Temperature Variation in Breast Tissue Model With and Without Tumor Based on Porous Media

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Abstract: *The human body is made by* 200 *different types of cells, which are separated by voids. Blood supplies the nutrients and minerals to all cells within the tissue through these voids. The breast tissue is treated as a porous media in the study. Tumor includes the vascular (blood) and the extra-vascular (solid) regions. The porosity of a tumor is higher than normal tissue. The present work deals with the temperature variation of normal and tumorous breast tissue based on porous media. The finite element method is used to solve the two-dimensional bio-heat equation. The results show that the temperature profile of normal breast tissue in the porous media model is almost identical with the conventional bio-heat model at correction factor is equal to* 0.6*. The temperature of tumor region in the porous media model is slightly lower than the conventional bio-heat model. When the porosity is increased, the temperature of normal breast tissue is increased. But in tumorous breast tissue, the temperature is slightly increased in skin surface to anterior part of the tumor and slightly decreased in tumor region. The temperature of normal and tumorous breast tissue is increased when metabolism, blood velocity, and room temperature are increased in the porous media model. The central temperature of the tumor region reaches a steady state faster than anterior and posterior temperature of both normal and tumorous breast tissue in conventional bio-heat model and porous media model.*

Keywords: Breast Tissue, Temperature variation, Tumor, Porous media, Finite element method

DOI: https://doi.org/10.3126/jnms.v4i1.37116

1 Introduction

The human body is made by 200 different types of cells [12]. The living tissue is an organization of similar cells supported by intercellular structures. When blood passes through the capillaries, it fills in the intercellular spaces between the tissue cells [4]. These intercellular spaces are known as voids (pores). The porous medium is the material containing a solid matrix with an interconnected void space consisting of a fluid phase. It is characterized by its porosity, which is defined as ratio of the void space to the total volume of the medium. Biological tissues are the group of dispersed cells, which are separated by voids. So, they can be treated as a porous medium. Blood supplies the nutrients and minerals to all cells within the tissue through the voids [8]. The compound matrix of the tissue and the blood vessel (arteries, veins, and capillary tubes) are considered as a porous medium [32], which is separated into two phases: solid and fluid. Breasts are made by fatty and fibrous connective tissue, glandular tissue, blood vessels, and lymph system [5]. So, breast tissue is treated as a porous media in the study.

Breast tumor is caused by uncontrolled and abnormal cell division in the breast tissue. Tumor includes the vascular(blood) and the extra-vascular(solid) regions. Due to the increased number of cells in both phases, the porosity in a tumor is higher than normal. So, the tumor is assumed highly porous organ [28]. Due to the different physiological conditions, different organs have different porosity. The liver and kidney have porosity about 0.3 and 0.35 respectively, and the porosity of the brain varies from negligible value $0.03 - 0.05$ to high as liver [28]. It is found from the literature that the porosity of tumor varies from $0.2 - 0.7$ [6, 28] for different size of tumors. It is also found that the higher tumor porosity corresponds to higher blood velocity [6].

In 1948, the Pennes bio-heat model [20] was initially developed for predicting heat transfer in the human

forearm based on his experimental results. From several decades, the porous media model has been applied to analyze heat transfer in biological tissues. In 1980 Chen and Holmes [2] developed a microvascular model, which was based on the porous media. Xuan and Roetzel [32] developed a bio-heat model of the human thermal system based on the heat transfer principle in porous media. Khaled and Vafai [8] reviewed the model based on porous media related to modeling flow and heat transfer in biological tissue. Nakayama and Kuwahara [14] presented a rigorous mathematical development of bio-heat transfer model in porous media based on volume averaging theory. Yang et al. [33] used the finite element method in the porous model of human forearm for heat transfer analysis. They found that when the porosity, metabolic heat generation, and environment temperature are increased, the temperature of muscle is increased. Majchrzak and Turchan[10], Tucci et al.[28] described the heat transfer equation in porous media model for simulating a biological tissue in hyperthermia therapy.

Many researchers studied the bio-heat transfer model in porous media. Some of them studied in cancer treatment as hyperthermia therapy. The present work aims to study the temperature variation of normal and tumorous breast tissue in porous media using the two-dimensional finite element method.

2 Methodology

2.1 Discretization

The geometry of female breast is hemispherical in shape, whose radius is assumed to be 72 mm [11, 15, 17], [23, 25]. The schematic diagram of two-dimensional element wise discretization of breast tissue with tumor is shown in Figure 1. The breast is divided into five layers: epidermis, dermis, subcutaneous tissue, glandular layer and muscle with thoracic wall, which are symmetrical about the central line (X-axis). The thickness of epidermis, dermis, subcutaneous, glandular, and muscle with thoracic wall are assumed to be 1.5 mm, 2 mm, 1.5 mm, 45 mm and 22 mm, respectively [23, 25].

