Exponentiated Inverse Chen distribution: Properties and applications

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Abstract

A new distribution having three parameters called Exponentiated Inverse Chen distribution is proposed in this study. Important statistical properties like Survival function, hazard rate function, skewness and kurtosis etc are studied. Some methods of estimation Least Square, Maximum likelihood and Cramer-Von Mises methods are used using R programming software. A data set is discussed and Validity of the model is tested by analyzing P-P and Q-Q plots. Different information criteria such as Akanke's information criterion, Bayesian information criterion, Corrected Akanke's information criterion, and Hannan – Quinn information criterion are applied for model comparisons. For testing the goodness of fit of the proposed model, Kolmogrov-Smirnov, Anderson darling and Cramer –Von Mises statistics are used. The proposed model called Exponentiated Inverse Chen distribution is more applicable as compared to some existing probability model. It is found that MLEs are better with respect to LSE and CVM. PDF curve of model has shown that it can have various shapes like increasing as well as decreasing, monotonically increasing , constant as well as inverted bathtub shaped based hazard function is seen. Applicability and suitability of model is evaluated. For this, we have considered a real-life dataset. We found distribution that EIC is much flexible.

Keywords: Chen distribution, Cramer-von Mises, Estimation, Hazard function

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Introduction

This study formulates a new model called Exponentiated Inverse Chen distribution. Various statistical properties like Survival function, hazard rate function, skewness and kurtosis etc are studied. Real data set is taken for testing the applicability and comparing the results with other published models showing that the proposed model fits better than the competitive models. Different information criteria values are studied which shows suitability of the model compared to considered models.

In literature, it is seen that probability models generated not present a good fit to distributions of real data associated to geology, biology, ecology, hydrology, reliability, life testing, engineering and risk analysis. Formulation of new extended distribution is essential for analysis of such problems. We can produce extended, flexible, and generalized distributions by adding some parameters as well as applying some changes to the baseline distribution. New distribution formed provides a better fit as compared to the challenging models. There are many techniques of getting new distribution using existing distribution. One of the methods is modification of the existing distribution. Sah Telee and Kumar (2023) introduced *Modified Generalized Exponential Distribution* by modifying the *Generalized exponential distribution*. Treyer (1964) gave inverse *Rayleigh distribution* which was modified by (Khan, 2014) adding one extra parameter α. Another technique is combining the distributions. Chaudhary et al. (2022) combined *Lomax distribution* with *Inverse exponential distribution* to derive new distribution called *Inverse exponentiated odd Lomax exponential distribution.*

Chen (2000) gave two-parameter lifetime distribution having bathtub shaped or increasing failure rate having cumulative distribution function (CDF) as

$$G(x;\beta,\lambda)=1-e^{\lambda(1-e^{x^{\beta}})};\ \beta,\lambda>0,\ x>0 \tag{1}$$

Corresponding probability density function (PDF) of the Chen distribution is given as

$$g(x;\beta,\lambda)=\beta\lambda x^{\beta-1}e^{x^{\beta}}e^{[\lambda(1-e^{x^{\beta}})]};\ \beta,\lambda>0,\ x>0 \tag{2}$$

Inverse Chen distribution (Srivastava & Srivastava, 2014) is more flexible model having various shapes of the hazard and density function having CDF & PDF given by

$$G(x)=\exp\left\{\lambda\left(1-e^{x^{-\alpha}}\right)\right\};x>0,(\alpha\lambda)>0 \tag{3}$$

$$g(x)=\alpha\lambda e^{x^{-\alpha}}x^{-(\alpha+1)}\exp\left\{\lambda\left(1-e^{x^{-\alpha}}\right)\right\};x>0,(\alpha\lambda)>0\ . \tag{4}$$

Dey et al., (2017) was presented the exponentiated Chen (EC) distribution with CDF,

$$F(x)=\left\{1-\exp\left[\lambda\left\{1-e^{x^{\beta}}\right\}\right]\right\}^{\alpha}\ ;(\alpha,\ \beta,\lambda)>0,\ x>0\,. \tag{5}$$

Extension of the inverse Chen distribution called exponentiated inverse Chen distribution is done in this article. The Kumaraswamy exponentiated Chen distribution is introduced by (Khan, et al., 2018). Using generalized Burr-Hatke differential equation, extended Chen (EC) distribution was derived. Tarvirdizade and Ahmadpour (2019) gave a model that was formulated by use of Weibull and Chen distributions. Joshi & Kumar (2020) has defined a flexible model using Chen distribution called Lindley-Chen distribution. Similarly another extension of Chen distribution has presented by (Joshi & Kumar, 2021) using Poisson family of distribution named Poisson Chen distribution.

Whole article is explained in different sections. Firstly, we present the formulated *exponentiated inverse Chen distribution* with some properties. Discussion of the parameter estimation and different test criteria to assess the potentiality of the proposed model is done in other sections.

