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Proofs Satisfy All the Figures which are satisfied with a Statement of a Theorem: A Geometrical Case Study

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Abstracts: *The objective of the study is to find the learning effectiveness of given the theorem of geometry of class10 in the book published by government of Nepal MOEST which has been the subject of debate among teachers and learners. The study primarily focus Makawanpur district and the informants were selected convinently for the study. There are 95 community schools and 41 private schools in the district. The teachers were selected from both Nepali and English mediums. I conducted the questionnaire survey of the selected teachers and a case study of students. So, there were 4o mathematic teachers for the data collection and five students. Primary data had been collected visiting personally to fill up a set questionnaire with six objectives yes/no and true/false questions. There was a pilot test before the collection of the data. It was found that the present diagram and used strategy of class 10 book has to be reviewed immediately before the new session starts for the coming academic session. The conclusion of the study is that the debatable proof of theorem can be replaced with the one that matches the principle of theorem "Figures which are satisfied with a statement of a theorem the proofs satisfy all of them".*

Keywords: equal, figure, parallelogram, proof, strategy, theorem

1. Introduction

The curriculum is being changed from time and again by CDC in Nepal. Textbooks of class 10 of Mathematics have been changed since 2074 BS. According to the Ministry of Education, Science, and technology CDC, the textbook is taken as an important material of teaching and learning processes (Awasthi et.al., 2074). A total of 441769 textbooks of compulsory Mathematics were printed in the year 2074 (Aryal et. al., 2018). Changing text is reasonable but a change of proof of a theorem of geometry is a matter of questioning. Some theorems can be proved with more than one strategy. Which is the best? There can be a question among the learners whether all proofs are universal truth or not. All truths are equal for evaluation. According to oxford language, the theorem is a general proposition not self-evident but proved by a chain of reasoning; a truth established through accepted truths (Hornby, 1995). Words about theorem from Oxford Advanced Learner's Dictionary are a rule or principle, especially in mathematics, that can be proven to be true. The above definitions mean that theorem is a statement that needed to be proven. In some cases, references to the previous theorem can be given to prove new theorems for reasons. Similarly, three and more than three figures of different shapes and sizes are drawn for experimental proof. So, proof that satisfies only a particular figure of the same statement among the satisfied figures of a theorem is the problem that encouraged the study.

From a point of view of learners, some of the proofs are easier, whereas some of them are difficult according to strategies. Although learners use their best strategy; the facilitator should give all strategies for further study to learners. Here I must accept the point that several proofs of the same theorem cannot be included in the textbook because of book size. While choosing the strategy, the point to be noted: what concepts are necessary to be practised or required for further lessons or for achieving the goal of the curriculum. For example, in the theorem, "Parallelograms which are on the same base and between the same parallels are equal in the area" a facilitator can use the strategy of base× height or

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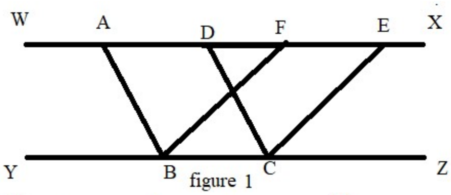
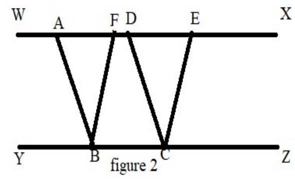
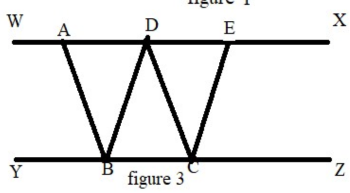
congruency. If the curriculum demands to be practised of equality of congruent triangles in the further lessons, the facilitator has to choose congruency strategy otherwise s/he can go with base × height in the case of finding only areas of them. Learners have to be given the concept of both strategies for generative knowledge. The statement of the theorem satisfies three types of figures. The first two figures 1 and 2 are very common while drawing according to the statement but the third one can be in the case of coincident; it happens if two points are overlapping. Some learners can think of figure no. 3







Different types of proofs have been found in textbooks and practice books. Here are some of them.