During the development of female breast, the size of lobules (milk glands) and ducts are increased. The sex

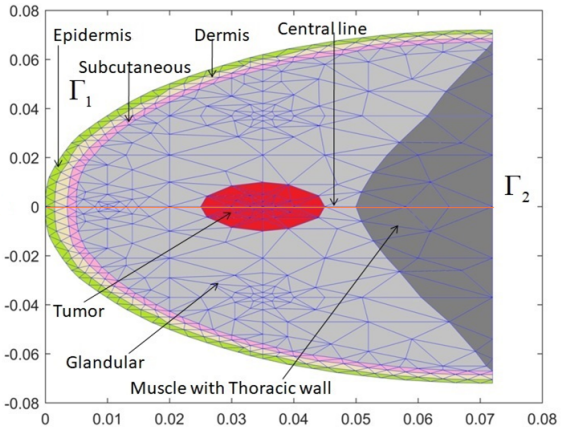

Figure 1: Schematic diagram of vertical cross-section area of the breast tissue from areola to the thoracic wall.

hormones (estrogen and progesterone) of female body are the major factor of increased cell division [13] in the breast. The damage of cell division causes tumors. Tumors may be benign or malignant (cancerous). Most of the breast tumor develops in milk ducts and lobules of the glandular layer. So, the tumor is assumed at the glandular layer in the central line of the breast. The size of a tumor is assumed to be 20 mm diameter with center $(35, 0)$. For numerical solutions, the finite element method is used in the study. The whole domain of the breast is divided into 862 triangular finite elements. The epidermis, dermis, subcutaneous, glandular, tumor, and muscle with the thoracic wall are divided into 128, 128, 130,

382, 52, and 42 triangular finite elements, respectively. The numerical solution is based on the finite element method.

2.2 Mathematical Formulation

The temperature variation of human body is related to heat transfer model [32]. Mathematical models of heat transfer in biological tissue is formulated as conventional bio-heat model and porous media model.

2.2.1 Conventional bio-heat model

The Pennes bio-heat model is one of the earliest model of heat transfer, which describes the energy balance equation between blood perfusion and metabolism in the living tissue. The simplified form of Pennes bio-heat equation [20] is:

$$\underbrace{\rho_t c_t \frac{\partial \Phi}{\partial t}}_{\text{Heat Storage}} = \underbrace{\text{div}(k_t \text{ grad } \Phi)}_{\text{Conduction}} + \underbrace{w_b \, c_b \, \rho_b(\Phi_B - \Phi)}_{\text{Blood Perfusion}} + \underbrace{m}_{\text{Metabolism}} \tag{1}$$

where, ρ_t is density of tissue $[kg/m^3]$, c_t is specific heat of tissue $[J/kg^oC]$, k_t is thermal conductivity of tissue $[W/m\ ^\circ C]$, w_b is volumetric blood perfusion rate per unit volume $[s^{-1}]$, c_b is specific heat of the blood $[J/kg\ ^\circ C]$, ρ_b is density of the blood $[kg/m^3]$, Φ_B is arterial blood temperature $[^\circ C]$, Φ is local temperature of tissue $[^\circ C]$, m is metabolic heat generation rate of healthy tissue $[W/m^3]$.

2.2.2 Porous media model

The biological tissues are treated as porous media since they are composed of dispersed cells separated by a connected void. The blood supplies nutrients and minerals to all cells through these voids within the tissue [8]. The porous tissue model has two phases: fluid phase (vascular) and solid phase (extra-vascular) [8, 14, 32]. The heat transfer equations for fluid phase and solid tissue phase [8, 32] are:
For fluid phase (blood phase),

$$p\rho_b c_b \left(\frac{\partial \Phi^b}{\partial t} + v_b \cdot \nabla \Phi^b \right) = \nabla \cdot (p k_b \nabla \Phi^b) + h(\Phi^b - \Phi^t) \tag{2}$$

