STRAIN BASED APPROACH IN FATIGUE DAMAGE MODELING OF BRITTLE MATERIAL-APPLICATION TO CONCRETE

Er. Indra Narayan Yadav[*1]

Abstract

Due to very good compressive strength of concrete, it is used widely in all over the world during three decades. The Formulation of Concrete is through combination of Cement, stone aggregate, sand and water according to their design mix based on the ultimate strength required for the structural component. The Mixing of concrete is as mortar, the layer of cement, sand and water is wrapped around the aggregate. When the load is applied to the concrete, the weaker zone i.e. mortar of cement, sand is weaker than stone aggregate, damage by formulation of crack before crack in aggregate. The Damage behavior of Concrete is thus to be analyzed according to their fatigue behavior.

Strain Based approach in Fatigue Damage Modelling of Brittle Material in Concrete is presented to describe the behavior and failure of con-crete by utilizing Damage Mechanics approach. Stiffness degradation and inelastic deformation are the essential features of concrete that develop due tothe formation of multitude of microcracks in the fatigue environment. Microcracking, which is anisotropic in nature, destroys the bond between material grains, and affects the elastic properties resulting in the reduction of material stiffness in elastic as well as plastic stage. This paper presents an anisotropic fatigue damage model for plain concrete subjected to cyclic tension. The model is developed, in strain space, using the general framework of internal variable theory of continuum thermodynamics and Damage Mechanics. It is argued that within the damage surface of given strain states the unloading-reloading cycles (fatigue loading) stimulate the nucleation and growth microcracks in concrete, which will result in stiffness degradation and inelastic deformation, and hence material is termed as damaged. Damage is reflected through the fourth-order stiffness tensor involving a damage parameter whose increment is governed by the consistency equation associated with a cycle dependent damage surface in strain space. The model is capable of predicting stiffness degradation, inelastic deformation and strength reduction under fatigue loading and compared against experimental result.

By increasing the number of loading cycles, the strength of concrete gradually decreases and the limit surface is allowed to contract and form new curves representing residual strengths. The magnitude of loading, load range, and the

[*1] *Assistant Professor, Department of Civil Engineering, Thapathali Campus, Institute of Engineering, Tribhuvan University, Nepal*
Corresponding author
emails: indra_narayan_yadav@yahoo.com

load path are known to influence the fatigue life and hence are addressed in this formulation. In this paper, a strength softening function is proposed in order to address the re-duction in the strength of concrete due to fatigue. Separate softening functions are also proposed to account for the deformation characteristics in concrete under cyclic loading. Numerical simula-tions predicted by the model in both uniaxial and biaxial stress paths show a good correlation with the experimental data available in the literature.

Keywords: *Fatigue; anisotropic; damage; Concrete; Thermodynamics; stiffness; microcracks*

Introduction

Reinforced concrete structures such as bridges, hydraulic foundations, pressure vessels, crane beams are subjected to long term cyclic loading. The effect of cyclic loading is to develop permanent damage in the concrete materials as a result of which failure happens under the stress having value less than the ultimate strength of concrete. Concrete, a heterogeneous material comprising the mixture of cement, sand and aggregate, exhibits several mutually interacting inelastic mechanisms such as microcrack growth and inelastic flow even under small amplitude of cyclic load when applied in large number of cycles. As a consequence, concrete does not guarantee endurance fatigue limit like metal as described in Miner's hypothesis .

The presence of permanent damage at fatigue failure has been documented by a number of investigations.

Developed fatigue damage model for ordinary concrete subjected to cyclic compression based on mechanics of composite materials utilizing the concept of dual nature of fatigue damage, which are cycle dependent and time dependent damage. The model was capable of capturing the cyclic behavior of plain concrete due to progressive creep strain with the increase in number loading cycles. used accelerated pavement testing results for carrying out cumulative fatigue damage analysis of concrete pavement. In , they reported that Miner hypothesis does not accurately predict cumulative fatigue damage in concrete. The experimental work of clearly showed that increase of damage in the material takes place in about last 20% of its fatigue life. presented a theoretical model to predict the fatigue process of concrete in alternate tension-compression fatigue loading using double bounding surface approach described in strain-energy release rate by constructing the damage-effective tensor.