Exponentiated Inverse Chen (EIC) distribution

A three parameters exponentiated inverse Chen distribution with CDF and PDF is

$$F(x)=1-\left[1-\exp\left\{\lambda\left(1-e^{x^{-\alpha}}\right)\right\}\right]^{\theta}\ ;(\alpha,\lambda,\theta)>0,\ x>0 \tag{6}$$

$$f(x)=\left(\frac{\alpha\lambda\theta}{x^{(\alpha+1)}}\right)\exp\left\{x^{-\alpha}+\lambda\left(1-e^{x^{-\alpha}}\right)\right\}\left[1-\exp\left\{\lambda\left(1-e^{x^{-\alpha}}\right)\right\}\right]^{(\theta-1)} \tag{7}$$

Survival rate function

Survival function S(x) of EIC is

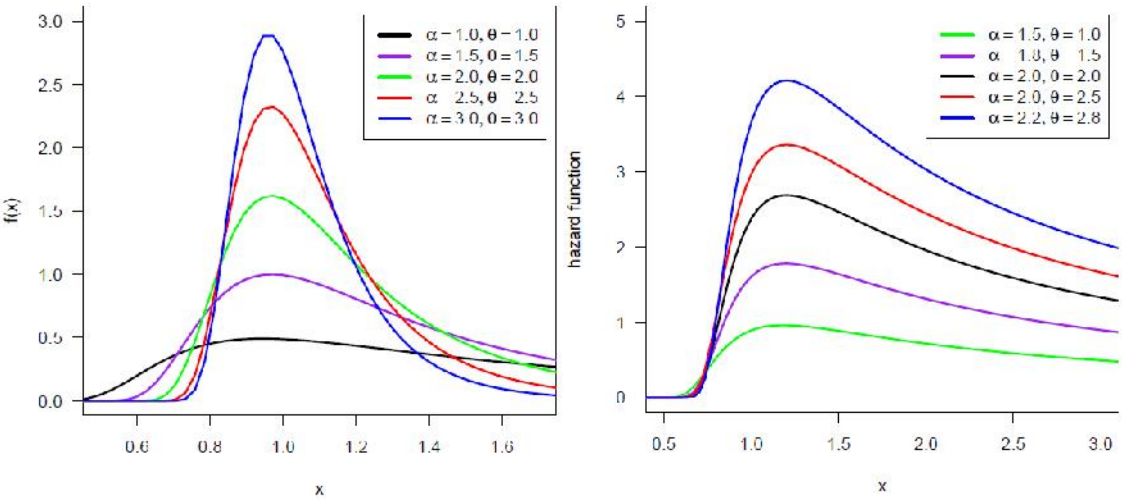
$$S(x)=\left[1-\exp\left\{\lambda\left(1-e^{\,x^{-\alpha}}\right)\right\}\right]^{\theta} \tag{8}$$

Hazard rate function

$$h(x)=\frac{f(x)}{R(x)}=\alpha\lambda\theta\,x^{-(\alpha+1)}\exp\left\{x^{-\alpha}+\lambda\left(1-e^{\,x^{-\alpha}}\right)\right\}\left[1-\exp\left\{\lambda\left(1-e^{\,x^{-\alpha}}\right)\right\}\right]^{-1} \tag{9}$$

Figure 1, displays pdf and hrf plots of EIC for some values of α, λ & θ.

Figure 1
Pdf and hrf for some values of parameters.



Quantile function

The Quantile function is

$$Q(u)=\left[\ln\left\{1-\frac{1}{\alpha}\ln\left\{1-(1-u)^{(1/\beta)}\right\}\right\}\right]^{(-1/\theta)} \quad ;0<u<1 \tag{10}$$

Random deviate generation

$$x=\left[\ln\left[1-\frac{1}{\alpha}\ln\left\{1-(1-v)^{1/\beta}\right\}\right]\right]^{-1/\theta} \quad ;0<v<1 \tag{11}$$

Skewness and Kurtosis

Bowley's skewness is

$$\text{Skewness}=\frac{Q(0.75)+Q(0.25)-2*Q(0.5)}{Q(0.75)-Q(0.25)}\text{ , and} \tag{12}$$

Octiles based kurtosis defined by (Moors, 1988) is

$$K_u(M)=\frac{Q(0.875)-Q(0.625)+Q(0.375)-Q(0.125)}{Q(3/4)-Q(1/4)}, \tag{13}$$

Parameters estimation

Here, parameters of the EIC are estimated by using some estimation techniques.

