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Amalgamation of Mathematics in Economics: A Historical and Theoretical Perspective

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ABSTRACT

The integration of mathematical models into economics has evolved into a cornerstone of contemporary economic research. This progression, marked by increasing algebraic usage, reflects the discipline's shift toward a structured, rigorous framework. Despite its value, the adoption of mathematical methods faced challenges and debates over their practical efficacy, highlighting the dynamic interplay between theoretical insights and practical applications to shape modern economic analysis. The paper addresses the lack of a simple and consolidated framework of essential mathematical concepts and their applications in economics. Focusing on that problem, this paper aims to examine the evolution of mathematics in classical economics. Moreover, role of mathematics in solving contemporary economic problems related to decision-making and forecasting, and its contribution to improving economic efficiency are also considered. To fulfill objectives, a qualitative method with descriptive analysis is used to examine the foundational works of classical economists and its application in current economic issues. Mathematical models and matrix algebra are used to forecast trends, maximize resources, and examine interdependencies among economic variables in Nepal, while game theory directs strategic policies in areas like hydropower trade. The findings reveals that mathematical tools and models have transformed economic analysis, offering a robust framework for addressing complex policy issues and promoting sustainable development, particularly in Nepal and beyond.

1 Introduction

Economics, as a social science, investigates the optimal allocation of resources such as land, labour, capital, machinery, raw material to fulfill society's needs and desires. It focuses on efficiently distribution of resources to produce goods and services. Meanwhile, mathematics, which originated from practical activities like counting, measuring, and analyzing shapes, has evolved into a discipline rooted in logical reasoning and numerical computation. Over centuries, mathematics has advanced from addressing practical problems to solving abstract and idealized concepts. Since the 17th century, it has played a pivotal role in technological innovation and the physical sciences. More recently, mathematics has also become indispensable in the quantitative analysis of the biological sciences, demonstrating its versatility and far-reaching applications (Tarasov, 2019) .The integration of mathematics into economics has transformed the discipline over time. In 1930, only 10% of publications in prestigious journals such as The Economic Journal and The American Economic Review employed algebraic methods. By 1980, this figure had risen dramatically to 75%, reflecting the growing reliance on mathematical approaches in economic research (Reiss, 2000)William Stanley Jevons and Gustav Schmoller with respect to the issue of whether mathematics is or is not an adequate language to express economic relationships. First, Menger's and Jevons's respective methodologies are identified as Aristotelian which means, inter alia, that economic properties are real, are naturally related to each other, exist as part of the observable world and can be separated (in thought or otherwise. This shift highlights the profound influence of mathematical formalism on the field of economics.

Mathematics enhances the rationality and precision of economic arguments and conclusions. It enables the visualization and quantification of complex economic concepts, hypotheses and challenges. By systematically expressing ideas involving size, quantity and time, mathematics provides a powerful toolkit for addressing economic problems. Tools like input-output tables and linear programming models demonstrate how mathematical methods are used to analyze and solve complicated economic phenomena (Grubel & Boland, 1986). Moreover, quantitative techniques allow economists to compare current analyses with historical data, ensuring assessments and conclusions are grounded in reliable calculations. Graphical and mathematical representations have become central to articulating economic theories, bridging abstract ideas and real-world applications. The application of mathematics to economics dates back to the very beginning of human society, however, the late 19th century, with pioneers such as William Stanley Jevons, Léon Walras, and Irving Fisher establishing a foundation for mathematical economics, the mathematics took a huge leap in contributing the economic concepts. These early contributions introduced a structured approach to economic analysis that continues to shape modern tools and methods (Reiss, 2000)William Stanley Jevons and Gustav Schmoller with respect to the issue of whether mathematics is or is not an adequate language to express economic relationships. First, Menger's and Jevons's respective methodologies are identified as Aristotelian which means, inter alia, that economic properties are real, are naturally related to each other, exist as part of the observable world and can be separated (in thought or otherwise.

Classical economics, emphasizes the self-regulating nature of markets, rational behavior and the interaction of supply and demand in resource allocation by using figures. While classical economics initially relied on qualitative reasoning, the incorporation of mathematical tools like functions, graphs and equations has expanded its scope and precision. Techniques such as the Keynesian IS-LM model, optimization theory, and game theory extend classical concepts, providing structured frameworks for interpreting and solving complex economic problems. For instance, market equilibrium economics can be determined through solving linear equations representing demand and supply (Dewett, K.K., Varma, 2003). For more instance, the interaction between the real market equilibrium (represented by IS curve) and money market equilibrium (represented by LM curve) provides the general equilibrium in the entire economy (Froyen, 2014). These approaches highlight how mathematics deepens our understanding of economic relationships and provides exact solutions to persistent questions.

