_n} \wedge \mu_{C_1} \wedge \mu_{C_2} \wedge \mu_{C_3} \wedge ... \wedge \mu_{C_m}$$

.

Thus, decision is the confluence of goal and constraints.

Suppose a reputed corporation posts a job posting for a vacant position. Candidates $x_1, x_2, \cdots, x_n$ apply for the available position. They form a discrete set of alternatives, say, $\{x_1, x_2, x_3, \cdots, x_n\}$. The selection committee looks for certain attributes in candidates, such as experience, youth, competence, and so on in specific field, which is referred to as goals, $G_i, i = 1, 2, 3, \cdots, m$. The selection committee also intended to set some limits, such as low pay, which can be considered constraints, $C_j, j = 1, 2, \cdots, p$. At the conclusion of the interviewing process, each candidate's x_k is graded on a scale of 0 to 1 with the help of a membership function based on their goals and constraints. The grade of the candidate x_k for the goals G_i are denoted by g_{ki}, and the candidate's grades for the constraints C_j are denoted by c_{kj}. Then, the fuzzy sets corresponding to goals G_i and constraints C_j are

$$G_i = \{(x_1, g_{1i}), (x_2, g_{2i}), (x_3, g_{3i}), \cdots, (x_n, g_{ni})\}, i = 1, 2, \cdots, m,$$

and

$$C_j = \{(x_1, c_{1j}), (x_2, c_{2j}), (x_3, c_{3j}), \cdots, (x_n, c_{nj})\}, j = 1, 2, 3, \cdots, p.$$

According to Bellman-Zadeh[1], the decision is fuzzy set is

$$D = G_1 \cap G_2 \cap G_3 \cap \cdots \cap G_m \cap C_1 \cap C_2 \cap C_3 \cap \cdots \cap C_p$$

with grade

$$\mu_D = \mu_{G_1} \wedge \mu_{G_2} \wedge \cdots \wedge \mu_{G_n} \wedge \mu_{C_1} \wedge \mu_{C_2} \wedge \cdots \wedge \mu_{C_m},$$

i.e,

$$\mu_l = min\{g_{l1}, g_{l2}, \cdots, g_{lm}, c_{l1}, c_{l2}, \cdots, c_{lp}\}, \quad 1 \leq l \leq n.$$

3 Applications

Here are some examples of how the Bellmen-Zadeh method can be used to measure excellence for selecting best candidates.

3.1 Application of Bellmen-Zadeh method in measuring excellency

Suppose an educational institute has declared to provide scholarship to support higher study. Excellent result in different subjects: S_1, S_2, S_3, S_4 and S_5 is the major factor in selecting students for getting scholarships. But the term excellent creates a fuzzy environment and makes it difficult to select a student in a crisp environment. So, fuzzy logic is more suitable here. The marks obtained by the five students in different subjects are given below:

Studets	Subjects				
	S_1	S_2	S_3	S_4	S_5
A	90	93	94	87	88
B	92	89	93	86	96
C	91	97	88	92	84
D	88	97	90	91	90
E	95	92	86	90	92

Table 1: Marks of students.

The obtained marks are out of 100 and an excellent result is considered when the marks are more than 80. So the domain of the marks is [80, 100]. To select a student for a scholarship, we use the Bellmen-Zadeh maximin method [1] by converting the crisp set of obtained marks into a fuzzy set. For this we need the corresponding membership values of marks, which is obtained by the following membership function

$$\mu_X(x) = \begin{cases} 0, & \text{for } 0 \leq x < 80, \\ \frac{x-80}{90-80}, & \text{for } 80 \leq x < 90, \\ 1, & \text{for } 90 \leq x \leq 100. \end{cases}$$

Using the membership function defined above, we calculate the grading value of the marks obtained by the students in different subjects

$$\mu_A(90) = 1, \mu_A(93) = 1, \mu_A(94) = 1, \mu_A(87) = 0.7, \mu_A(88) = 0.8$$
$$\mu_B(92) = 1, \mu_B(89) = 0.9, \mu_B(93) = 1, \mu_B(86) = 0.6, \mu_B(96) = 1$$
$$\mu_C(91) = 1, \mu_C(97) = 1, \mu_C(88) = 0.8, \mu_C(92) = 1, \mu_C(84) = 0.4$$
$$\mu_D(88) = 0.8, \mu_D(97) = 1, \mu_D(90) = 1, \mu_D(91) = 1, \mu_D(92) = 1$$
$$\mu_E(95) = 1, \mu_E(92) = 1, \mu_E(86) = 0.6, \mu_E(90) = 1, \mu_E(92) = 1$$

Students	Subjects				
	S_1	S_2	S_3	S_4	S_5
A	1	1	1	0.7	0.8
B	1	0.9	1	0.6	1
C	1	1	0.8	1	0.4
D	0.8	1	1	1	1
E	1	1	0.6	1	1

Table 2: Grading marks.