Maximum Likelihood Estimation (MLE)

Let, $x_1, x_2, ..., x_n$ is a sample from $EIC(\theta,\alpha,\beta)$ and likelihood function is,

$$L(\Theta;x_1,x_2...x_n)=f(x_1,x_2,...x_n/\Theta)=\prod_{i=1}^{n}f(x_i/\Theta)$$

$$L(\theta,\alpha,\beta)=\theta\alpha\beta\prod_{i=1}^{n}x_i^{-(\theta+1)}\exp\left\{x_i^{-\theta}+\alpha\left(1-e^{x_i^{-\theta}}\right)\right\}\left[1-\exp\left\{\alpha\left(1-e^{x_i^{-\theta}}\right)\right\}\right]^{\beta-1} \quad ;x>0$$

Log-likelihood function is:

$$l(\sim, ª, ¯ \mid \underline{x}) = n\ln\sim + n\ln ª + n\ln ¯ - (\sim +1)\sum_{i=1}^{n}\ln(x_i) + \sum_{i=1}^{n} x_i^{-\sim} + ª\sum_{i=1}^{n}\left(1-e^{x_i^{-\sim}}\right)$$
$$+ (¯ -1)\sum_{i=1}^{n}\ln\left[1-\exp\left\{ª\left(1-e^{x_i^{-\sim}}\right)\right\}\right] \quad (14)$$

Differentiating (14), we get,

$$\frac{\partial l}{\partial\sim} = \frac{n}{\sim} - \sum_{i=1}^{n}\ln x_i - \sum_{i=1}^{n}\left(x_i^{-\sim}\ln x_i\right)$$
$$+ ª\sum_{i=1}^{n}\left[x_i^{-\sim}e^{x_i^{-\sim}}\ln x_i\left[1-(¯ -1)\exp\left\{ª\left(1-e^{x_i^{-\sim}}\right)\right\}\left\{1-\exp\left\{ª\left(1-e^{x_i^{-\sim}}\right)\right\}\right\}^{-1}\right]\right]$$

$$\frac{\partial l}{\partial ª} = \frac{n}{ª} + \sum_{i=1}^{n}\left(1-e^{x_i^{-\sim}}\right) - (¯ -1)\sum_{i=1}^{n}\left[\left(1-e^{x_i^{-\sim}}\right)\exp\left\{ª\left(1-e^{x_i^{-\sim}}\right)\right\}\left\{1-\exp\left\{ª\left(1-e^{x_i^{-\sim}}\right)\right\}\right\}^{-1}\right]$$

$$\frac{\partial l}{\partial ¯} = \frac{n}{¯} + \sum_{i=1}^{n}\ln\left[1-\exp\left\{ª\left(1-e^{x_i^{-\sim}}\right)\right\}\right]$$

Applying $\frac{\partial l}{\partial\sim} = \frac{\partial l}{\partial} = \frac{\partial l}{\partial ª} = 0$ and by simultaneous solution for all parameters, ML estimators of the $EIC(\sim, ª, ¯)$ distribution can be obtained. Using R software, we can obtain the estimated parameters. The observed information matrix is,

$$¤ = \begin{bmatrix} ¤_{11} & ¤_{12} & ¤_{13} \\ ¤_{21} & ¤_{22} & ¤_{23} \\ ¤_{31} & ¤_{32} & ¤_{33} \end{bmatrix}$$

Where

$$¤_{11} = \frac{\partial^2 l}{\partial\sim^2},\ ¤_{12} = \frac{\partial^2 l}{\partial\sim\partial ª},\ ¤_{13} = \frac{\partial^2 l}{\partial\sim ¯}$$
$$¤_{21} = \frac{\partial^2 l}{\partial ª\partial\sim},\ ¤_{22} = \frac{\partial^2 l}{\partial ª^2},\ ¤_{23} = \frac{\partial^2 l}{\partial ª\partial ¯}$$
$$¤_{21} = \frac{\partial^2 l}{\partial ª\partial\sim},\ ¤_{22} = \frac{\partial^2 l}{\partial ª^2},\ ¤_{23} = \frac{\partial^2 l}{\partial ª\partial ¯}$$

Since $\Omega = (\sim, ª, ¯)$ and $\hat{\Omega} = (\hat{\sim}, \hat{ª}, \hat{¯})$ are parameter space and MLE of Ω, then $(\hat{\Omega}-\Omega) \rightarrow N_3\left[0, (U(\Omega))^{-1}\right]$. Variance covariance matrix can be obtained as below using Fisher's information matrix $U(\Omega)$ and Newton-Raphson algorithm by maximizing the likelihood function,

$$\left[U(\Omega)\right]^{-1} = \begin{bmatrix} \text{variance}(\hat{\alpha}) & \text{covariance}(\hat{\alpha},\hat{\lambda}) & \text{covariance}(\hat{\alpha},\hat{\theta}) \\ \text{covariance}(\hat{\alpha},\hat{\lambda}) & \text{variaance}(\hat{\lambda}) & \text{covariance}(\hat{\lambda},\hat{\theta}) \\ \text{covariance}(\hat{\alpha},\hat{\theta}) & \text{covariance}(\hat{\lambda},\hat{\theta}) & \text{variance}(\hat{\theta}) \end{bmatrix} \quad (15)$$

Hence 100(1-α) % C.I. of MLEs, for are

$$\hat{\sim} \pm Z_{(\sim/2)}S.E.(\hat{\sim}),\ \hat{ª} \pm Z_{(\sim/2)}S.E.(\hat{ª}) \text{ and, } \hat{¯} \pm Z_{(\sim/2)}S.E.(\hat{¯})$$

where $Z_{\alpha/2}$ is upper percentile of SNV.