Despite the recognized importance of mathematics in economics, research articles focusing on this intersection remain limited, particularly those offering a structured, practical approach. Existing literature often lacks clarity and accessibility for students, making it challenging to grasp some common fundamental mathematical concepts and their applications in economics. Advanced econometric tools and specific models are well-studied, yet there is a need for research that simplifies and consolidates these ideas. This study seeks to bridge this gap by presenting essential mathematical concepts and their applications in economics in a simplified, comprehensive framework. It aims to serve as an accessible guide for learners, enhancing their understanding of how mathematics underpins and enhances economic theory and practice.

1.1 Objectives

The general objectives of the study are examining the growing role of mathematics in classical economics and its contribution to understanding, analyzing, and solving complex modern economic problems. However, the specific objectives of the study are as follows;

- To analyze the application of mathematical tools, such as functions, simultaneous equations, matrix algebra, calculus, logarithms in addressing economic problems since the classical period.
- To explore how mathematical methods improve decision-making and prediction in areas like demand and supply analysis, market equilibrium, resource allocation, and trade.
- To demonstrate the role of mathematical knowledge in enhancing the efficiency in the economy.

1.2 Literature Review

Adam Smith, widely regarded as the father of economics, made significant contributions to economic theory primarily in his book "The Wealth of Nations," published in 1776 AD. Since, Adam Smith and his followers in the Classical School of economics were not believing strongly on mathematical models, he used mathematics in limited areas compared to later economists, but he laid important conceptual foundations that later scholars had used to formalize mathematical methods. Their primary goal was to create economic theory based on observation and reasoning rather than mathematical equations. Smith's work primarily used qualitative analysis and logical reasoning rather than formal mathematical models. He contributed in economic concept about division of labour, invisible hand, value theory, market mechanism and many more in qualitative manner. Smith's analysis was more descriptive and were using logical arguments and examples of the real world. His discussion on the division of labour and productivity could be seen as an early understanding of marginal productivity, which was formalized mathematically later on. Similarly, the concept of "invisible hand" was developed into general equilibrium theory and a mathematical model by economist Leon Walras and Vilfredo Pareto later on (Arrow & Debreu, 1954)exchange and consumption. In addition the assumptions made on the technologies of producers and the tastes of consumers are significantly weaker than Wald's. Finally a simplification of the structure of the proofs has been made possible through use of the concept of an abstract economy, a generalization of that of a game.

Prof. Dr. Alfred Marshall, leader of neo-classical economists, contributed in economics by his innovative use of mathematics to develop economic models. His integration of mathematical concepts provided a foundation for many economic theories and methodologies. Marshalls' Book "Principles of Economics" published in 1890 A.D. was a keystone of economic literature, where he extensively used mathematics to clarify economic principles. Demand and supply analysis was one of the most important contributions, where he used mathematical curves to represent demand and supply functions. By the interaction of these two functions further he suggests to determine the market equilibrium, equilibrium price and quantity.

He introduced the concept of elasticity of demand and supply to measure the responsive of one dependent variable to the change in independent variables. This relation is expressed by Marshall as presented in Eq.1.

$$Ep = -\frac{\triangle Q}{\triangle P} \times \frac{P}{Q} \quad \dots (1)$$

Furthermore, he used mathematics to define and calculate consumer surplus and producer surplus, which measures the benefits to consumers and producers in a market. similarly, he developed the marginal concept as the additional values made on dependent value obtained from the extra one unit of an independent value. Under this concept, he guided to derived marginal values of utility, cost, revenue and so on. He formalized the law of diminishing returns, and used mathematical functions to describe the relationship, and to show how it affects production decisions and cost structures (Ahuja, 2004).

Augustin Cournot, who published "Researches into the Mathematical Principles of the Theory of Wealth" in 1838, was a pioneer of mathematical economics. Cournot used mathematical equations in his book to describe how firms compete in a market and determine output levels based on goods demand and supply. John Stuart Mill was another economist who made significant contributions to the use of mathematics in economics in the 1800s. Mill used mathematical models to analyze the relationship between supply and demand and to explain the laws of diminishing marginal utility in his book "Principles of Political Economy" published in 1848 (Cournot & Bacon, 2020).

In 1881, Francis Ysidro Edgeworth applied Jeremy Bentham's felicific calculus to economic behavior, allowing each decision's outcome to be converted into a change in utility. Edgeworth based his exchange model on three assumptions: individuals are self-interested, individuals act to maximize utility, and individuals are free to reconstruct with another third-party independently. Edgeworth displayed the contract curve on an economy with two participants in a box named Edgeworth box. The contract curve describes the set of solutions where both individuals can maximize utility (Edgeworth, 1881). William Stanley Jevons who used math first to study how people buy things. In his 1871 book, "The Theory of Political Economy," he used equations to explain how people decide what to buy based on the satisfaction they get from different goods and services. Leon Walras, created the idea of general equilibrium theory. He used math to show how different markets in an economy interact and how prices and amounts are set through equations that all work together. Other important economists who used math in their work in the early 1900s include Vilfredo Pareto, Irving Fisher, and John Bates Clark. Pareto used math to study how income and wealth are spread across society and how the equilibrium is attained in the economy (Pareto, 2014). Fisher used equations to explore the link between the amount of money and price levels in the economy and he developed the equation of exchange (Persons, 2012). Clark, in his 1899 book "The Distribution of Wealth," used math to look at what determines how income and wealth are divided in society.