In the past few years, a number of damage constitutive models have been published to model the observed mechanical behavior of concrete under monotonic and cyclic loading. The need for such models arises from the physical observation that two dominant microstructural patterns of deformation in concrete are inelastic flow and microcracking. The inelastic flow component of deformations is modeled by using plasticity theories whereas the nucleation and propagation of microcracks and microvoids is incorporated in the constitutive models with the use of damage mechanics theories. The progressive development of cracks and microcracks alters the elastic properties (degradation of elastic moduli) due to energy dissipation and concrete material becomes more compliant.

This paper presents a class of damage mechanics theory to model the fatigue damage and failure of concrete caused by multitude of cracks and microcracks whereby anisotropic damaging behavior is captured through the use of proper response function involving damage parameter in material stiffness tensor. The increment of damage parameter is obtained from consistency equation in cycle dependent damage surface in strain space. The model is also capable of capturing the inelastic deformations that may arise due to misfits of crack surfaces and development of sizable crack tip process zone.

Formulation

For an inelastic damaging process, with an assumption of small deformations which is valid for brittle materials and low frequency fatigues where thermal effects could be ignored, a constitutive relation between stress and strain tensors can be deduced from Thapa and Yazdani [9] utilizing fourth order material stiffness tensor as :

$$\boldsymbol{\sigma} = \frac{\partial A}{\partial \boldsymbol{\varepsilon}} = \boldsymbol{E}(k) : \boldsymbol{\varepsilon} - \boldsymbol{\sigma}^i(k) \tag{1}$$

where, the stress and strain tensors are given by $\boldsymbol{\sigma}$ and $\boldsymbol{\varepsilon}$ respectively, A denotes Helmholtz Free Energy, $\boldsymbol{E}$ denotes the material stiffness tensor that depends on the state of micro cracks and k is the cumulative scalar fatigue damage parameter. Here, the tensor contraction operation is designated by ":" and the stress tensor corresponding to the inelastic damage is given by $\boldsymbol{\sigma}^i$. Assuming "N" being the fatigue cycle number, taking rate form of Eq. (1) by differentiating with respect to "N":

$$\begin{aligned} \dot{\boldsymbol{\sigma}} &= \boldsymbol{E}(k) : \dot{\boldsymbol{\varepsilon}} + \dot{\boldsymbol{E}}(k) : \boldsymbol{\varepsilon} - \dot{\boldsymbol{\sigma}}^i(k) \\ &= \dot{\boldsymbol{\sigma}}^e + \dot{\boldsymbol{\sigma}}^D - \dot{\boldsymbol{\sigma}}^i(k) \end{aligned} \tag{2}$$

where, $\dot{\boldsymbol{\sigma}}^e$ denotes the stress increment when further damage in the material is prevented, $\dot{\boldsymbol{\sigma}}^D$ is the stress relaxation rate due to further micro cracking (elastic damage) and $\dot{\boldsymbol{\sigma}}^i(k)$ designates for the stress tensor rate corresponding to the irreversible deformation due to micro cracking. It is further assumed that, damage during fatigue loading degrades elastic properties and affects the stiffness tensor. The damage is recorded in the fourth order material stiffness tensor **E**. To account for induced anisotropy, the following additive decomposition of **E** is adopted:

$$\boldsymbol{E}(k) = \boldsymbol{E}^O + \boldsymbol{E}^D(k) \tag{3}$$

where, $\boldsymbol{E}^O$ is the stiffness tensor of the initial undamaged or uncracked material and $\boldsymbol{E}^D(k)$ is the overall stiffness degradation caused by damage during fatigue loadings. Further, the rates of stiffness tensor $\dot{\boldsymbol{E}}(k)$ and that of the inelastic stress tensor $\dot{\boldsymbol{\sigma}}^i$ are expressed as fluxes in thermodynamics state sense and are expressed in terms of evolutionary equations as,

$$\dot{\boldsymbol{E}}^D = -\dot{k}\boldsymbol{L} \text{ and } \dot{\boldsymbol{\sigma}}^i = \dot{k}\boldsymbol{M} \tag{4}$$

where, **L** and **M** are the fourth and second order response tensors that determine the direction of the elastic and inelastic damage processes.