Proof: 1

Statements	Reasons
In triangle ABF and DCE i. AB = DC ii. ∠BAF = ∠CDE iii. ∠AFB = ∠DEC iv. The two triangles ABF and DCE are congruent, which means that their areas are equal. v. Trapezium ABCE-ΔDCE=Trapezium ABCE-ΔABF vi. Parallelogram ABCD= Parallelogram FBCE	a. opposite sides of a parallelogram are equal b. corresponding angles c. corresponding angles d. By the SAA criterion e. Δs with the equal area are subtracted from the same trapezium. f. Whole part axiom

Proved
(Note: in case of figure iii point F is D)

Proof: 2

Statements	Reasons
In triangle ABF and DCE i. AB = DC ii. ∠BAF = ∠CDE iii. ∠AFB = ∠DEC iv. The two triangles ABF and DCE are congruent, which means that their areas are equal. v. ΔABF+Trapezium FBCD=ΔDCE+ Trapezium FBCD vi. Parallelogram ABCD= Parallegram FBCE	a. opposite sides of a parallelogram are equal b. corresponding angles c. corresponding angles d. By the SAA criterion e. Δs with equal are added to the same trapezium f. Whole part axiom

Proved
(Note in case of figure iii point F is D)

Proof: 3

Statements	Reasons

International Research Journal of MMC (IRJMMC)

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i. Area of parallelogram ABCD=BC×Height ii. Area of Parallelogram FBCE = BC×Height iii. Area of Parallelogram ABCD= parallelogram FBCE	a. Area of a parallelogram is base×Height b. Same as “a”. c. Axiom of equality (WX//YZ)

Proved

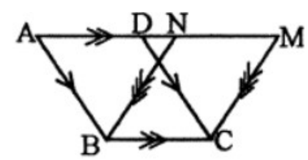
(Note in case of figure iii points F is D)

While writing the textbook, the best strategy needs to be applied. Nepal government had started a New Education System Plan since 2028 BS. “Parallelograms which are on the same base and between the same parallels are equal in the area” is theorem number 1 in the textbook of class ten Mathematics (Awasthi et.al., 2074, p. 143). From the beginning of the New Education System Plan 2028, the proof of the statement of the theorem “Parallelograms which are on the same base and between the same parallels are equal in the area” was proved with the strategy of proof no. 1. In 2074 BS, proof no. 2 has been applied. In the exam of SEE in 2075 (Nepal Board), a question from the theorem was asked with figure 1.

१६. दिइएको चित्रमा AM||BC, AB||DC र NB||MC छन् भने प्रमाणित गर्नुहोस् :

i) ΔABN ≅ ΔDCM

ii) ▱ABCD को क्षेत्रफल = ▱NBCM को क्षेत्रफल



In the given figure, AM||BC, AB||DC and NB||MC. Prove that:

i) ΔABN ≅ ΔDCM

ii) Area of ▱ABCD = Area of ▱NBCM

There were rare cases; that year students solved the question. It was found dissatisfaction of the students after taking the exam. The same was found while answer sheets were being examined by the teacher. The strategy of proof no.3 satisfies all the three types of figures but sometimes out of 4 marks 2 marks was allocated for congruency of two triangles. A study has been held for finding the solution to the nationwide problem in the geometry of Mathematics of class ten. The problem of teachers and newcomers students from compulsory mathematics encouraged me to study the burning problem.

> *The strategy of Proof: Consider parallelograms ABCD and BCFE, both on-bases BC such that the opposite sides AD and EF are contained in the same line that is parallel to the line Consider two cases, where (i) point D falls between points A and E, and (ii) where the point E falls between points A and D. For case (i), argue why △ABE ≃△DCF. Suppose that segment BE intersects segment CD at point H in case (i). Then add the area of △BCH to the area of △ABE and compare this to the sum of the areas of △BCH and △DCF. Now subtract the area of △HDE from both of these. What results? How much of the argument for case (i) can be applied to case (ii)?* (Lodder, 2016)

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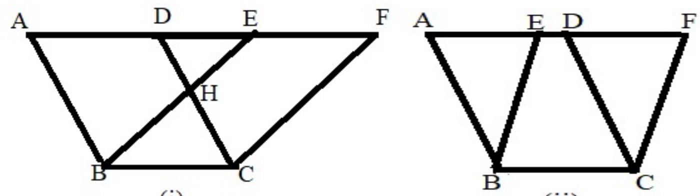

(i) (ii)

According to the above note given in indentation, learners gain more knowledge according to the figures. Proofs satisfy all the figures which are satisfied with a statement of a theorem. In the condition of the given question of problem-solving types, corollary, and/or figures the above strategies can be applied. In a Maths book of class 9, published in 2001 AD the first edition (English version) proof of the theorem was according to the principle of the theorem. Originally this book was written by Shmbhu Narayan Vaidya and revised by CDC in 2060 (Vaidya, 2001). The photocopy of the theorem has been pasted below. Before starting the theorem all the knowledge of Definitions, Postulates and Theorems should give to learners previously.