Least Square Estimation

We have also used the LSE to estimate the α, λ and θ of EIC. Minimizing the function below

$$M\left(X;\alpha,\lambda,\theta\right)=\sum_{i=1}^{n}\left[F(X_i)-\left(\frac{i}{n+1}\right)\right]^2 \qquad (16)$$

Let us consider that $F(X_{(i)})$ is CDF of order statistics $X_{(1)} < X_{(2)} < \ldots < X_{(n)}$ taking $\{X_1, X_2, \ldots, X_n\}$ as a sample from F (.). By minimizing, LSE can be obtained

$$M\left(X;\alpha,\lambda,\theta\right)=\sum_{i=1}^{n}\left[1-\left[1-\exp\left\{\lambda\left(1-e^{x_i^{-\alpha}}\right)\right\}\right]^{\theta}-\frac{i}{n+1}\right]^2; x\geq 0, \left(\alpha,\theta,\lambda\right)>0 \qquad (17)$$

Differentiating (17), we get,

$$\frac{\partial M}{\partial \alpha}=2\lambda\theta\sum_{i=1}^{n}x_i^{-\alpha}\ln(x_i)e^{x_i^{-\alpha}}\left[1-\left[Z(x_i)\right]^{\theta}-\frac{i}{n+1}\right]\left[Z(x_i)\right]^{\theta-1}\exp\left\{\lambda\left(1-e^{x_i^{-\alpha}}\right)\right\}$$

$$\frac{\partial M}{\partial \lambda}=2\theta\sum_{i=1}^{n}\left[1-\left[Z(x_i)\right]^{\theta}-\frac{i}{n+1}\right]\left[Z(x_i)\right]^{\theta-1}\exp\left\{\lambda\left(1-e^{x_i^{-\alpha}}\right)\right\}\left(1-e^{x_i^{-\alpha}}\right)$$

$$\frac{\partial M}{\partial \theta}=-2\theta\sum_{i=1}^{n}\left[1-\left[Z(x_i)\right]^{\theta}-\frac{i}{n+1}\right]\left[Z(x_i)\right]^{\theta}\ln\left\{Z(x_i)\right\}$$

Here, $Z(x_i)=1-\exp\left\{\lambda\left(1-e^{x_i^{-\alpha}}\right)\right\}$

Weighted LSE is defined by minimization of

$$M\left(X;\alpha,\lambda,\theta\right)=\sum_{i=1}^{n}w_i\left[F(X_{(i)})-\left(\frac{i}{n+1}\right)\right]^2 \qquad (18)$$

Where, $w_i=\left(Var(X_{(i)})\right)^{-1}=\frac{\left(n+2n+1\right)*\left(n+2\right)}{i*\left(n+1-i\right)}$ is weight.

Estimation by Cramer-Von Mises (CVME)

By minimizing function (19), CVME can be calculated.

$$K\left(X;\alpha,\lambda,\theta\right)=\frac{1}{12n}+\sum_{i=1}^{n}\left[F\left(x_{i:n}\mid\alpha,\lambda,\theta\right)-\left(\frac{2i-1}{2n}\right)\right]^2$$

$$=\frac{1}{12n}+\sum_{i=1}^{n}\left[1-\left[1-\exp\left\{\lambda\left(1-e^{x_i^{-\alpha}}\right)\right\}\right]^{\theta}-\left(\frac{2i-1}{2n}\right)\right]^2 \qquad (19)$$

Differentiating (19), we get,

$$\frac{\partial K}{\partial \alpha}=2\lambda\theta\sum_{i=1}^{n}x_i^{-\alpha}\ln(x_i)e^{x_i^{-\alpha}}\left[1-\left[Z(x_i)\right]^{\theta}-\frac{2i-1}{2n}\right]\left[Z(x_i)\right]^{\theta-1}\exp\left\{\lambda\left(1-e^{x_i^{-\alpha}}\right)\right\}$$

$$\frac{\partial K}{\partial \lambda}=2\theta\sum_{i=1}^{n}\left[1-\left[Z(x_i)\right]^{\theta}-\frac{2i-1}{2n}\right]\left[Z(x_i)\right]^{\theta-1}\exp\left\{\lambda\left(1-e^{x_i^{-\alpha}}\right)\right\}\left(1-e^{x_i^{-\alpha}}\right)$$