To progress further, specific forms of response tensor **L** and **M** must be specified. Since the damage is highly directional (i.e. anisotropic), the response tensor should be formulated to achieve such anisotropy. For addressing anisotropy and formulating response tensor, lets decompose the strain tensor into its positive and negative cones. The positive and negative cones of the strain tensor holds the corresponding positive and negative Eigen values of the system. Note that. $\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^{+} + \boldsymbol{\varepsilon}^{-}$ Guided by the experimental results and observations for glass/epoxy woven fabric composite materials in tension by Hansen [3], where majority of damage is shown to take place in the direction of applied strain and in tension regimes (i.e. the cleavage mode of cracking as shown in Fig.1), and further assuming that there is no coupling between cleavage type cracks in orthogonal direction, the following form of response tensors are proposed.

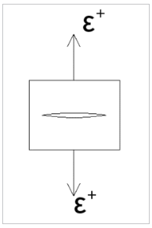


Figure 1. Cleavage mode of cracking

$$\boldsymbol{L} = \frac{\boldsymbol{\varepsilon}^{+} \otimes \boldsymbol{\varepsilon}^{+}}{\boldsymbol{\varepsilon}^{+} : \boldsymbol{\varepsilon}^{+}} \qquad (5)$$

$$\boldsymbol{M} = \frac{\alpha \boldsymbol{\varepsilon}^{+}}{(\boldsymbol{\varepsilon}^{+} : \boldsymbol{\varepsilon}^{+})^{1/2}} \qquad (6)$$

where, the symbol "⊗" is the tensor product operation. Here, it should be noted that the material used in Hansen's work was assumed to be quasi-isotropic laminate, with the lay-up sequence of $[(+45^0\#-45^0)/(90^0\#0^0)]_s$, made up of plain woven glass/epoxy prepreg fabrics. So, the strength values in different directions are considered equal initially.

In this paper, we proposed an alternate and new damage evolution law based on the second invariant of the positive cone of the strain tensor based on the fact that the damage in each cycle depends on current elastic stiffness E, number of cycles N and second invariant of the positive cone of strain tensor.

$$k = E \int_0^N A \left(\frac{\varepsilon^{+} : \varepsilon^{+}}{{\varepsilon_O}^2} \right)^m N^B dN \qquad (7)$$

where, A, B and m are material constants, N is the number of cycles and ε_O is the reference strain level. Differentiating with respect to number of cycles N, the increment damage in one cycle is given as,

$$\dot{k} = EA \left(\frac{\boldsymbol{\varepsilon}^{+} : \boldsymbol{\varepsilon}^{+}}{{\varepsilon_O}^2} \right)^m N^B \qquad (8)$$

Substituting Eqs. (5), (6) and (8) into Eq. (4), yields

$$\dot{\boldsymbol{E}}^D = -EA \left(\frac{\boldsymbol{\varepsilon}^{+} : \boldsymbol{\varepsilon}^{+}}{{\varepsilon_O}^2} \right)^m N^B \frac{\boldsymbol{\varepsilon}^{+} \otimes \boldsymbol{\varepsilon}^{+}}{\boldsymbol{\varepsilon}^{+} : \boldsymbol{\varepsilon}^{+}} \qquad (9)$$

$$\dot{\boldsymbol{\sigma}}^i = EA \left(\frac{\boldsymbol{\varepsilon}^{+} : \boldsymbol{\varepsilon}^{+}}{{\varepsilon_O}^2} \right)^m N^B \frac{\alpha \boldsymbol{\varepsilon}^{+}}{(\boldsymbol{\varepsilon}^{+} : \boldsymbol{\varepsilon}^{+})^{1/2}} \qquad (10)$$