For solid tissue phase,

$$(1-p)\rho_t c_t \frac{\partial \Phi^t}{\partial t} = \nabla \cdot ((1-p) k_t \nabla \Phi^t) + h(\Phi^t - \Phi^b) + (1-p)m \tag{3}$$

where, p is porosity (ratio of vascular volume to total volume), which is generally less than 0.1 [8, 14], Φ^b is local arterial blood averaged temperature $[^\circ C]$, Φ^t is local tissue averaged temperature, v_b is blood velocity, k_b is effective thermal conductivity of blood, k_t is effective thermal conductivity of tissue, h is local volumetric heat transfer coefficient [32]. Metabolic heat generation term does not appear in blood phase since it occurs only in the tissue [8].
For the thermal equilibrium condition, $\Phi^b = \Phi^t = \Phi$ [10] The combined (2 and 3) form of heat transfer equations in porous tissue model is:

$$C \, \frac{\partial \Phi}{\partial t} = \nabla \cdot (k \, \nabla \Phi) - H + (1-p)m \tag{4}$$

where, $C = (1-p)\rho_t c_t + p\rho_b c_b$, $k = (1-p)k_t + pk_b$ and $H = p\rho_b c_b v_b \cdot \nabla \Phi^b$.
Here, H represents the blood perfusion term. Pennes [20] considered the heat exchange between blood and tissue takes place in the capillary bed. The blood is supplied to the capillary by arterioles and drain out by venule. In the Pennes assumption, the blood perfusion term H is replaced by $-\rho_b c_b w_b(\Phi_B - \Phi)$. Khaled and Vafai [8] used the perfusion term $H = -\beta \, \frac{p\rho_b c_b (v_b)_{avg}}{\delta} \, (\Phi_B - \Phi)$ on the study of heat transfer in biological tissue based on porous media model. Where, $(v_b)_{avg}$ is average blood velocity within blood capillaries $[m/s]$, and δ is average spacing $(0.5 - 1.0\ mm$ [29, 30]) between the transverse blood vessels, β

is correction factor needs to be multiplied in perfusion term of pennes bio-heat equation, which vary from $0.6 - 0.7$ for muscle tissue [8, 30]. In breast tissue, the perfusion rate of glandular layer and muscle layer are similar [15]. So, we used $\beta = 0.6$ in our study. The heat transfer equations in porous media (4) can be written as:

$$C\,\frac{\partial \Phi}{\partial t} = \nabla \cdot (k\,\nabla \Phi) + \beta\,\frac{p\rho_b c_b (v_b)_{avg}}{\delta}\,(\Phi_B - \Phi) + (1-p)m \tag{5}$$

2.3 Boundary Condition

2.3.1 Boundary condition on external part

The heat is lost by convection, radiation, and sweat evaporation from outer surface of the body. Skin is the outer surface of the breast, which is assumed to be exposed to the environment. So, the total heat flux in this boundary is calculated by mixed boundary condition:

$$\Gamma_1 : (1-p)k_t\frac{\partial \Phi^t}{\partial \eta} + pk_b\frac{\partial \Phi^b}{\partial \eta} = h_{cr}(\Phi - \Phi_\infty) + S$$

At the boundary, $p = 0$ since the thickness of most superficial skin surface is very thin and approaches to zero [32]. So,

$$\Gamma_1 : k_t\frac{\partial \Phi}{\partial \eta} = h_{cr}(\Phi - \Phi_\infty) + S \tag{6}$$

where, η is the normal direction to the surface boundary, h_{cr} is heat transfer coefficient due to convection and radiation $[W/m^2\,{}^\circ C]$, Φ_∞ is room temperature $[^\circ C]$, $S = LE$, E is sweat evaporation rate $[kg/m^2 sec]$, and L is Latent heat of evaporation $[J/kg]$.

2.3.2 Boundary condition on internal part

The core temperature of the body is maintained at $37^\circ C$ ($\pm\ 0.6^\circ C$). Thoracic wall of the breast tissue is attached with body core. So, Dirichlet boundary condition is used in this part:

$$\Gamma_2 \quad : \quad 37^\circ C \tag{7}$$

2.4 Variational Form

The variational form of the partial differential equation (1) and (5) together with boundary condition (6) are:

For conventional bio-heat model,

$$\begin{aligned} I[\Phi(x,y,t)] \;=\; & \frac{1}{2}\iint_\Omega \left[k_t\left(\left(\frac{\partial \Phi}{\partial x}\right)^2 + \left(\frac{\partial \Phi}{\partial y}\right)^2\right) + w_b c_b \rho_b (\Phi_B - \Phi)^2 - 2m\Phi \right. \\ & + \left. 2\rho_t c_t \Phi \frac{\partial \Phi}{\partial t}\right] dxdy + \frac{1}{2}\int_{\Gamma_1}\left[h_{cr}\,(\Phi - \Phi_\infty)^2 + S\Phi\right] d\Gamma_1 \end{aligned} \tag{8}$$

For porous media model,

$$\begin{aligned} I[\Phi(x,y,t)] \;=\; & \frac{1}{2}\iint_\Omega \left[k\left(\left(\frac{\partial \Phi}{\partial x}\right)^2 + \left(\frac{\partial \Phi}{\partial y}\right)^2\right) + R(\Phi_B - \Phi)^2 - 2(1-p)m\Phi \right. \\ & + \left. 2C\Phi \frac{\partial \Phi}{\partial t}\right] dxdy + \frac{1}{2}\int_{\Gamma_1}\left[h_{cr}\,(\Phi - \Phi_\infty)^2 + S\Phi\right] d\Gamma_1 \end{aligned} \tag{9}$$

where,
$C = (1-p)\rho_t c_t + p\rho_b c_b$, $k = (1-p)k_t + pk_b$, and $R = \beta\,\frac{p\rho_b c_b (v_b)_{avg}}{\delta}\,(\Phi_B - \Phi)$.