$$\frac{\partial K}{\partial \theta}=-2\theta\sum_{i=1}^{n}\left[1-\left[Z(x_i)\right]^{\theta}-\frac{2i-1}{2n}\right]\left[Z(x_i)\right]^{\theta}\ln\left\{Z(x_i)\right\}$$

here, $Z(x_i)=1-\exp\left\{\lambda\left(1-e^{x_i^{-\alpha}}\right)\right\}$

Applying, $\frac{\partial K}{\partial \sim} = 0, \frac{\partial K}{\partial a} = 0 \; and \; \frac{\partial K}{\partial ^{-}} = 0$ simultaneously we can get estimated parameters.

Applications to real dataset

The data set describes the survival times of 44 patients. These patients who treated radio therapy suffer from Head and Neck cancer disease. The data set which is reported by (Efron, 1988) is presented below.

12.20, 23.56, 23.74, 25.87, 31.98, 37, 41.35, 47.38, 55.46, 58.36, 63.47, 68.46, 78.26, 74.47, 81.43, 84, 92, 94, 110, 112, 119, 127, 130, 133, 140, 146, 155, 159, 173, 179, 194, 195, 209, 249, 281, 319, 339, 432, 469, 519, 633, 725, 817, 1776

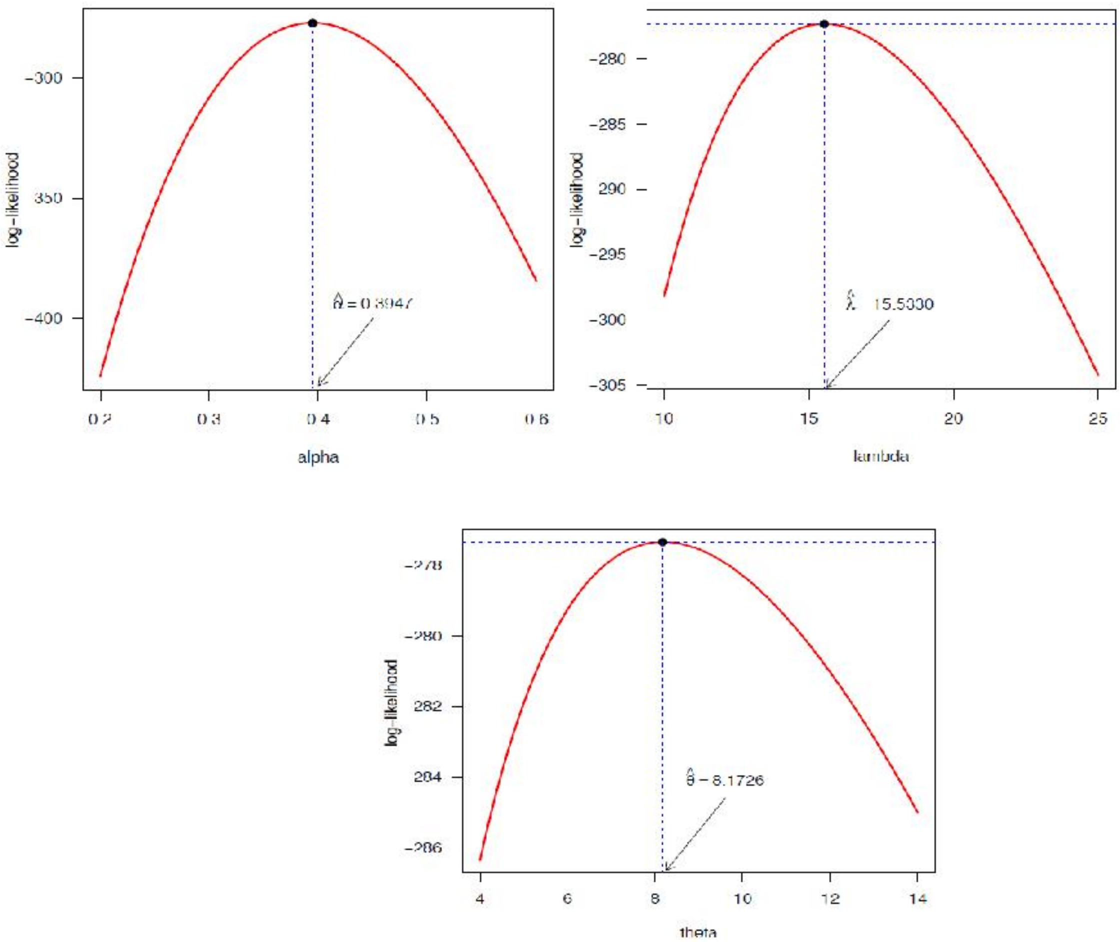
By use of R software of the optim () function(R Core Team, 2020), calculation of MLEs of EIC by maximizing the log likelihood function defined in equation (14) (Mailund, 2017).