Substituting Eqs. (3) and (4) into Eq. (2), we get:

$$\dot{\boldsymbol{\sigma}} = \boldsymbol{E}:\dot{\boldsymbol{\varepsilon}} + \left[-EA\left(\frac{\boldsymbol{\varepsilon}^+:\boldsymbol{\varepsilon}^+}{\varepsilon_O{}^2}\right)^m N^B \frac{\boldsymbol{\varepsilon}^+ \otimes \boldsymbol{\varepsilon}^+}{\boldsymbol{\varepsilon}^+:\boldsymbol{\varepsilon}^+}\right]:\boldsymbol{\varepsilon} - EA\left(\frac{\boldsymbol{\varepsilon}^+:\boldsymbol{\varepsilon}^+}{\varepsilon_O{}^2}\right)^m N^B \frac{\alpha\boldsymbol{\varepsilon}^+}{(\boldsymbol{\varepsilon}^+:\boldsymbol{\varepsilon}^+)^{1/2}} \quad (11)$$

Numerical Simulation

The proposed model has four material parameters A, B, m and α. In order to check the validity of the model and to obtain the value of the parameters introduced, numerical simulation have been performed and the comparison is done with the experimental results by Hansen [3]. Due to scarcity of experimental data in the literature for the measurement of these parameters in the numerical simulation, judgments were used to obtain the numerical results.

On the basis of the experiment carried out by Hansen [3], the following constants were used for numerical simulation. A = 0.03, B = -0.74, m = 1.4, and α=0.0035. These parameters were estimated by comparing predicted results and experimental results over a range of applied stresses. The initial strain tensor was taken $\varepsilon^+ = [0.007 \quad 0 \quad 0]$ corresponding to the applied far-field stress tensor of $\sigma^+ = [155 \quad 0 \quad 0]$ MPa. The reference strain was fixed at 0.0156. In the case of BVID specimen, the initial uniform impact damage was considered to be 2.5% prior to fatigue.

In Fig.2, the complete static stress-strain behavior is illustrated and compared with experimental data by Hansen [3]. The similar trends are observed. The maximum stress level is seen to be 310MPa giving a strain of 0.02. After this peak value, the stress goes on decreasing as strain keeps on increasing. The material is said to be reached at failure strain at the value of 0.025 as also presented by the experiment. Similarly, Fig.3 clearly shows that the proposed model captures the initial damage due to the knee effect quite well. The model is useful in prediction of Phase I and Phase II but could not predict Phase III. The predicted S-N curve is compared with the experimental S-N curve by Hansen [3] for BVID specimen. The effect of material parameter "m" on predicted S-N curve is also shown in Fig.4. It can been seen that the trend of predicted and experimental S-N curves matches each other which is quite satisfactory.