The Lagrange linear interpolation functions are used to approximate the solution of the triangular finite elements [21, 22].

For minimization,

$$\frac{dI}{d\Phi_i} = 0 \tag{10}$$

where, Φ_i represents the temperature in i^{th} triangular mesh.

The system of linear equations (10) can be written in matrix form:

$$A\ \dot{\Phi} + F\ \Phi = B \tag{11}$$

where, $\dot{\Phi} = [\frac{\partial \Phi_i}{\partial t}]$, $\Phi = [\Phi_i]$, B are $q \times 1$ vectors, A and F are Capacitance, Conductance matrices with dimension $q \times q$, q is the total number of nodal points. To solve the system (11) with respect to time, we apply the Crank-Nicolson method as follows:

$$\left(A + \frac{\triangle t}{2} F\right) \Phi^{i+1} = \left(A - \frac{\triangle t}{2} F\right) \Phi^{i} + \triangle t\ B \tag{12}$$

where, $\triangle t$ is the time interval.

3 Results and Discussion

The temperature variation of the breast tissue is studied in both conventional bio-heat model and porous media model. The finite element method is used to solve two-dimensional bio-heat equation. The temperature of breast tissue with and without tumor is compared in the study. Keangin and Rattanadecho [6] used the values 0.5 for porosity, and $2.476 \times 10^{-5}\ m/s$ for blood velocity in 20 mm size of tumor. The same values are used in our study due to same tumor size. Tucci et al. [28] used the blood velocity 3×10^{-5} in their study of porous media model. This value is used in our study for normal tissue since tumor has weak blood perfusion due to lower porosity than normal tissue [6]. Since the porosity is less than 0.1, we used 0.04 porosity for the tissue in normal case. The parameter values of each layers of breast tissue are shown in Table 1 and 2.

Table 1: Values of thermal conductivity, perfusion and metabolism

Breast Tissue's Layers	Thermal Conductivity k $(W/m\ ^\circ C)$	Perfusion $w_b\ c_b\ \rho_b$ $(W/m^3\ ^\circ C)$	Metabolism s (W/m^3)
Epidermis	0.20934 [1, 23, 25]	0 [1, 23, 25]	0 [1, 23, 25]
Dermis	0.31401 [1, 23, 25]	800 [11, 23, 25, 27]	400 [11, 23, 25, 27]
Subcutaneous	0.41868 [1, 23, 25]	800 [11, 23, 25, 27]	400 [11, 23, 25, 27]
Glandular	0.48 [11, 23, 25, 27]	2400 [11, 23, 25, 27]	700 [11, 23, 25, 27]
Muscle with Thoracic wall	0.48 [11, 23, 25, 27]	2400 [11, 23, 25, 27]	700 [11, 23, 25, 27]
Tumor	0.55 [26]	48000 [11, 23, 25, 27]	1400 [11, 23, 25, 27]

Table 2: Parameter values used in Model

Parameters	Values
Heat transfer coefficient (h_{cr}) ($W/m^2\ ^\circ C$)	13.5 [17, 25, 27]
Latent heat of evaporation (L) (J/kg)	2.4×10^6 [1, 23, 25]
Sweat evaporation rate (E) ($kg/m^2 sec$)	3.0806×10^{-6} [18, 24]
Room temperature (Φ_∞) ($^\circ C$)	25 [18]
Porosity (p)	0.4 for tumor [6]
Blood velocity for tumor (m/s)	2.476×10^{-5} [6]
Effective thermal conductivity of blood (k_b) ($W/m\ ^\circ C$)	0.24 for normal [16] 0.5 for tumor [10]
Density of tissue (ρ_t) (kg/m^3)	1050 [19, 23]
Density of blood (ρ_b) (kg/m^3)	1000 [28]
Specific heat of tissue (c_t) ($J/kg^\circ C$)	3475.044 [1, 23]
Specific heat of blood (c_b) ($J/kg^\circ C$)	3500 [7, 16]
average spacing (δ) (m)	0.001 [29]
correction factor (β)	0.6 [8, 30]

3.1 Steady state results of temperature of breast tissue

Figure 2 and 3 exhibit the temperature profiles of the normal breast tissue in conventional bio-heat model and porous media model respectively. The results show that the temperature of the breast tissue is continuously increasing from the skin surface to the body core in both models, which is the real phenomenon of the normal breast tissue [3, 23]. It is observed that the temperature profile of the porous media model is slightly higher than the conventional bio-heat model due to the porosity.