Using math in economics in the early 1900s marked a big change in how economists thought and led to the development of more advanced mathematical models in the years that followed. Hermann Heinrich Gossan, a Prussian economist, believed that math was not just a tool to handle the complexity of the economic world, but also a crucial part of pure economics. In his 1854 work (Gossen, 1983), he used the complexity of economics to support his mathematical approach as, "For the justification of this framework, it suffices to observe that economics concerns itself with the interplay of a variety of forces and that it is impossible to determine the resultant effect without calculus. For this reason, it is impossible to present the true system of economics without the aid of mathematics - a fact that has long been recognized in the case of pure astronomy, pure physics, mechanics and so forth."

The Leontief matrix and national income models were examined in a 2022 study by Sheeja K. on the subject of “Matrix Algebra and Economic Models,” which looks into the use of matrix algebra in economic modeling. The primary objective of this study is to comprehend how these models depict interdependencies among economic sectors and predict results. Both the open and closed Leontief models use input-output approaches to evaluate industrial linkages, whereas the National Income model determines national income based on government spending, investment, and consumption. Matrix equations are methodologically solved using Gaussian elimination, matrix inversion, or Cramer’s rule. These models’ outcomes show how accurately they forecast economic patterns, such as shifts in the demand for private automobiles during the COVID-19 pandemic. Ultimately, matrix-based approaches offer significant role in solving contemporary problems (Sheeja, 2022).

In summary, Adam Smith established the qualitative foundation of economics, which was subsequently developed by the use of mathematical models to express economic ideas by Cournot, Mill, Edgeworth, and Jevons. The theories of equilibrium and income distribution were established by Walras and Pareto, while Marshall introduced elasticity, consumer surplus, and marginal analysis. Recent research highlights previous contributions as the basis for present developments. On recent time there has been a rise in empirical research and the establishment of mathematical models and tools that continue to improve economic decision-making and analysis. However, historical integration of mathematics into economics served as the basis for contemporary developments.

2 Methodology

This study categorizes the use of mathematics in economics into two periods: the classical era and the modern era. Classical economics, spanning from ancient times to the early 20th century, focused on foundational principles such as trade, wealth accumulation, and qualitative theories like Adam Smith’s The Wealth of Nations. This era gradually embraced basic mathematical tools, with contributions from figures like Marshall, Cournot, and Jevons, who introduced algebraic and calculus-based approaches to supply, demand, and marginal analysis. Modern economics, emerging in the mid-20th century, shifted toward formalization and computational methods. It integrated advanced tools like econometrics, game theory, and optimization techniques in more advanced way though they are introduced already in classical era, evolving into data-driven disciplines powered by big data, machine learning, and AI to analyze complex economic systems and behavior. Fig. 1 and Fig. 2 provide the mathematical classification of theoretical economic eras. In this paper, only relationship between mathematics and economics within the classical economics has been explored and some empirical practices in contemporary Nepalese economy are evaluated. Among the numerous connections between mathematics and economics explored by classical era economists, this study moved on the direction of interpretivism ontology. These relationships are crucial and provide many directives to formulate and apply economic policies. This study uses a qualitative approach to explore how mathematics has been applied to explain some economic theories. Tables and diagrams are used to illustrate mathematical applications.

