A General study of Cesaro sequence spaces with Matrix transformations

Jagat Krishna Pokharel
Associate Prof. Sanothimi Campus, Bhaktapur
E-mail: jagatpokhrel.tu@gmail.com

Abstract

In this paper, the researcher utilize the non-absolute type of Cesaro sequence space to transform the Cesaro sequence spaces and establish the necessary and sufficient conditions for the existence of an infinite matrix in the spaces $\ell\infty$ and C, respectively. When the sequences $x\epsilon X$ satisfy the condition that the series $\sum_{k=1}^{\infty} x_k$ converges, the sequence space H becomes a non-absolute Banach space that fulfills the fundamental requirements for transforming the Cesaro sequence space X_p into the corresponding spaces of ℓ_∞ and all convergent sequences. As a consequence of the matrix transformation, some theorems are derived.

Keywords: *Sequence Space, Dual Space, Transformation, Infinite Matrix, Absolute Type, Convergent*

Introduction

In Ng and Lee (1978) it has been taken into account that the amount of space available by H of the entire real sequence $\{x_k\}$ the series for which $\sum_{k=1}^{\infty} x_\text{k}$ convergent. We will determine the H terms of Kothe theory, and establish conditions that are both necessary and sufficient for a transformation using a matrix to H should be translated into space ℓ_∞, C of convergent sequences in general and of all bounded sequences. These findings will be used to determine cnecessary and sufficient conditionsto transform the Cesaro sequence spaces with an infinite matrix later in this note into the l∞ and the space C respectively.

Definition:- If a Sequence $X = \{x_k\}$ Its absolute value, too, belongs to a specific space $|X| = \{|x_k|\}$. Otherwise, it is said that the space is non-absolute. The non-absolute type of aequence space has a number of undesirable characteristics.

Definition:- Let $A = \left(a_{n,k}\right)$ be an infinite complex number matrix $a_{n,k}\,(n, k = 1,2, \dots)$ and P and Q be two subset of space \`S\` of complex sequence and A is a matrix transformation from P to Q that is defined by the letter A, for every sequences $X = (x_k) \in P$ .The sequence $A(x) = A_n(x)$ is in Q where

$$A_n(x) = \sum_{k=1}^{\infty} a_{n,k} x_k$$

The class of all such P to Q matrix transformations will be denoted by (P, Q).

Literature Review

Towards the end of the nineteenth century and the beginning of the twentieth century, functional analysis was developed. Its evolution was largely a reaction to the effort by many individual, because so many people were working so hard to understand physical phenomena at the time, questions that emerged during the investigation of differential and integral equations were very fascinating.

Functional Analysis' basic theme is to treat functions as "points" or "elements" in some abstract space, so that rather than working with individual functions as points (the tradition in classical analysis). When dealing with functions, think of them as points in a space with a general structure.

Functional Analysis is a branch of abstract mathematics whose central focus is the examination of linear spaces endowed with various limit-related structures generated by topology, norm, para-norm, a family of semi-norm or by some other means and operators acting upon them. In the beginning, Banach Hahn Mazur applied functional analysis to summability theory, and later, it was studied by many distinguished mathematicians, including Kojiman, Steinhaus, Schur, Mazur, Orlicz, Wilansky, Maddox, and many others..

A linear sequence space with elements in another linear space is referred to as a sequence space. Summability is the research of linear transformations in sequence spaces. An early version of the summability theory was proposed in a letter by Goottfried Wilhelm Leibniz $(1646 - 1716)$ to C.Wolf(1713) in which he attributed the sum $\frac{1}{2}$ to the oscillatory series: $1 - 1 + 1 - 1 + 1 - \ldots$.

Since $s = 1 - (1 - 1 + 1 - \ldots) = 1 - s$ or $s = \frac{1}{2}$

Similarly, squaring L.H.S.

$$(1 - 1 + 1 - 1 + \ldots)(1 - 1 + 1 - 1 + \ldots) = \frac{1}{4}$$

i.e.,

$$1 - (1.1 + 1.1) + (1.1 + 1.1 + 1.1) - \ldots = \frac{1}{4}$$

Which gives $1 - 2 + 3 - 4 + \ldots = \frac{1}{4}$.

The theory of summability is concerned with the generalization of the notion of the sum of a series which is usually affected by auxiliary series. In particular, in the above example the original oscillatory series $1 - 1 + 1 - 1 + 1 - \ldots$is divergent but the new series $1 - 2 + 3 - 4 + \ldots$ is convergent.

ILAM Vol. 19, Issue 1, August 2023

The notion of convergence of an infinite series was first resolved satisfactory by the french mathematician A.L Cauchy, Frobenious in 1980 introduced a generalized method of summability by arithmetic means by Ernesto Cesaro in 1890 as the (C,K) method of summability. Towards the 19^{th} century, Mathematicians were motivated by problems like those in summability theory to study general theory of sequence and transformation on them..