The temperature profiles of the tumorous breast tissue in conventional bio-heat model and porous media model are shown in Figure 4 and 5, respectively. The graphs show that the temperature of tumor region in porous media model is slightly lower than the conventional bio-heat model. Wessapan and Phadungsak [31] found similar behavior in comparisons of the temperature profiles in temperature ablation in tumor treatment in the conventional bio-heat model and porous media models. The comparison of temperature profiles of the normal and tumorous breast tissue in the conventional bio-heat model and porous media model is exhibited in Figure 6. The results show that the temperature profiles of the tumorous breast tissue is higher than normal in both conventional bio-heat model and porous media model. There is small variation in skin surface temperature of normal and tumorous breast tissue due to far location of tumor (35 mm) from areola. It is found from the literature that the temperature of breast skin surface is higher when tumor is close to areola and vice-versa [27, 23]. The skin surface temperature of breast tissue in conventional bio-heat model is almost identical with porous media model in both normal and tumorous breast tissue due to value of correction factor $\beta = 0.6$. When the value of β decreases from 0.6 the temperature of normal and tumorous breast tissue in porous media model are lower than conventional

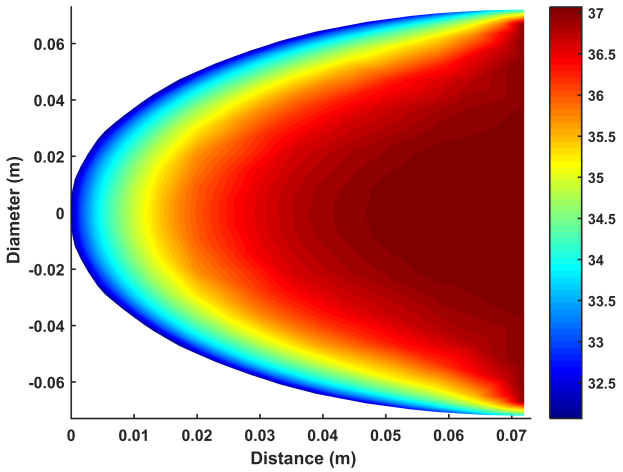

Figure 2: Temperature profiles of normal breast tissue (conventional model).

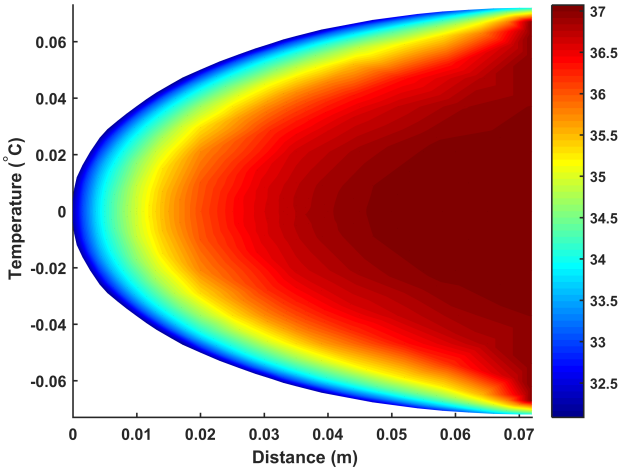

Figure 3: Temperature profiles of normal breast tissue (porous media model).

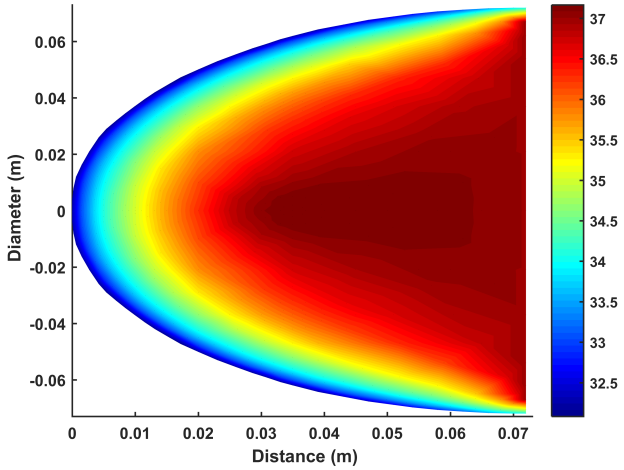

Figure 4: Temperature profiles of tumorous breast tissue (conventional model).