The fuzzy set corresponding to the marks obtained in subjects S_1, S_1, S_3, S_4, and S_5 respectively are

$$\begin{aligned}
G_1 &= \{(A,1),(B,1),(C,1),(D,0.8),(E,1)\}\\
G_2 &= \{(A,1),(B,0.9),(C,1),(D,1),(E,1)\}\\
G_3 &= \{(A,1),(B,1),(C,0.8),(D,1),(E,0.6)\}\\
G_4 &= \{(A,0.7),(B,0.6),(C,1),(D,1),(E,1)\}\\
G_5 &= \{(A,0.8),(B,1),(C,0.4),(D,1),(E,1)\}
\end{aligned}$$

The selection committee has to give their decision for excellency by using Bellmen-Zadeh [1] method,

$$\begin{aligned}
D &= G_1 \cap G_2 \cap G_3 \cap G_4 \cap G_5\\
&= \{(A,0.7),(B,0.6),(C,0.4),(D,0.8),(E,0.6)\}
\end{aligned}$$

By Max-min method, we conclude that D has best preference with membership value of 0.8.

3.2 Application for selecting best candidates

Suppose an organization has to fill a post, for which five candidates, say A, B, C, D and E are shortlisted. These candidates form a set of alternatives, say $\{A, B, C, D, E\}$. The selection committee looks for certain attributes in candidates, such as computer skills, experience in years, academic qualifications, and youthfulness. These qualities can be considered the goal of the problem. The domain of experience is [5, 10]. The selection committee has taken an exam for computer skills and with domains [30, 40]. Also, the organization imposed another condition that the salary should be moderated from Rs 30,000 to Rs 40,000, and this can be taken as a constraint. The marks obtained in computer skills, experience, scored percentage at bachelor level, and age in years are given below:

Candidates	Experience(E)	Computer skill(C)	Qualification(Q)	Youngness(Y)
A	8	37	76	29
B	6	38	79	26
C	9	33	75	33
D	5	38	72	25
E	7	35	75	32

Table 3: Result in different sectors.

The shortlisted candidates who are ready to work for domain [30000 , 40000] of salary and their response for it is:

Candidates	A	B	C	D	E
Salary (Rs.)	37000	38000	36000	35000	39000

Table 4: Salary in Rs.

The corresponding membership value for each components E, C, Q and Y are respectively calculated by defining the functions

$$\mu_E(x) = \begin{cases} 0, & \text{for} \quad 0 \le x < 4, \\ \frac{x-4}{10-4}, & \text{for} \quad 4 \le x < 10, \\ 1, & \text{for} \quad 10 \le x \ ; \end{cases}$$

$$\mu_C(x) = \begin{cases} 0, & \text{for} \quad 0 \le x < 30, \\ \frac{x-30}{40-30}, & \text{for} \quad 30 \le x < 40, \\ 1, & \text{for} \quad x \ge 40 \ ; \end{cases}$$

$$\mu_Q(x) = \begin{cases} 0, & \text{for} \quad 0 \le x < 70, \\ \frac{x-70}{80-70}, & \text{for} \quad 70 \le x < 80, \\ 1, & \text{for} \quad x \ge 80 \ ; \end{cases}$$

$$\mu_Y(x) = \begin{cases} 0, & \text{for} \quad 0 \le x < 20, \\ \frac{35-x}{35-20}, & \text{for} \quad 20 \le x < 35, \\ 0, & \text{for} \quad x \ge 35. \end{cases}$$

The membership values in different sectors using are

Candidates	E	C	Q	Y
A	0.67	0.7	0.6	0.4
B	0.33	0.8	0.9	0.6
C	0.83	0.3	0.5	0.13
D	0.16	0.8	0.2	0.83
E	0.5	0.5	0.5	0.2

Table 5: Membership values.