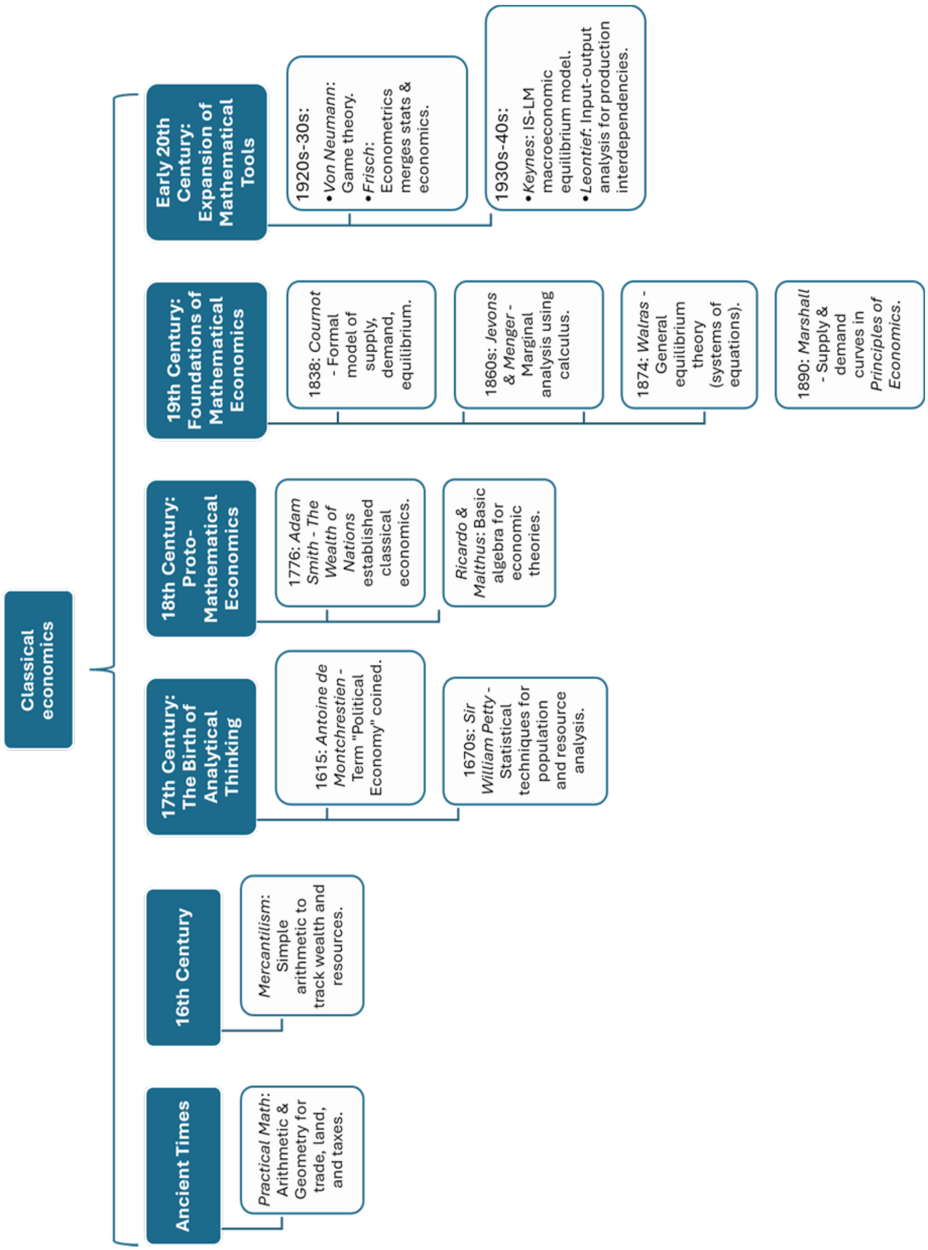

Fig. 1. Classical era as per the evolution of mathematical usage in economics

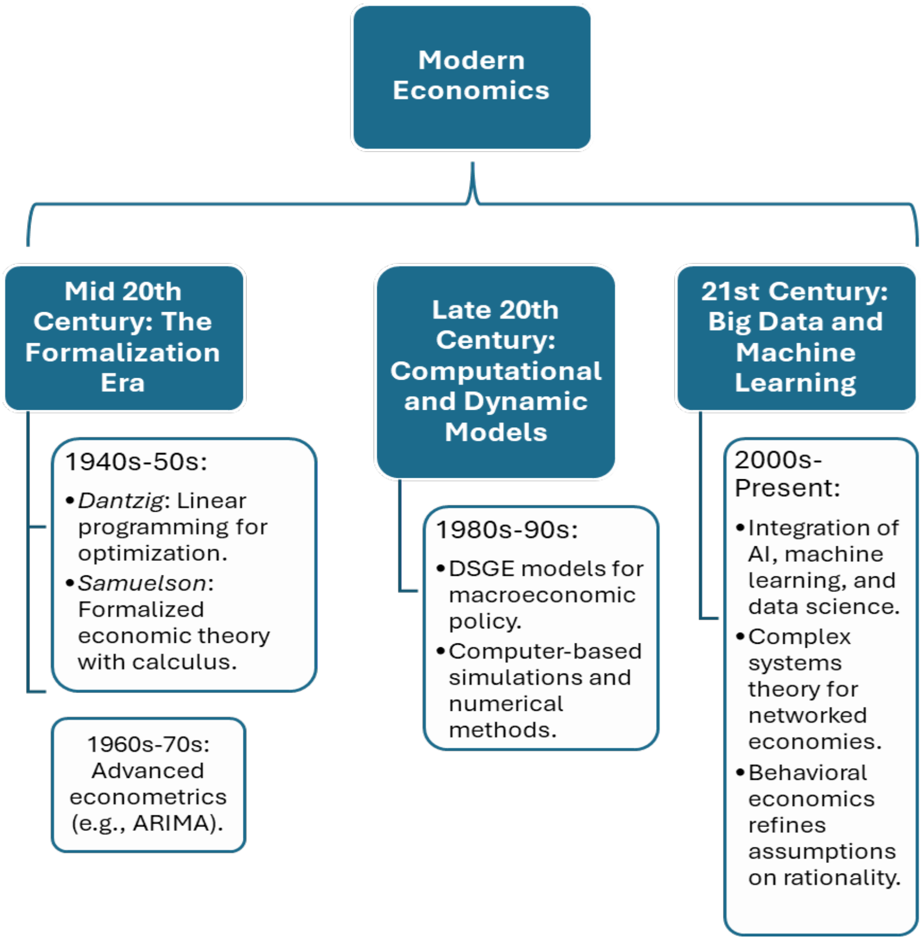

Fig. 2. Modern era as per the evolution of mathematical usage in economics

This research conducts an extensive review of academic literature, including books, journal articles, historical papers, and dissertations. The review was focused on seminal contributions and was carried out using sources from JSTOR, Google Scholar, and institutional databases. Descriptive statistics were employed to explore the use of mathematics in economics and to systematically present the findings.

3 Exploration and discussion: Mathematics in Economics

Initially economics depend on verbal reasoning with minimal mathematics, but modern economics extensively uses mathematical models to establish theories and solve economic issues. Mathematical knowledge is now essential for decision making, forecasting and enhancing efficiency of economy. Some amalgamation of mathematics in economics are detailed in the following sections:

3.1 Mathematical Function in Economics

A function is an operation that is associated with each element of one set (domain set) with one or more elements of another set (range set). A function describes the relation between two or more than two variables. As a positive science, economics shows the cause-and-effect relationship between the several economic variables in the form of mathematical function (Koutsoyiannis, 1979). Some functional relationships in economics are;

$Qd = f(P)$, $Qs = f(P)$, $C = f(Y_d)$ etc. where, Q_d = Quantity demanded, Q_s = Quantity supplied, Y_d = Disposable income, C = consumption and P = price. By equalizing the demand and supply function, static equilibrium in micro economics is obtained. Thus, at the equilibrium position (Ahuja, 2004) and efficient allocation of resources are calculated easily. The market will be in efficient position, when the total surplus of the economy is maximum and the total surplus is the sum of consumer's surplus and producer's surplus.

3.2 Simultaneous Equations in Economics

Simultaneous equations are two (or three or more) equations with two (or three or more) unknown variables that can only be satisfied by pair values of these two (or three or more) variables at the same time (Christ, 2001). With the use of simultaneous equations, many economic models are developed where multiple variables are independent (LaFrance, 1985). For instance, demand equation and supply equation can be used to determine the market equilibrium (Eq. 2, 3) and Eq. 4 is for the market equilibrium.