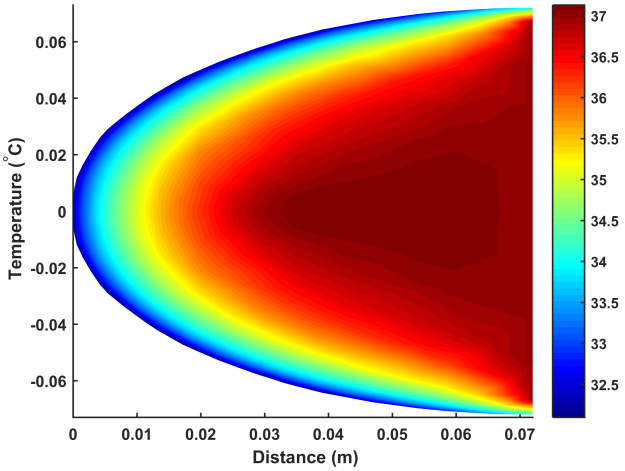

Figure 5: Temperature profiles of tumorous breast tissue (Porous media model).

bio-heat model and vice-versa, which is shown in Figure 7.

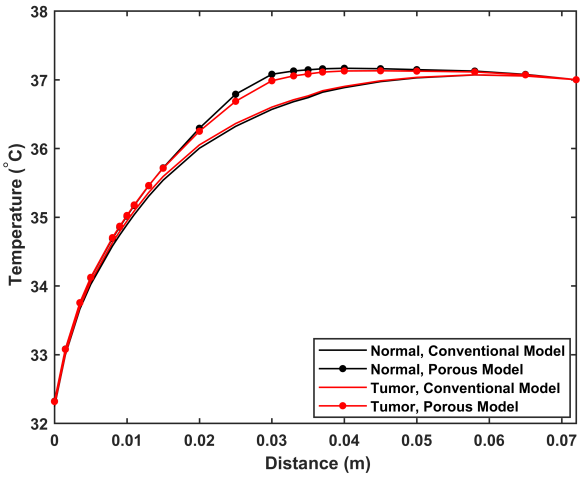

Figure 6: Comparison of temperature profiles of the breast tissue.

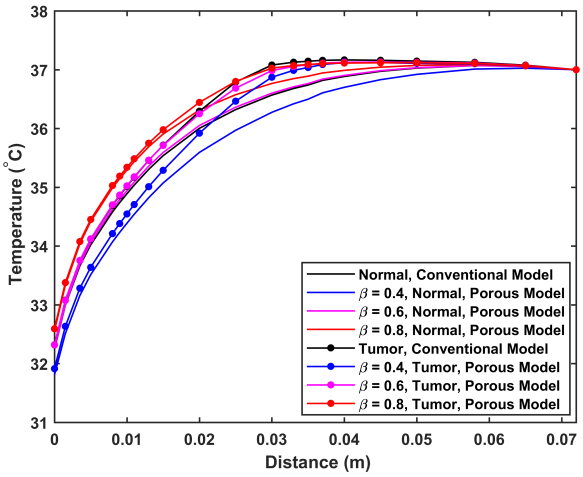

Figure 7: Comparison of temperature profiles of normal and tumorous breast tissue in conventional bio-heat model and porous media model at different correction factor β.

Porosity is an important parameter of the porous media model. Figure 8 represents the graph of temperature profiles of normal and tumorous breast tissue in porous media model at different porosity $0.04, 0.05, 0.06$ for normal and $0.5, 0.6, 0.7$ for tumor. The results show that the temperature of the normal breast tissue is increased in each layers of the breast tissue when the porosity of tissue is increased. The similar behavior of temperature in porous media model of human forearm was shown by Yang et al. [33]. In case of tumorous breast tissue, the temperature is slightly increased in skin surface to anterior part of the tumor and slightly decreased in tumor region when porosity of tumor is increased due to higher porosity of tumor than normal. When tumor porosity is increased, the blood flow is increased [28] and increase in blood perfusion rate is sufficient to decrease the temperature in living tissue [9]. The temperature profiles of normal and tumorous breast tissue in porous media model at different metabolic heat production, blood velocity and room temperature are shown in Figures 9, 10 and 11, respectively. The different metabolic heat production $700, 1000, 1400\ W/m^3$ for normal and $8724.4, 16000, 24000\ W/m^3$ for tumor are used in Figure 9. The different blood velocities 3×10^{-5}, 3.5×10^{-5}, $4 \times 10^{-5}\ m/s$ for normal and 2.476×10^{-5},