Table 1
MLE and S.E. for α, λ and θ

Parameter	MLE	S.E.
alpha	0.39472	0.09778
lambda	15.53296	4.47920
theta	8.17259	5.32860

Figure 2 displays the graph of profile log-likelihood function of parameters .It is clear ML estimates is unique.

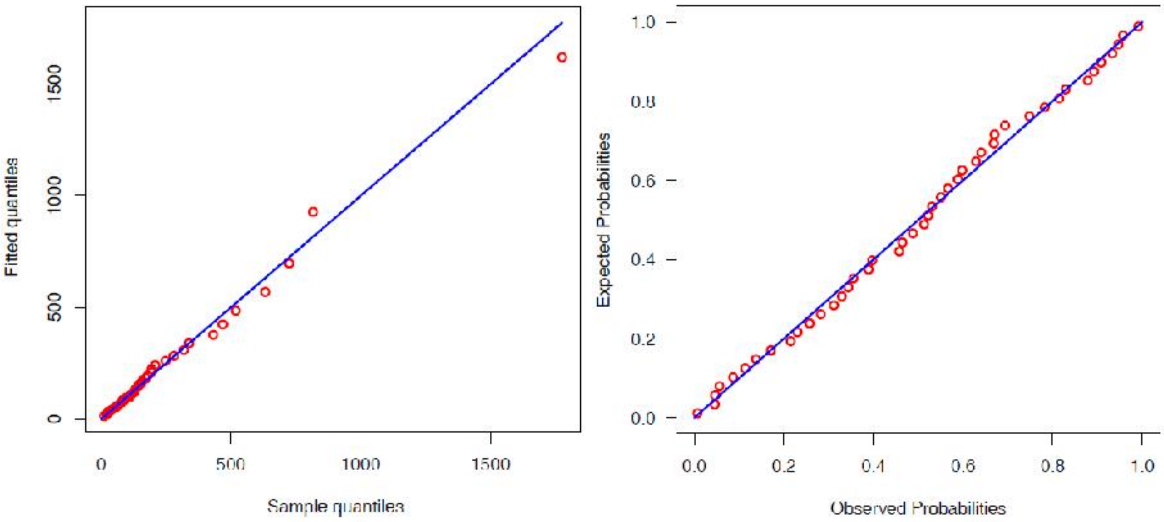
Figure 2
Profile log-likelihood function of parameters



Graph of P-P plot and Q-Q plots are shown in Figure 5.3. It is found that EIC model has better fitting to the data taken in consideration.

Figure 3

The QQ (Left) & PP (Right) plot of model.



The estimated value of the parameters of EIC model with LL, AIC, and KS values with p- values below

Table 2

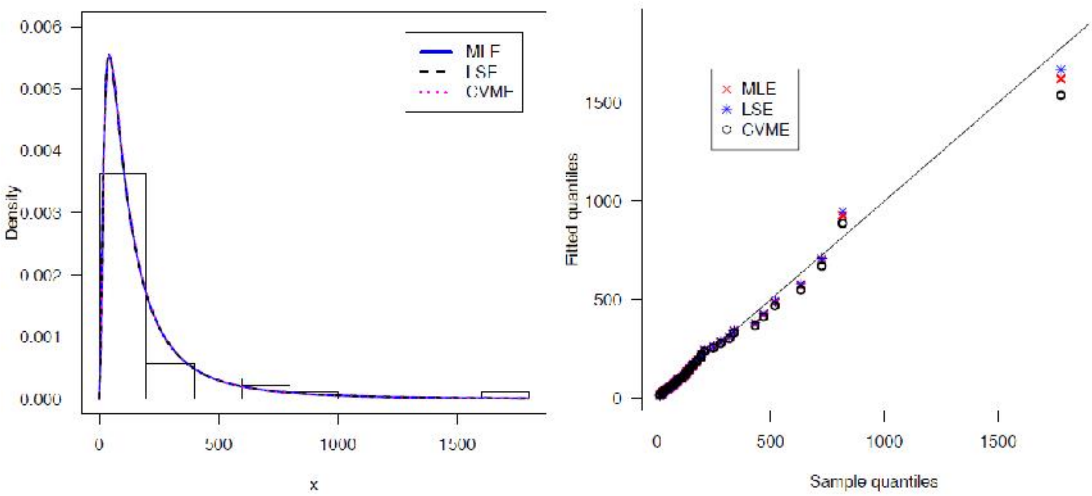
Estimated parameters, LL, &AIC with p- values

Methods	$\hat{\sim}$	$\hat{\ }$	$\hat{a}$	LL	AIC	KS(p-value)
MLE	0.3947	15.5330	8.1726	-277.3498	558.6997	0.0571(0.9972)
LSE	0.3984	15.6113	7.8590	-277.3540	558.7079	0.0588(0.9958)
CVE	0.4256	17.5576	7.4056	-277.4133	558.8266	0.0536(0.9989)

Figure 4 shows the plotted histogram versus pdf of fitted model. It also displays Q-Q plot.