Summability theory was used to further investigate sequence space. The theory of identifying limits, also known as summability theory or short summability, is based on functional analysis, function theory, topology, and functional analysis. The cesaro refers to (also known as cesaro average) the order $\{X_n\}$ are the terms of sequence $\{c_n\}$ where $c_n = \sum_{i=1}^{n} x_n$ is consisted of the first n elements, with the arithmetic mean $\{x_n\}$.

This idea bears Ernesto Cesaro's name. Convergent sequences are preserved along with their limits by the Cesaro Means operation by Cesaro Summability view in divergent sequences theory. If the series is referred to as Cesaro summable if the Cesaro means sequence is convergent. There are numerous instances where the Cesaro Means Sequence converges, but not the original Sequence.

for example, sequence $\{x_n\} = \{(-1)^n\}$ which is Cesaro summable.

Let ω be the set of all sequences of all real or complex numbers and $\ell\infty, c_1, c_0$ denote the spaces of all bounded, convergent and null sequences $x = (x_k)$ with the usual norm $\| x \|_\infty = sup|x_k|$, where $k \in \mathbb{N} = 1,2,\ldots$ the set of positive integers. Also by bs, cs, ℓ_1 and ℓ_p ; we denote the spaces of all bounded convergent, absolutely summable, and p- absolutely summable sequences respectively.

Let, $\bar{X} = (x_k) = (x_k)_{k=1}^{\infty}$ and ω will denote the difference- classes of all sequences $\bar{X} = (x_k), k \geq 0$ over the field $\mathbb{C}$ of complex number.

1. $c_0 = \{\bar{X} = (x_k) \in \omega: |x_k| \to 0 \text{ as } k \to 0)\} \to$ (Thespaceofnullsequence).
2. $c_1 = \{\bar{X} = (x_k) \in \omega \;\exists\; \ell \in \mathbb{C}$, s.t. $|x_k - \ell| \to 0$ as $k \to 0\}$ → (The space of convergent sequences).
3. $\ell_\infty = \{\bar{X} = (x_k) \in \omega;\ \sup_k |x_k| < \infty\}$ → (The space of bounded sequence).
4. $\ell_p = \{\bar{X} = (x_k) \in \omega: \sum_{k=1}^{\infty} |x_k|^p < \infty\}, 0 < p < \infty.$ → (the space of absolutely p- summable sequence).

   If X denote the normed space and x_k^s are the elements of X ,then the Shiue (1970) introduced the following sequence space.
5. $c_0(X)$=$\{\bar{X} = (x_k)$: $x_k \in X, K \geq 1$,$\| x_k \| \to 0$ as $K \to 0\}$.

6. $c_1(X) = \{\bar{X} = (x_k): x_k \in X, K \geq 1, \exists\, \ell \in X$, s.t $\parallel x_k - \ell \parallel \to 0$ as $K \to 0\}$.
7. $\ell_\infty(X) = \{\bar{X} = (x_k): x_k \in X, K \geq 1, \sup_k \parallel x_k \parallel < \infty\}$.
8. $\ell_p(X) = \{\bar{X} = (x_k): x_k \in X, k \geq 1, \sum_{k=1}^{\infty} \parallel x_k \parallel^p < \infty\}: 0 < P < \infty$..
   Kizmaz (1981) defined the difference sequence spaces $\ell_\infty, C(\Delta), C_o(\Delta)$ as follows:
   $X(\Delta) = \{x = (x_k): (\Delta x_k) \in (\Delta X)\}$, for $X \in \ell_\infty, c, c_o$,
   where $\Delta x = (\Delta x_k) = (x_k - x_{k+1})$ for all $k \in \mathbb{N}$.
   In 1981, Kizmaz introduced and studied X- valued difference sequence spaces $c_0(\Delta), c_1(\Delta)$ and ℓ_p as follows.
9. $c_0(\Delta X) = \{\bar{X} = (\Delta x_k): \Delta x_k \in X, \parallel \Delta x_k \parallel \to 0$ as $k \to \infty\}$.
10. $c_1(\Delta X) = \{\bar{X} = (\Delta x_k): \Delta x_k \in X$ and $\exists\, \ell \in X$ suchthat $\parallel \Delta x_k - \ell \parallel \to 0$ as $K \to \infty\}$.
11. $\ell_\infty(\Delta X) = \{\bar{X} = (\Delta x_k): \Delta x_k \in X, \sup_{k\geq 1} \parallel \Delta x_k \parallel < \infty)\}$,where $\Delta x = \Delta x_k = (\Delta x_k - \Delta x_{k-1})$.
    Mikail & Colak (1995) introduced and studied Δ^2 difference sequence spaces of Banach space X valued sequences $c_0(\Delta^2 X), c_1(\Delta^2 X)$ and $\ell_\infty(\Delta^2 X)$ as given below:
12. $c_0(\Delta^2 X) = \{\bar{X} = (\Delta x_k): \Delta x_k \in X, \parallel \Delta^2 x_k \parallel \to 0$ as $K \to 0\}$.
13. $c_1(\Delta^2 X) = \{\bar{X} = (\Delta x_k): \Delta x_k \in X$, and $\exists\, \ell \in x$ suchthat $\parallel \Delta^2 x_k - \ell \parallel \to 0$ as $K \to \infty\}$
14. $\ell_\infty(\Delta^2 X) = \{\bar{X} = (\Delta x_k): \Delta x_k \in X: sup_{k\geq 1} \parallel \Delta^2 x_k \parallel < \infty\}$,
    where $\Delta^2 x_k = (\Delta x_k) = (\Delta x_k - \Delta x_{k+1})$.