$$Qd_x = a - bp_x \quad \text{... (2)}$$

$$Qs_x = c + dp_x \quad \text{... (3)}$$

At equilibrium, $$Q_d x = Q_s x \quad \text{... (4)}$$

Similarly, to determine the equilibrium national income and interest rate two simultaneous equations of IS curve and LM curve are used (Eq. 5 and 6). To illustrate JM Keynes' concepts from the book General Theory of Employment, Interest, and Money (1936) in a graphical expression, John Hicks developed the IS-LM curve hypothesis in 1937. Particularly when considering the interaction between the money market and goods market, Hicks's work contributed to the formalization and simplification of Keynesian economics (Hicks, 1937). The IS-LM model represents the demand side of the economy and emphasizes the short-term analysis of monetary and fiscal policy variables (Black & Dowd, 2005). In the equations below, IS curve is provided as the real market equilibrium and LM curve is provided as the money market equilibrium. Also, 'i' is the interest rate and 'Y' is the national income. 'k' and 'h' are coefficients dependent on the real market. The solution of these equations provides the equilibrium national income.

$$\text{IS curve} = k_1 Y + h_1 i \quad \text{... (5)}$$

$$\text{LM curve} = k_2 Y - h_2 i \quad \text{... (6)}$$

Graphical methods are limited to solving problems involving only two variables. However, problems with more than two variables can be effectively resolved using simultaneous equations and their related techniques.

3.3 Matrix Algebra in Economics

Matrix is defined as a rectangular array of numbers or any other symbols in rows and columns and enclosed by two capital brackets (Arfken et al., 2013). One of the most prominent use of Matrix algebra has been observed in input output analysis (Theil, 1983). Leontief's 1936 input-output model was the first to popularize matrix algebra in economics, which represented inter-industry relationships using matrices (Leontief, 1936). Now, it is one of the emerging branches of economics study which is a quantitative technique and is used to examines the interdependencies between different economic variables of different economic sectors (Munroe & Biles, 2005).