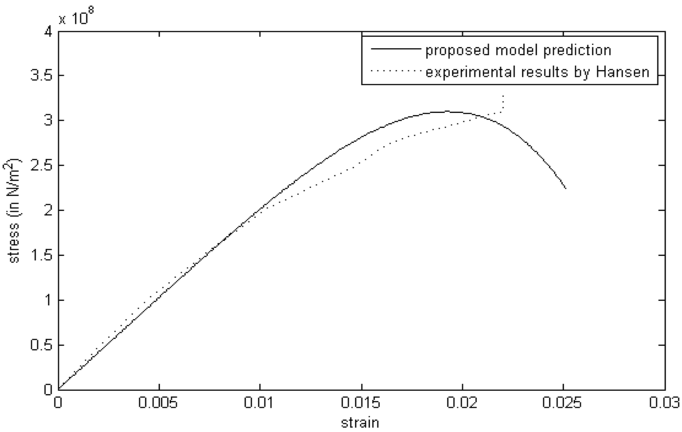


Figure 2. Static monotonic stress-strain curve

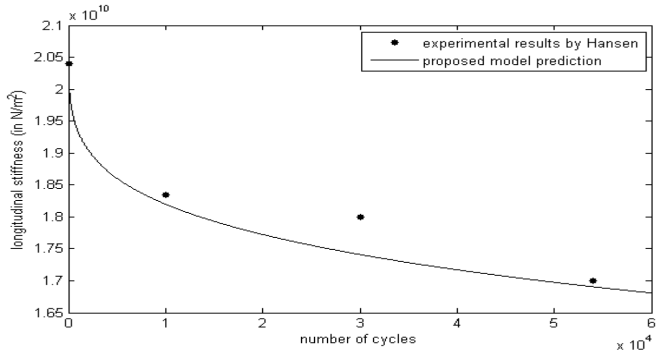


Figure 3. Longitudinal stiffness reduction with number of cycles

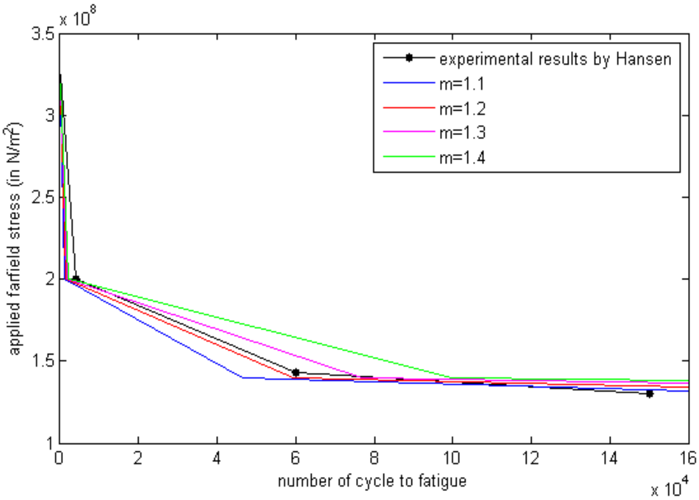


Figure 4. Comparison of experimental S-N curve with predicted S-N curves

The fatigue stress-strain curves are shown in Fig.5 and 6. Fig.5 illustrates the elastic degradation as well as a permanent deformation due to inelastic behavior. However, Fig.6 illustrates the nature when the composite is treated as perfectly elastic with only reversible deformations, though the damage is progressively accumulating as evidenced by degradation of longitudinal stiffness. This behavior corresponds to an idealized case whereby crack faces close perfectly upon unloading and is achieved in the model by letting $\alpha=0$. But in most of the heterogeneous materials like glass-fiber composites, permanent deformations take place which has been replicated in Fig.5. The inelastic damage accumulation i.e. the cumulative permanent deformation under fatigue loading is also illustrated in Fig.7.

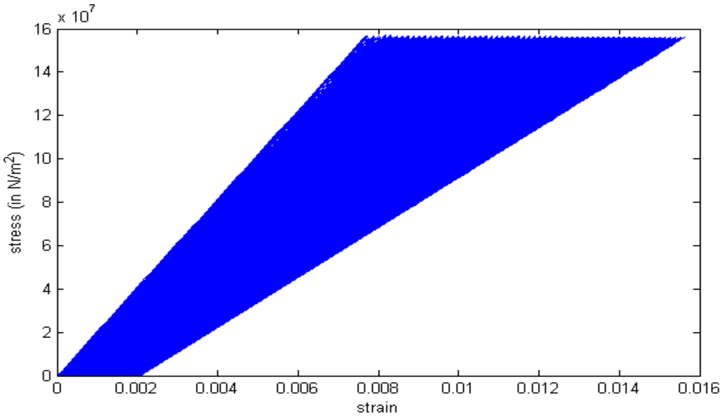


Figure 5. Stress-strain behavior of woven composite during inelastic damage accumulation (Nf=48993, εf=0.0156, σf=155MPa)

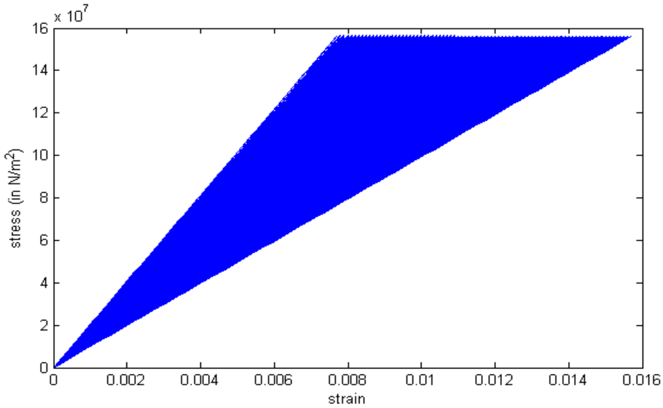


Figure 6. Stress-strain behavior for elastic damaging process (Nf=129355, εf=0.0156, σf=155MPa)