According to Mr Godfrey (1926), the three types of figures can be drawn which are satisfied with the statement. The differences are I) AB and QC not intersected II) A and Q overlapped and III) AB and QC intersected

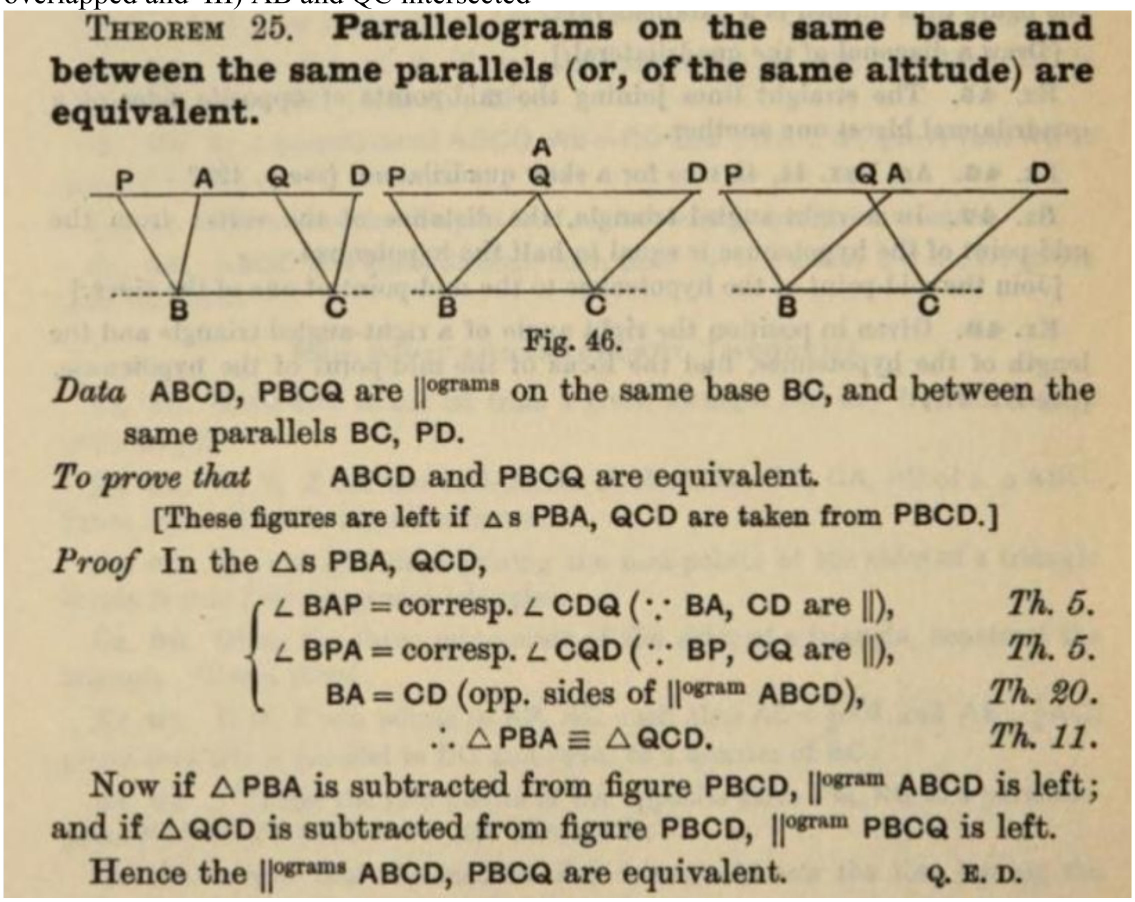
THEOREM 25. **Parallelograms on the same base and between the same parallels (or, of the same altitude) are equivalent.**

Fig. 46.

Data ABCD, PBCQ are ||ograms on the same base BC, and between the same parallels BC, PD.

To prove that ABCD and PBCQ are equivalent.

[These figures are left if △s PBA, QCD are taken from PBCD.]