Figure 4

Histogram and pdf of fitted distributions (left) & fitted quantile versus sample quantile (right) of different estimation methods of EIC.



In this part of study, applicability of EIC taking a real dataset used previously is presented. Model's potentiality is compared with these models.

Exponentiated Exponential Poisson (EEP) model:

The pdf of EEP (Ristić and Nadarajah, .2014) is

$$f(x)=\left(\frac{\sim\ a}{1-e^{-a}}\right)e^{-\ x}\left(1-e^{-\ x}\right)^{(\sim-1)}\exp\left\{-a\left(1-e^{-\ x}\right)^{\sim}\right\}\quad ;x>0,\sim>0,a>0$$

Chen distribution

Chen distribution (Chen, 2000) has pdf as

$$f_{CN}(x)=a\ \ x^{\ -1}e^{x}\ \exp\left\{a\left(1-e^{x}\ \right)\right\}\quad ;(a,\ )>0,x>0.$$

Generalized Gompertz model

Generalized Gompertz model (El-.Gohary et al., 2013) has pdf

$$f_{GGZ}(x)={}^{-}a\,e^{\sim x}e^{-\frac{a}{\sim}\left(e^{\sim x}-1\right)}\left[1-\exp\left(-\frac{a}{\sim}\left(e^{\sim x}-1\right)\right)\right]^{-1}$$

Weibull Extension model:

Weibull extension model (Tang et al., 2003) has pdf

$$f_{WE}(x)=a\ \left(xa^{-1}\right)^{-1}\exp\left(xa^{-1}\right)\ \exp\left\{-a\sim\left(\exp\left(xa^{-1}\right)\ -1\right)\right\}$$

Different information criteria for evaluation of the applicability of the EIC are tabulated in Table 3.

Table 3

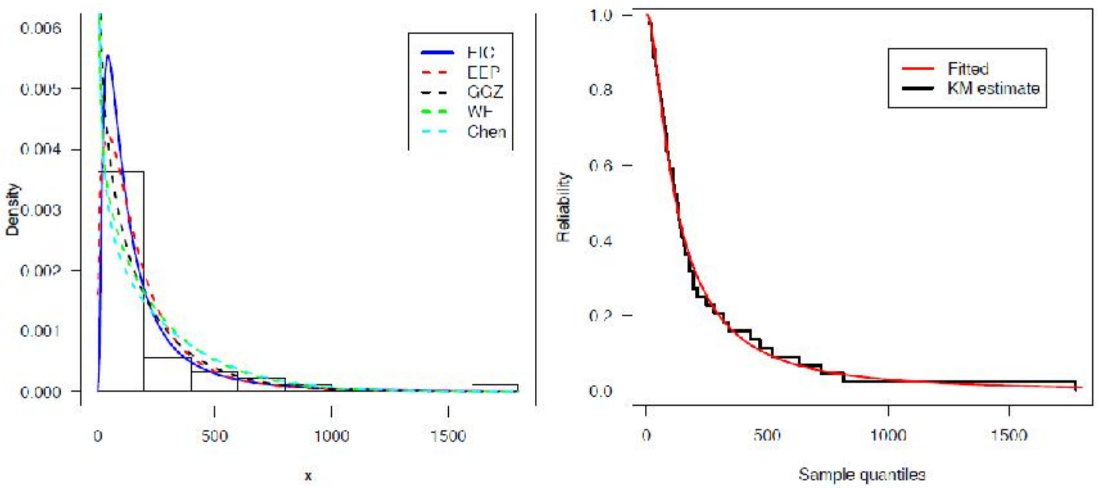
LL, AIC, BIC, CAIC and HQIC

Distribution	LL	AIC	BIC	CAIC	HQIC
EIC	-277.3498	558.6997	562.2681	558.9924	560.0230
EEP	-278.9665	563.9330	569.2855	564.5330	565.9179
GGZ	-283.4765	572.9530	578.3055	573.5530	574.9380
WE	-285.2362	576.4725	581.8251	577.0725	578.4575
Chen	-288.0750	580.1501	583.7185	580.4428	581.4734

We have displayed the graph of goodness-of-fit of EIC and models defined below:

Figure 5

The Histogram and the density function of fitted distributions and fitted KM



Goodness-of-fit of the EIC model with other competing model are compared by calculating value of Kolmogrov-Smirnov (KS), Creamers –von Mises (W) and Anderson Darling (A^2) test statistic in Table 4. EIC has the minimum test statistic values .That is, *p*-values of EIC model is higher indicating the better and consistent fit than considered models.