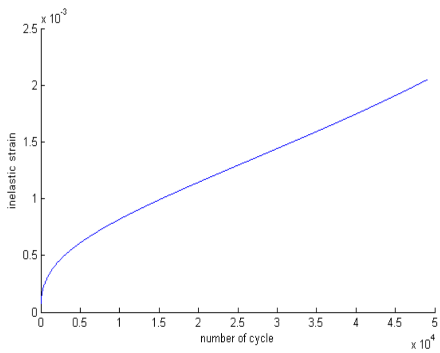
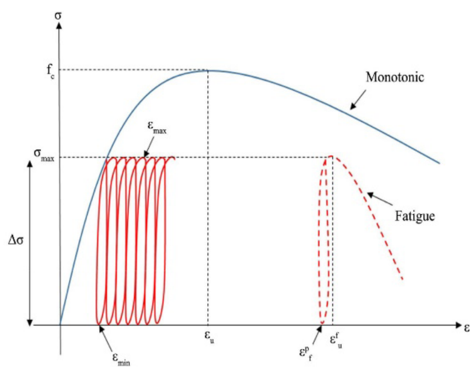
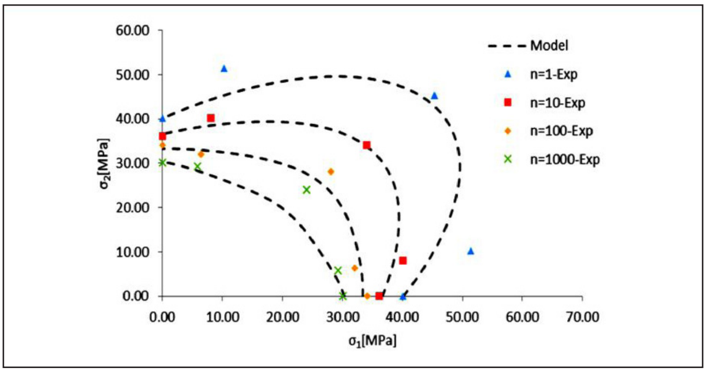