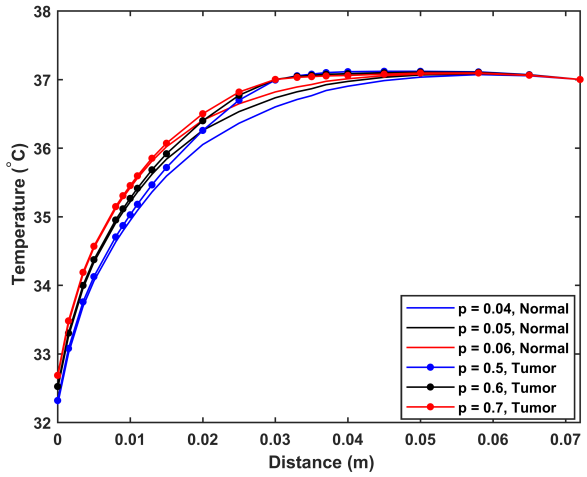

Figure 8: Temperature profiles of normal and tumorous breast tissue in porous media model at different porosity (p).

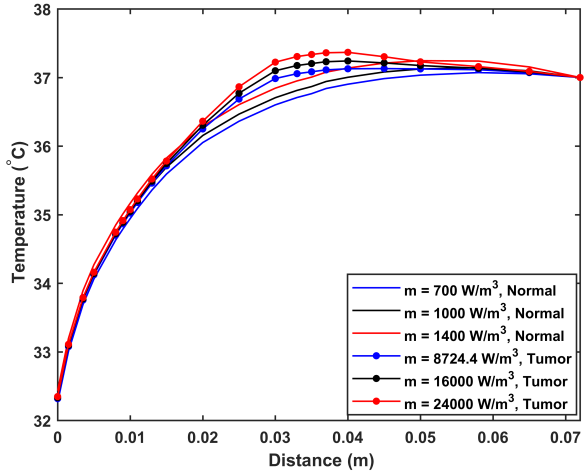

Figure 9: Temperature profiles of normal and tumorous breast tissue in porous media model at different metabolism (m).

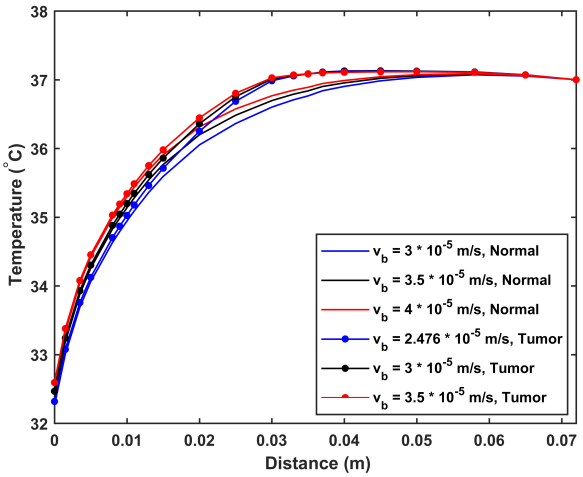

Figure 10: Temperature profiles of normal and tumorous breast tissue in porous media model at different blood velocities (v_b).

3×10^{-5}, 3.5×10^{-5} m/s for tumor are used in Figure 10 and different room temperatures $21°C$, $25°C$, $28°C$ are used in Figure 11 for comparison of temperature profiles of normal and tumorous breast tissue in porous media model. The results show that the temperature of breast tissue is increased when metabolism, blood velocity and room temperature are increased.

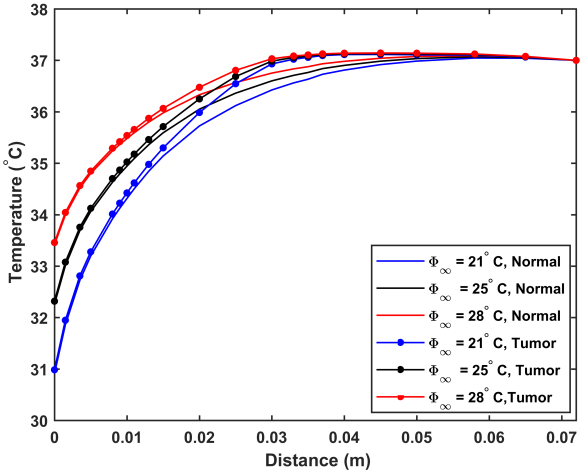

Figure 11: Temperature profiles of normal and tumorous breast tissue in porous media model at different room temperature (Φ_∞).

3.2 Unsteady state results of temperature of breast tissue

Figures 12 and 13 represent the temperature profiles of normal and tumorous breast tissue in time dependent case. The skin surface temperatures of normal and tumorous breast tissue in conventional bio-heat model and porous media model are shown in Figures 12. It is observed from the graph that the skin surface temperature of breast tissue in conventional bio-heat model and porous media model take around 5000 seconds to reach steady state. It takes long time due to large domain 72 mm between areola and thoracic wall. In our previous work [23], we found that the tissue takes long time to achieve steady state temperature for larger domain size.