Through an analysis of the relationships between production factors and output, the input-output model examines the interdependencies among economic sectors. It aids in the evaluation of trade and environmental implications, forecasting of economic growth, sectoral linkages, and policy consequences. While emphasizing replacement opportunities and indirect effects across industries, the model helps with resource allocation, including capital, labor, and material requirements, forecasts production and import levels, and facilitates national income accounting. Any economy will naturally undergo sectoral shifts over time. The economic structure of Nepal has changed from being mainly dependent on agriculture to significantly on service sector (Kafley & Joshi, 2023). To explore the use of matrix algebra in economics, one illustration is made in this paper. For instant, let the economic structure of Nepal can be defined by three sectors as foreign employment, tourism and agriculture sector. These sectors are not only vital individually but are also intricately interlinked, forming a dynamic economic structure.

Foreign employment primarily generates remittances, which substantially contribute to national income and enhance household purchasing power. Tourism, encompassing earnings from foreign visitors, drives economic activities in hospitality, transportation, and other service industries. Agriculture, serving as the backbone of Nepal's rural economy, includes crop production, livestock, and related activities. The interactions among these sectors are notable: agriculture supplies raw materials to the tourism sector, while tourism stimulates demand for agricultural product. Simultaneously, remittances elevate household incomes, further fueling demand for both agricultural goods and tourism services.

Input-output analysis is a useful method for assessing the relationships between these three important sectors: tourism, foreign remittances, and agriculture. Employing an input-output framework with matrix algebra offers a robust methodology to analyze these interdependencies, providing insights into their collective contributions to the Nepalese economy. The input output matrix (A) of order 3×3 for these three sectors can be expressed as;

$A = [a_{ij}]_{\text{i = 1, 2, 3 and j = 1, 2, 3.}}$

The matrix represents proportions of output from sector 'j' used as an input in sector 'i'. The total output equation (X) is calculated as; $X = (I-A)^{-1}D$. Where, 'I' is the identity matrix of order 3×3, D is the final demand of three sectors which includes domestic consumption demand, exports demand and investments demand, and $(I-A)^{-1}$ is the Leontief Inverse Matrix

Leontief inverse $(I-A)^{-1}D$ captures both direct inputs and the ripple effects across sectors, highlighting the interdependencies in the economy. Agriculture depends significantly on remittance-financed consumption, while tourism indirectly drives demand for agricultural products and services. Additionally, remittances increase household disposable income, further boosting demand for tourism and agricultural outputs. Policy simulations, such as analyzing the impact of increase in remittance inflows or growth in tourism earnings due to government promotion, can reveal their multiplier effects on agriculture and overall economic output. In scenarios like these, matrix algebra assists in quantifying the effects of investments on tourism and agriculture, identifying important sectoral links, and designing strategies to reduce dependency on remittances.

3.4 Differential Calculus, Integral Calculus and Optimization Theory in Economics

Differential calculus is an important branch of mathematics and frequently used in economics theory. It is the most common type of math found in economics. It includes the use of various formulas to measure limits, functions and derivatives. Many economists use derivatives to calculate the marginal values like marginal utility, marginal cost, marginal revenue, marginal profit, elasticity of demand etc. especially exterior differential calculus (Chiappori & Ekeland, 2004).

For an instance, we can derive some marginal values as presented in Eq. 7, 8 and 9.

$$\text{Marginal utility (MU)} = \frac{dTU}{dQc} \quad \ldots (7)$$

$$\text{Marginal cost (MC)} = \frac{dTC}{dQp} \quad \ldots (8)$$

$$\text{Marginal revenue (MR)} = \frac{dTC}{dQs} \quad \ldots (9)$$

Here, Q_c, Q_p and Q_s are quantity consumed, quantity produced, and quantity sold respectively.

In Mathematics, optimization refers to the selection of the best option from some of the available alternatives. Economics is closely related with the optimization problem i.e., maximization and minimization. In economics it refers to the maximization or minimization of an economic function by selecting the best combination of inputs or independent variables. It gives the equilibrium position of a household, a firm or a policy maker. Differential calculus is the key element to solve the optimization problems of economics such as satisfaction maximization, profit maximization, cost minimization, output maximization (Cox, 2008). In the economic field, the calculation employs calculus' second derivative to optimize economic functions to solve these economic optimization problems.

For instant, in the economy, there are four functions: the total cost function (C), total output function (Q) which is also equal to quantity produce (Q_p), the total revenue function (R), and the profit function (π). The profit function: is analyzed using the second derivative so that profit optimization is obtained in the economy. If ($\pi'' < 0$), then profit maximization in the economy is obtained. , then is minimum, profit (loss) minimization is obtained; and if , then is a turning point. Similarly, a consumer who has the utility function , which is continuous and differentiable, can use optimization techniques to find the utility maximizing level of consumption.