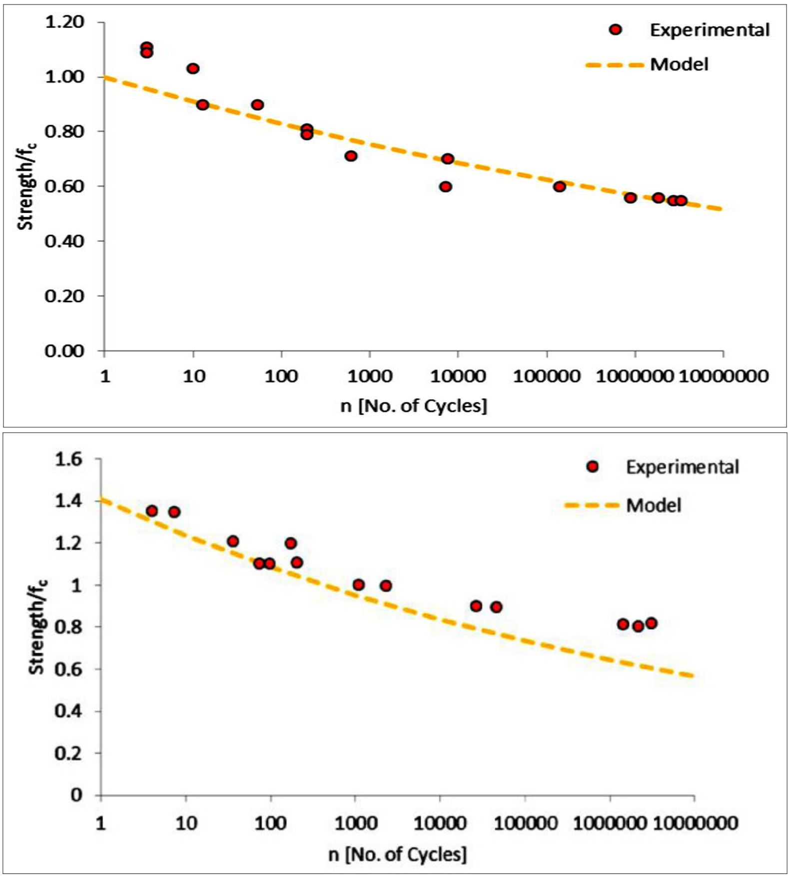
Figure 7. Inelastic damage accumulation



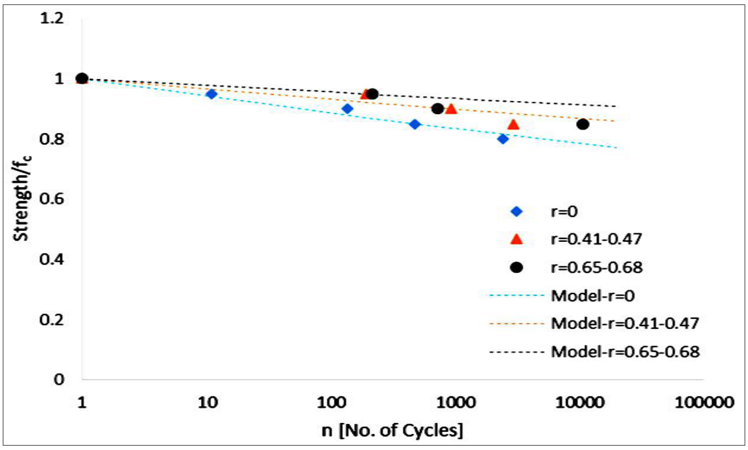
Schematic representation of stress-strain of concrete under mono-tonic and cyclic loading.



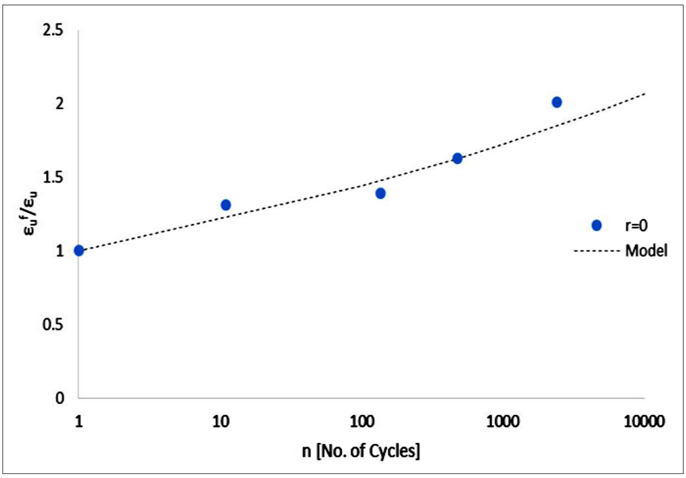
Graph Residual Strength Surface for Various number of Cyclic loading, experimental data by Neslon et al.



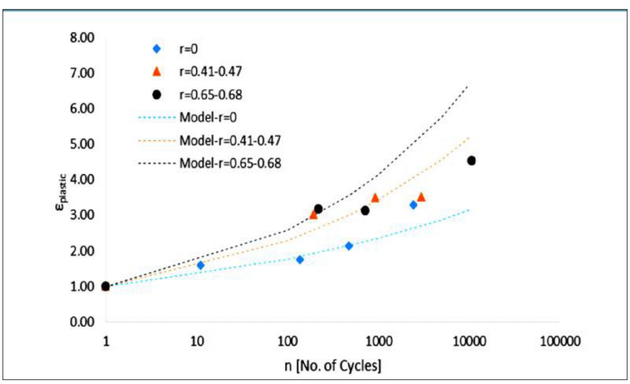
Graph regarding S-n curve for concrete under biaxial cyclic loading with stress ratio 1.0, experimental data by Yin and Hsu