Proof In the △s PBA, QCD,

$$\left\{\begin{array}{ll} \angle \text{BAP} = \text{corresp.} \angle \text{CDQ} \ (\because \text{BA, CD are } \|), & \textit{Th. 5.} \\ \angle \text{BPA} = \text{corresp.} \angle \text{CQD} \ (\because \text{BP, CQ are } \|), & \textit{Th. 5.} \\ \text{BA} = \text{CD (opp. sides of } \|^{\text{ogram}} \text{ ABCD)}, & \textit{Th. 20.} \end{array}\right.$$

$$\therefore \triangle \text{PBA} \equiv \triangle \text{QCD}. \qquad \textit{Th. 11.}$$

Now if △PBA is subtracted from figure PBCD, ||ogram ABCD is left; and if △QCD is subtracted from figure PBCD, ||ogram PBCQ is left.

Hence the ||ograms ABCD, PBCQ are equivalent. Q. E. D.

(Godfrey et. al, 1926)

You may recall the method of finding the areas of these two parallelograms by counting the squares (AREAS OF PARALLELOGRAMS AND TRIANGLES). But it can not be a theorem.

International Research Journal of MMC (IRJMMC)

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Volume 3(1) ISSN 2717-4999 (Online) 2717-4980 (Print)

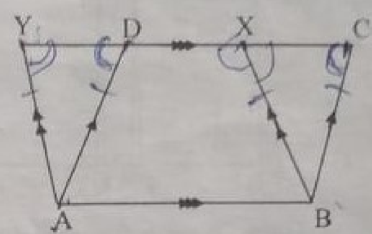
12 Area of triangle and quadrilateral

Theorem – 12

Parallelograms on the same base and between same parallel lines are equal in area.

Given : The parallelograms ▱ABCD and ▱ABXY are on the same base AB and between the same parallel AB and CY in which AD // BC and AY // BX.

To prove : Area of ▱ABCD = Area of ▱ABXY.



Proof:

S.No.	Statements	S.No.	Reasons
1.	In ΔADY and ΔBCX	1.	
(i)	AY = BX	(i)	Opposite sides of parallelogram ABXY.
(ii)	$\angle ADY = \angle BCX$	(ii)	The corresponding angles made by the transversal YC intersecting the parallel lines AD and BC are equal.
(iii)	$\angle AYD = \angle BXC$	(iii)	the transversal YC intersects AY // BX and corresponding angles are equal.
2.	$\Delta ADY \cong \Delta BCX$	2.	2. S.A.A. axiom
3.	Area of ΔADY = Area of ΔBCX	3.	The area of congruent triangles are equal.
4.	Area of trapezium ABCY – area of ΔADY = Area of trapezium ABCY – area of ΔBCX	4.	Subtracting triangles having equal area from trapezium ABCY.
5.	▱ABCD = ▱ABXY	5.	Axioms of subtraction.

Note: This theorem can be proved by using formula of area. Proved.

(Vaidya, 2001)

This proof matches with proof number 1.

International Research Journal of MMC (IRJMMC) Open Access

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1.1 Objectives

The study enables to review justification of the theorem in the textbook and learning effectiveness of the students which is debatable and problematic among the teachers and learners.

2. Materials and Methods Used

A questionnaire was developed for the research. The teachers from the makawnapur district were convinently selected and their information was collected. The teachers were selected as much possible as to cover the community as weel as the private school teachers regarding the equal number os sex. Primary data were collected from them and analysed . The researcher visited the respondents personally to collect the data. To find the learning effectiveness, five students were connviniently selected and they were given the three different figures to solve and their answers were collected and analysed. The results are discussed; the findings and recommendations are listed.

After the data collection and analysis, it was again desirable to discuss the issues with selected experienced and renowned maths teachers from the Makawanpur Multiple Campus namely T N Bhattarai, Sonelal Jha, Binod Sharma, Rabin Koirala, and Rudra Bd Bogati who agreed that the justification of the theorem given in the text doesnot satify in all respective cases. They concluded that it's only the case that given example can fit the theorem but can not exactly carry the acceptable implication in all cases and the students lack the knowledge and ability to effectively justify the theorem. So, it is also seriously incorporated while analysing the data and suggestions for further recommendations.

3. Result and Discussion

According to the medium of language, there are English and Nepali medium schools at Hetauda in Makawanpur. In Makawanpur, there are 594 schools among them, 520 schools are community and 74 are institutional (Aryal et. al., 2018). The district is a stratified sampling for secondary schools which represent all types of schools of Nepal. It has schools in a remote area as well as in town.