Constraint maximization and minimization are common issues in economics. These can be solved using constraint optimization rules. In microeconomics, examples include utility maximization under a budget constraint, profit maximization under an expenditure constraint, and cost minimization under a production constraint are taken as the common optimization problems. Lagrangians multiplier rule is one of the important techniques, used in constraint optimization solutions (Le Van & Cagri Saglam, 2004. Again, to find the total values from the given marginal values, integral calculus is used, which is also a part of the differential calculus. To find the values of consumer's surplus and producer's surplus definite integral calculus is used. This has been further elaborated by Fig. 3.

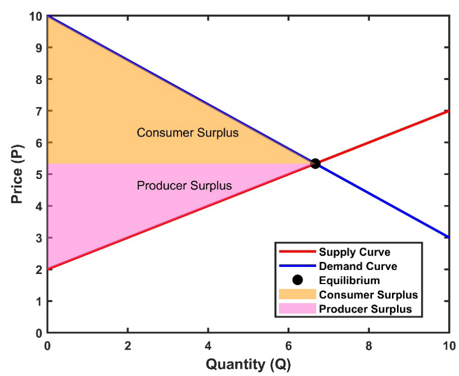

Fig. 3. Total Surplus of the Economy (the values are hypothetical)

We can use the following formulas (Eq. 10, 11 and 12) to get the values of consumer's surplus producer's surplus and total surplus of the economy.

Consumer's surplus = $\int_0^Q \textit{demand function} - \textit{price} \times \textit{quantity}$ ... (10)

Producer's surplus = price × quantity $\int_0^Q \textit{Supply function}$ ... (11)

Total surplus of the economy = consumer's surplus + producer's surplus ... (12)

$$= \left(\int_0^Q demand\ function - price\ \times quantity + price\ \times quantity - \int_0^Q \sup ply\ function \right)$$

$$= \left(\int_0^Q demand\ function - \int_0^Q \sup ply\ function \right)$$

3.5 Logarithms in Economics

If be the exponential function then is the solution of the given exponential function (Parajuli, 2010). In this function 'x' is an independent variable, 'a' is a positive constant but is not equal to one. In economics exponential functions are used in modeling, data transformation and analysis. It is essential to solve the complex economic variables' relationship, to analyze the growth theories by converting exponential growth trends into linear equation. With the help of logarithm, many economic models such as investment functions, production functions, demand supply relationship, growth models etc.; which are expressed in exponential functions, can be solved by logarithm technique. By taking natural logarithm of such economic functions, exponential functions can transfer into linear equations. For instance, the cobb-Douglas production function (Eq. 13) (Cohen, 2014).

$Q = AK^aL^b$ ... (13)

Which can also be written in linear form (Eq. 14).

$LnQ = \ln A + a.\ln K + b.\ln LQ = AK^a\ L^b$ ... (14)

From this linear equation of production function, estimation of parameters become possible and can use regression techniques on it.

In economics many models are developed by establishing the relationship between multiple independent variables and one dependent variable. By taking logarithms of such relationships change the multiplicative form in to an additive form; which simplify in both analysis and interpretation of the effect of independent variables on dependent variable. For illustration, total revenue is the product of price and quantity sold of a commodity i.e. , GDP can be expressed as the sum of the multiple of price and quantity product of various goods/ services of various sectors, i.e. (Eq. 15),

GDP = Market value of all product produced. ... (15)
i.e, (agricultural sectors, Industrial sectors and tertiary sectors)

Taking the logarithm on both sides, we get Eq. 16.

ln(GDP) = ln(agricultural sector contribution) + ln(industrial sectors contribution) + ln(tertiary sector contribution) ... (16)

3.6 Game Theory in Economics

Game theory is concerned with prediction of outcome of a game of strategy in which the player has incomplete or no information about the opponent's intention. It is a self-contained discipline that is used in applied mathematics, social sciences, most notably economics, as well as biology, engineering, political science, and other disciplines (Camerer, 2009). Game theory in economics originated with John von Neumann and Oskar Morgenstern's seminal work, Theory of Games and Economic Behavior (1944), which provided a mathematical framework for analyzing strategic interactions and decision-making in competitive and cooperative environments. Their work marked the foundation of modern game theory, extending economic analysis beyond individual optimization to strategic behavior (Leonard, 2010). The mathematical study of strategy and conflict in which an agent's success in making choices depends on the choices of others is known as game theory. Game theory, a mathematical framework, plays a crucial role in analyzing strategic interactions in Nepal among individuals, organizations, and the government. It has direct use in oligopoly markets where decision changes by one firm affect the market situation of other firms and other firms also must change their strategy. Game theory has also been applied to the development of theories of ethical or normative behavior. Scholars in economics and philosophy have used game theory to better understand rational decision-making. Prisoner's dilemma, Nash equilibrium are two most commonly used concepts of game theory in decision making of two opponents. It aids in policymaking, resource distribution in federal governance, trade negotiations, price war regulation, and monopoly control.