Table 4

Test statistics for testing goodness of fit with p-values

Models	KS ;p	A^2 ; p	W ;p
EIC	0.0335; 0.9987	0.1115; 0.9999	0.0161; 0.9994
EEP	0.0380 ; 0.9930	0.1486; 0.9987	0.0220; 0.9951
GGZ	0.0405; 0.9850	0.1798; 0.9950	0.0262; 0.9871
WE	0.0442; 0.9636	0.2630; 0.9631	0.0394; 0.9370
Chen	0.0725; 0.5115	0.7137; 0.5472	0.1279; 0.4652

Conclusion

This study explains a distribution named Exponentiated Inverse Chen distribution having three parameters. Detailed analysis of different statistical characteristics of model is also explained. We have presented expressions for its hazard rate function, survival function, the quantile function and skewness & kurtosis. Maximum likelihood estimation, Cramer-Von Mises estimation, and Least Square estimation methods are used to estimate the parameter. A real data set is taken for testing applicability of model. Validity of the model is tested graphically by plotting P-P plot, Q-Q plot and Empirical Versus Fitted CDF plot. Different information criteria show that model fits better to data set better compared to comparative models. Statistically goodness of fit of the model is tested showing that the models superior to the competitive models.

References

Chaudhary, A. K., Sah Telee, L. B. S., & Kumar, V. (2022). Inverse Exponentiated Odd Lomax Exponential Distribution: Properties and applications. *Nepalese Journal of Statistics*, *6*(01), 29-50.

Chen, Z. (2000). A new two-parameter lifetime distribution with bathtub shape or increasing failure rate function. *Stat Probab Lett*, 49(2), 155–161.

Dey, S., Kumar, D., Ramos, P. L., & Louzada, F. (2017). Exponentiated Chen distribution: Properties and estimation. *Communications in Statistics-Simulation and Computation*, *46*(10), 8118-8139.

Efron, B. (1988). Logistic regression, survival analysis, and the Kaplan-Meier curve. *Journal of the American statistical Association*, *83*(402), 414-425.

El-Gohary, A., Alshamrani, A., & Al-Otaibi, A. N. (2013). The generalized Gompertz distribution. *Applied Mathematical Modelling*, *37*(1-2), 13-24.

Joshi, R. K., & Kumar, V. (2020). Lindley-Chen Distribution with applications. *International Journals of Engineering, Science & Mathematics (IJESM)*, *9*(10), 12-22.

Joshi, R. K., & Kumar, V. (2021). Poisson Chen Distribution: Properties and application. *International Journal of Latest Trends in Engineering and Technology*, *18*(4), 001-012

Khan, M. S. (2014). Modified inverse Rayleigh distribution. *International Journal of Computer Applications*, *87*(13), 28-33.

Khan, M. S., King, R., & Hudson, I. L. (2018). Kumaraswamy exponentiated Chen distribution for modelling lifetime data. *Appl. Math*, *12*(3), 617-623.

Mailund, T. (2017). *Functional Programming in R: Advanced Statistical Programming for Data Science, Analysis and Finance*. Apress, Aarhus N, Denmark ISBN-13 (pbk): 978-1-4842-2745-9 ISBN-13 (electronic): 978-1-4842-2746-6 DOI 10.1007/978-1-4842-2746-6

Moors, J. J. A. (1988). A quantile alternative for kurtosis. *Journal of the Royal Statistical Society: Series D (The Statistician)*, *37*(1), 25-32.

Ristić, M. M., & Nadarajah, S. (2014). A new lifetime distribution. *Journal of Statistical Computation and Simulation*, *84*(1), 135-150.

Srivastava, P. K., & Srivastava, R. S. (2014). Two Parameter Inverse Chen distribution as Survival Model. *International journal of Statistika and Mathemtika*, *11*(1), 12-16.

Tang, Y., Xie, M., & Goh, T. N. (2003). Statistical analysis of a Weibull extension model. *Communications in Statistics-Theory and Methods*, *32*(5), 913-928.

Tarvirdizade, B., & Ahmadpour, M. (2019). A new Extension of Chen Distribution with applications to lifetime data. *Communications in Mathematics and Statistics, 9*, 23-38.

Team, R. C. (2020). R: A language and environment for statistical computing. R Foundation for Statistical Computing.

Sah Telee, L. B., & Kumar, V. (2023). Modified Generalized Exponential distribution. *Nepal Journal of Mathematical Sciences*, *4*(1), 21–32. https://doi.org/10.3126/njmathsci.v4i1.53154

Treyer, V. N. (1964). A distribution law of random variables for the reliability computations for wearable parts of machines and devices. *Doklady Acad. Nauk Belorus. SSR (In Russian), 8*, 47-50.