The fuzzy set corresponding to the marks obtained in sectors E, C, Q and Y respectively are

$$\begin{aligned} G_1 &= \{(A, 0.67), (B, 0.33), (C, 0.83), (D, 0.16), (E, 0.5)\}, \\ G_2 &= \{(A, 0.7), (B, 0.8), (C, 0.3), (D, 0.8), (E, 0.5)\}, \\ G_3 &= \{(A, 0.6), (B, 0.9), (C, 0.5), (D, 0.2), (E, 0.5)\}, \\ G_4 &= \{(A, 0.4), (B, 0.6), (C, 0.13), (D, 0.67), (E, 0.2)\}. \end{aligned}$$

Also to calculate the grading value for the salary, let us define membership function as

$$\mu_S(x) = \begin{cases} 1, & \text{for} \quad 0 \le x < 30000 \\ \frac{40000-x}{40000-30000}, & \text{for} \quad 30000 \le x < 40000 \\ 0, & \text{for} \quad x \ge 40000 \end{cases}$$

And the fuzzy set corresponding to salary is

$$C = \{(A, 0.3), (B, 0.2), (C, 0.4), (D, 0.5), (E, 0.1)\}.$$

The selection committee has to give their decision for the best candidate. For this, we use Bellmen-Zadeh [1] method.

$$\begin{aligned} \text{Fuzzy decision } (D) &= G_1 \cap G_2 \cap G_3 \cap G_4 \cap C \\ &= \{(A, 0.3), (B, 0.2), (D, 0.13), (E, 0.1)\}. \end{aligned}$$

Then by Maximin method, A with the largest grade 0.3 is selected for the vacant post.

4 Conclusions

Making a decision is a problem-solving process that leads to a certain action. It is a choice between numerous methods for achieving a goal. In this paper, we discussed how we can select the best person from a set of people of the same environment using the Bellman-Zadeh method of decision making using maximin method. Also, we examined how to select the best of the best for a specific outcome.

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References

[1] Bellman, R. E., and Zadeh, L. A., 1970, Decision-making in a fuzzy environment, *Management science*, 17(4), B-141.

[2] Baruah A. and Tripathy B. C., 2010, Norlund and Riesz mean of sequences of fuzzy real numbers, *Applied Mathematics Letters*, 23, 651-655.

[3] Cheng, C. B., 2004, Group opinion aggregation based on a grading process: A method for constructing triangular fuzzy numbers, *Computers and Mathematics with Applications*, 48(10-11), 1619-1632.

[4] Chen, J. and Ye, J., 2017, Some single-valued neutrosophic dombi weighted aggregation operators for multiple attribute decision-making, *Symmetry*, 9(6), 82.

[5] Das, P. C., 2018, Fuzzy normed linear sequence space , *Proyecciones (Antofagasta, On line)*, 37(2), 389-403.

[6] Hussain, A., Chun, J. and Khan, M., 2021, A novel multi-criteria decision making (MCDM) approach for precise decision making under a fuzzy environment, *Soft Computing*, 25(7), 5645-5661.

[7] John, T. M. and Kuriakose, S., 2011, Decision making under uncertainty in a fuzzy environment, *International Journal of Fuzzy Mathematics and Systems*, 1(3), 273-279.

[8] Leelavathy T. and Kowsalya M., 2019, Fuzzy sequencing problem with uncertain processing time, *Journal of Physics,* Conf. Ser 1377 012045.

[9] Lu, J., Zhang, S., Wu, J. and Wei, Y., 2021, COPRAS method for multiple attribute group decision making under picture fuzzy environment and their application to green supplier selection, *Technological and Economic Development of Economy*, 27(2), 369-385.

[10] Kripa K. and Govindarajan R., 2016, Fuzzy sequencing problem using generalized triangular fuzzy numbers, *International Journal of Engineering Research and Application*, 5, 61-64.

[11] Paudel, G. P. and Pahari, N. P., 2021, On fundamental properties in fuzzy metric space, *Academic Journal of Mathematics Education*, 4(1), 20-25.

[12] Paudel, G. P., Pahari, N. P. and Kumar, S., 2022, Generalized form of p-bounded variation of sequences of fuzzy real numbers, *Pure and Applied Mathematics journal*, 11(3), 47-50.

[13] Sahoo L., 2017, Solving Job Sequencing Problems with Fuzzy Processing Times, *IJARIIE-ISSN(O)*, 4.

[14] Stojic, G., 2011, Using fuzzy logic for evaluating the level of countries economic development, *PANOECONOMICUS,* 59(3), 293-310.

[15] Yuan, G. J., 2018, Fuzzy Logic Sets and Fuzzy Logic, Theory and Applications, New Delhi; Pearson.

[16] Zadeh, L. A., 2008, Is there a need for fuzzy logic?, *Information Sciences*, 178(13), 2751-2779.