Table 1 *Types of school*

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Nepali Medium schools	36	90.0	90.0	90.0
	English Medium School	4	10.0	10.0	100.0
	Total	40	100.0	100.0	

According to the ratio of English and Nepali medium schools, 36 schools from Nepali medium and 4 from English medium had been selected; the total number of respondents is 40. A teacher of Mathematics subject from a school was selected in random sampling.

Table 2 *Sex of respondents*

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Male teachers	37	92.5	92.5	92.5
	Female teachers	3	7.5	7.5	100.0
	Total	40	100.0	100.0	

Female teachers who teach Mathematics are small in number. So, out of 40 teachers, 3 female and 37 male teachers were selected for collecting primary data. The female teachers

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were found only in the valley and in the Nepali medium schools. It is 92.5% male and 7.5% female teachers.

Table 3 *Teaching experience of the teachers (respondents)*

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	1 to 5 years	15	37.5	37.5	37.5
	6 to 10 years	3	7.5	7.5	45.0
	above 10	22	55.0	55.0	100.0
	Total	40	100.0	100.0	

The table shows that 15 respondents experience one to five years, 3 teachers have teaching experience of 6 to 10 years, and 22 have been teaching more than 10 years. A large number of teachers have long experience.

S.N.	Question	True		False	
		Number of teachers	Percentage	Number of teachers	Percentage
1	Proofs satisfy all the figures which are satisfied with a statement of a theorem	38	95%	2	5%

The question in the above table is the principle of the theorem. Among the forty respondents, 95% know the principle of the theorem. Only 5% of respondents are unknown about the principle of the theorem. Among the Mathematics teachers, 95% agree to take that proof of theorem must satisfy all the figures which are satisfied by the statement of it.

S.N.	Questions	Yes		No	
		Number of teachers	Percentage	Number of teacher	Percentage
1	Does figure 2 satisfy the statement?	40	100%	0	0%
2	Does figure 1 satisfy the statement?	40	100%	0	0%
3	Does proof 1 satisfy both figures?	40	100%	0	0%
4	Does proof 2 satisfy both figures?	0	0%	40	100%

In the question, 'Does the figure satisfy the statement?', 100% of respondents said 'Yes'. Similarly, in response to the question **'**Does figure 1 satisfy the statement?', 100% agreed. All the respondents said 'Yes' for the answer to the question 'Does proof 1 satisfy both figures?' and 'No' for the question 'Does proof 2 satisfy both figures?'. Among the respondents, 100% of teachers accepted that both figures 1 and 2 satisfy the statement "Parallelograms on the same base and between the same parallels are equal in area." But only proof number 1 satisfies both. Proof number 1 not only satisfies figure 1 and figure 2 but also 3. In another word, proof number 1 satisfies all the figures which are satisfied by the statement 'Parallelograms on the same base and between the same parallels are equal in area.'

S.N.	Question	Proof no. 1		Proof no. 2	
		Number of teachers	Percentage	Number of teachers	Percentage
1	Which proof satisfies both?	40	100%	0	0%

The answer of 100% of respondents is 'Proof no. 1' in the question of 'Which proof satisfies both?'. In other words, proof number 1 satisfies both figures of the theorem.

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3.1 Findings

1. A set of statements and reasons that satisfies only a particular figure among all satisfied figures of the same statement of a theorem cannot be the proof.
2. The strategies to prove the theorem "Parallelograms which are on the same base and in the same parallels are equal in the area" are both proof 1 and proof 3.
3. The strategy of subtraction of congruent triangles from the whole trapezium turn by turn to show the parallelogram equal is one of the best.
4. The strategy of base×height is correct but it lacks the concept of congruency.
5. The theorem (no.1) of the textbook of class-10 Mathematics (CDC) published in 2074, lacks the principle of the theorem.

On the case of the students, they easily solved only one of the given figures which was dealt in the book but unluckily could not solve other two figure of the same theorem. It disclosed that the students had the limited knowledge related to the justification of the theorem and they did not know the exact proof. The students could not apply the strategy that they had studied because it was not rightly applicable for other two figures.

4. Conclusion

The proof of the theorem number one from class 10 compulsory Mathematics, published in 2074 by Nepal Government, Education, Science and Technology Ministry, Curriculum Development Center, Sanothimi, Bhaktapur, has to be reviewed and applied the strategies of the proof of the theorem that matches the principle of the theorem. In other words, the strategy used in the book written by Shambhu Narayan Vaidya and revised by CDC in 2060 should be applied to the theorem because it has adversely affected in students' learning. The alternative method of the book (base × height) matches the principle of the theorem but it lacks the concept of congruency.

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