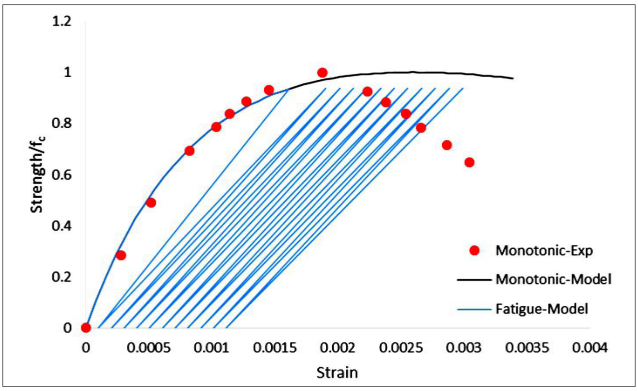
Graph regarding S-n curves for concrete under uniaxial loading with various stress ranges, experimental data by Awad



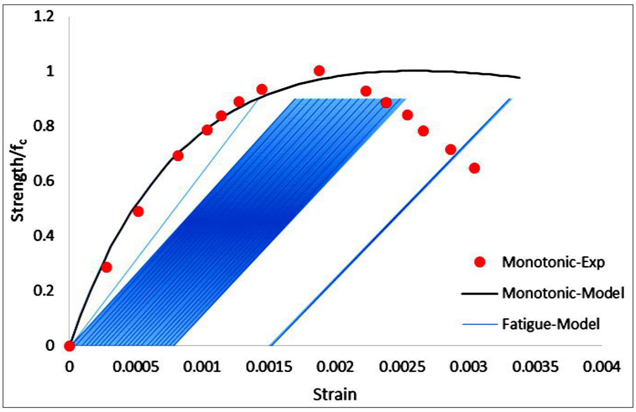
Graph regarding Ultimate strain versus number of cycles for concreteunder uniaxial cyclic loading, experimental data by Awad



Plastic strain versus loading cycles under uniaxialcyclic loading with various stress ranges, experimental data by Abadh



Stress-strain curves under cyclic (σmax/fc = 0.95)and monotonic uniaxial loading, experimental data by Awad



Conclusion

An anisotropic-inelastic fatigue damage model is established for concrete fatigue utilizing Continuum Damage Mechanics formulation in strain-space. Under cyclic fatigue loading, the limit surface is allowed to contract and form new surfaces identified as residual strength surface. This is accomplished by proposing a softening function that is based on amplitude, stress ratio, and load path. By including these parameters, the effects of strange range and the load paths on the fatigue life of concrete are studied and predicted. Furthermore, to capture the effects of fatigue loading on stress-strain behavior of concrete, two additional strain softening functions are proposed for changes in ultimate and residual (plastic) strains.[2]

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3. *U. Hansen, "Damage Developement in Woven Fabric Composites during Tension-Tension Fatigue," Journal of Composite Materials, vol. 33, no. 7, pp. 614-639, 1999.*

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The influencing factors on ultimate and plastic strains such as amplitude, load path, and load range are incorporated into the proposed softening functions. The distributed progressive weakening of the composite due to the formation of micro-cracks and micro-voids is reflected through the reduction of fourth-order material stiffness tensor. The damage mechanics formulation is cast within the general framework of the internal variable theory of thermodynamics considering rate-independent behavior, infinitesimal deformations and Helmholtz Free Energy (HFE) as an energy potential. The proposed damage evolution law consists four damage parameters. For the numerical simulation, the parameters, A, B, m and α are determined from the sensitivity study and the values are selected such that they best fit the experimental data. However, this model is quite simple as it considers only uniaxial loading condition and has a potential to be extended for multiaxial loading conditions.[3]

23. *K. B. Thapa and S. Yazdani, "Combined damage and plasticity approach for modeling brittle materials with application to concrete," International Journal of Civil and Structural Engineeering, vol. 3, no. 3, pp. 513-525, 2013.*
24. *D. Bhandari and K. B. Thapa, "Constitutive Modeling of Concrete Confined by FRP Composite Jackets utilizing Damage Mechanics Theory," in ResearchGate, India, 2013.*
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