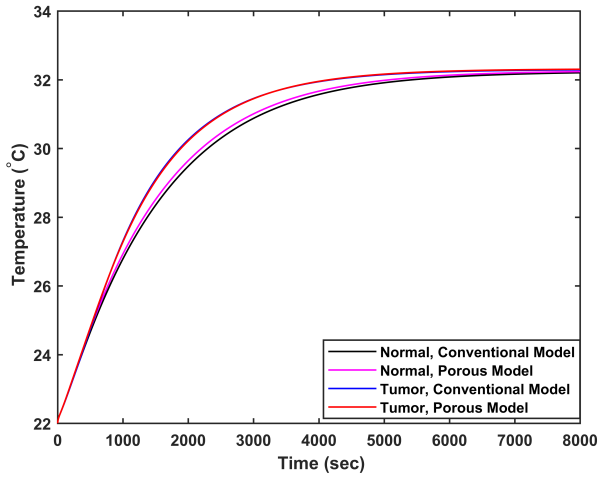

Figure 12: Skin surface temperature profiles of normal and tumorous breast tissue.

The anterior, central and posterior temperature of tumor region reach steady state at $36.55°C$, $36.97°C$, $36.93°C$ for conventional bio-heat model and $36.50°C$, $37.01°C$, $36.41°C$ for porous media model, and both model reach steady state after 2000, 1000 and 1500 seconds respectively. This shows that the posterior temperature of tumor region reaches steady state faster than anterior due to shortest distance of posterior part of tumor from body core than anterior part. The central temperature of the tumor region reaches steady state faster than anterior and posterior temperature due to high impact of blood flow and metabolic heat production in center of tumor than other.

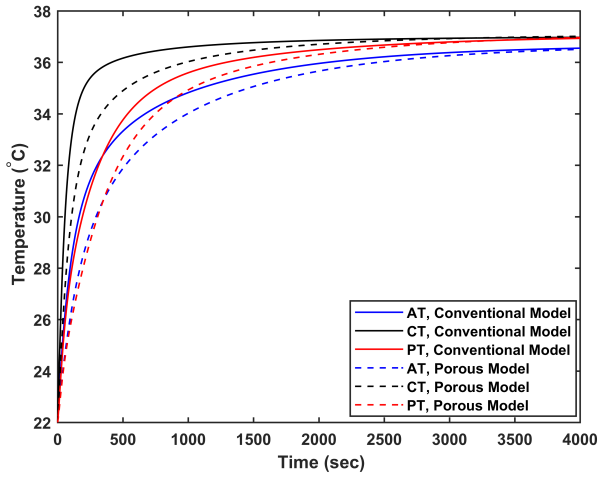

Figure 13: Anterior (AT), central (CT) and posterior (PT) temperature profiles of tumor region.

4 Conclusion

The two-dimensional bio-heat equation is solved for normal and tumorous female breast tissue by using the finite element method. The temperature variation of breast tissue is studied in the conventional bio-heat model and porous media model. The results show that the temperature profile of the porous media model is almost identical with the conventional bio-heat model in normal breast tissue at correction factor $\beta = 0.6$. The temperature of normal and tumorous breast tissue in porous media model is lower than conventional

bio-heat model with $\beta < 0.6$ and vice-versa. The temperature of the normal breast tissue is increased in each layer of the breast tissue when the porosity is increased. The temperature is slightly increased in skin surface to anterior part of the tumor and slightly decreased in tumor region when porosity of tumor is increased. The temperature of breast tissue is increased when metabolism, blood velocity, and room temperature are increased in both normal and tumorous breast tissue in porous media model. When the room temperature is increased, it is significantly increased in the skin surface temperature of the breast tissue. Similarly, an increase in blood velocity and metabolism significantly increases the temperature in tumor region. An increase in porosity increases the temperature in normal tissue but decreases the temperature in tumor region. So, the parameter porosity has the greatest impact on the temperature of breast in porous media. It is observed from the graph that the skin surface temperature of breast tissue in conventional model and porous media model take around 5000 seconds to reach a steady state. The central temperature of tumor region reaches a steady state faster than anterior and posterior temperature of both normal and tumorous breast tissue in conventional bio-heat model and porous media model.

Acknowledgement

The University Grants Commission (UGC) Nepal, under the UGC PhD Fellowship (Award No: PhD /74 - 75/ S & T - 11) is funded to the first author for this study. We would like to gratefully acknowledge the University Grants Commission, Nepal. We also thank all anonymous reviewers and editors for their valuable comments and suggestions for the improvement of the paper.

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