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price war regulation, and monopoly control.

For instance, one of the emerging issues of Nepal's hydropower development and cross-border energy trade with Bangladesh can be illustrated with the game theory. In this, Nepal (producer) and Bangladesh (buyer) are two players in the game. Nepal can either adopt a high-price strategy, aiming to secure higher export prices or offer competitively low prices to establish long-term trade relationships. Bangladesh can choose a high import commitment to ensure energy security with a stable supply or a low import commitment to maintain flexibility and avoid dependency. This has been further elaborated by the table. 1.

Table. 1. Energy trade payoff matrix of Nepal and Bangladesh

	Bangladesh: high import commitment (HIC)	Bangladesh: low import commitment (LIC)
Nepal: high price (HP)	(MB, MB)	(HL, ML)
Nepal: low price (LP)	(HB, HB)	(LB, LB)

Note: MB = moderate benefit, HB = high benefit, ML = moderate loss, HL = high loss and, LB = lower benefit

Based on Bangladesh's import commitment and Nepal's pricing policy, the table shows the results of trade agreements between the two countries under various scenarios. Nepal and Bangladesh both have moderate payoffs when Nepal sets a high price and Bangladesh commitment for high import, balancing cost and energy security. However, if Bangladesh lowers its import commitment and Nepal keeps prices high, Bangladesh suffers minor losses from a constrained supply of energy, while Nepal suffers high losses from a decline in demand. On the other hand, a low-price approach combined with a strong commitment to imports produces the best results. Bangladesh benefits from reliable and affordable energy, while Nepal gains more income from secure trade and reasonable prices. Conversely, limited import commitment and cheap pricing lead to reduced payoffs for both parties, which reflects decreased trade volumes and benefits. Thus, the best payoff for both nations is: for Bangladesh to guarantee a steady, long-term import commitment and for Nepal to implement fair pricing strategy.

4 Conclusion

The use of mathematics in economics began with Adam Smith's foundational ideas. Economists like Jevons, Walras, and Marshall introduced mathematical models to improve precision and clarity. Pareto. Mathematical techniques like calculus and optimization helped to solve complex problems and improve predictions. Mathematical models now play a critical role in modern economic theory and policy-making. For the first, functions and simultaneous equations used by Marshall and walrus helped turn economic theory into mathematical models and made easier to examine equilibrium values. They are important tools for policymaking, solving economic problems, and creating sustainable policies in the field of agricultural subsidy programs by analyzing the equilibrium between production costs and market prices, promoting food security and farmer welfare, labour and other factors market equilibrium by determining their reasonable prices and so on.

Integral calculus shows the calculation of economic surpluses, market efficiency and welfare while differential calculus is essential for visualizing marginal values, analyzing dynamic models, and studying equilibrium conditions. Optimization techniques, on the other hand, focus on resource allocation, cost efficiency and profit maximization. Game theory, seen more valuable in export-oriented production and trade. It has greater potential to maximize revenue through Nash equilibrium. Furthermore, research on game theory could help to improve negotiations in public-private partnerships, regulate monopolies, and reduce price wars by encouraging regional collaboration. It can support in international aid, trade, and transit agreements also by promoting cooperation and maximizing mutual benefits. This could provide valuable insights to address complex economic and policy issues. Matrix algebra is used to illustrate the interdependency and linkages of different sectors of the economy. This provided a framework for designing strategies to reduce drawbacks of the economy. This approach offered valuable insights that may guide policymakers in making informed decisions, ultimately contributing to the sustainable economic development of Nepal. Since, matrix algebra provides precise solutions than other approaches, future research could explore several study areas like sectoral disaggregation, multiregional analysis, policy simulations, incorporation of environmental and social factors, and cross-country comparisons. Hence, matrix algebra can be used to further refine the understanding of Nepal's economy.

Finally, the use of mathematical tools and models has transformed economic analysis and offered a strong framework for dealing with complex policy and economic issues through classical era. This has great potential to promote sustainable economic development and sound decision-making in Nepal and beyond. These approaches enable policymakers to develop strategies that promote fiscal sustainability and growth by improving forecasting capacities, optimizing resource allocation, and enabling exact quantification of sectoral links. Moreover, the ongoing development and use of these mathematical techniques might reveal creative solutions to new local and global economic problems, promoting fair and sustainable development. This paper briefly explored the use of mathematics in economics with selected areas and examples. It highlights the need for further research on mathematical tools and models in modern economic systems to expand understanding of their relationship.

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