The study of Difference sequence spaces was initiated (Kizmazin, 1981) to the Banach space theory,recent text (Tripathy & Esi, 2006) have defined the generalized difference sequence spaces $c_0(\Delta^m), c(\Delta^m)$ and $\ell_\infty(\Delta^m), m \in \mathbb{N}$.

Orhan (1983) construct these spaces, in general Banach normed space X to be defined exact $c_0(\Delta^m, X), c(\Delta^m, X), \ell_p(\Delta^m, X)$ and $\ell_\infty(\Delta^m \Delta, X)$.

Kizmaz (1981) introduced a Banach space for a complex sequence $x = (x_k)$, for which $p \geq 1$, the sequence space ℓ_p is complete under the norm defined by $\parallel x \parallel = (\sum_{k=1}^{\infty} |x_k|^p)^{\frac{1}{p}}$.

Similarly, for $0 < p < 1, \ell_p$ is complete p-normed space with p- normed defined by $\parallel x \parallel = \sum_{k=1}^{\infty} |x_k|^p$

Methodology

For the study of cesaro sequence space on matrix transformation we have defined the sequence space with koth dual space and proved some related theorem.

ILAM Vol. 19, Issue 1, August 2023

Different forms of Cesaro sequence space can be proved with their matrix transformation. Some theorems and lemmas are proved as following

The associate space H

Let H represent the total space of all sequences **x∈X** as if the series $\sum_{k=1}^{\infty} x_k$ Convergent.
In H, a norm ρ is defined as

$$\rho(x)=\sup\{|\sum_{k=1}^{\infty} x_k|:n\geq 1\}.$$

The Kothe - dual A' is a normed linear space with the associate norm. The associate space is another name for the Kothe dual (Zaanen, 1967).
Let $y \in X'$ and defined norm as
$\|y\| = sup\{\varphi_{(n)}: n \geq 1\}$
Where,
$\varphi_{(n)} = sup\{|\sum_{k=1}^{n} x_k y_k|: x \in X \in and \ \|x\| \leq 1\}$
We simply assume that there occurs a sequence X for every K. As a result, $\|x\| \leq 1$ and $x_k \neq 0$

Theorem: - The Space H is a non-absolute Banach sequence.
Proof:
let $\{x^{(i)}\}$ be a Cauchy sequence in H, such that we have ϵ > 0, $\{\rho(x^{(i)} - x^{(j)}\} < \in$ for all $i, j \geq n_0(\in)$We write $x^{(i)} = \{x^{(j)}\}$.Then for fixed k, $\{x_k^i\}$ is convergent if $\lim_{i\to\infty} x_k^{(i)} = x_k$, then

$$\left|\sum_{k=1}^{m}\left(x_k^{(i)} - x_k\right)\right| <\in \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

For $i \geq n_0(\in)$ and all m= 1,2,3… Hence, we have
$\rho(x^{(i)} - x^{(j)} \leq \in$ for all $i \geq n_0(\in)$.
As a result, H is final.
Theorem: - The Space H is separable.
Proof:- For every x∈H, Such that $x = \{x_k\}$. Let $x^N = \{x_{1,} x_2, x_3 \ldots . x_N, 0, ..\}$. Then it's clear $\rho\{x - x^N\} \to 0$ as N→ ∞. If A is a dense real-number system subset that can be counted, then H is a countable dense subset of A. The associate space of H will be determined in the following steps. Assume V is the totality of all spaces, y ∈X as a result,
$\sum_{k=1}^{\infty}|(y_k - y_{k+1})| < \infty$ ……………. (ii)
Theorem: - The associate space H' of the space H coincides with the space V, and associate norm ρ' of ρ is equivalent to the norm
$\|y\| = |y| + \sum_{k=1}^{\infty}|(y_k - y_{k+1})|$ for all $y \in H'$

The outcomes will be discussed in this section [6] E. Malkowsky and S.D to establish conditions that are both necessary and sufficient for a matrix transformation to take place $A = \{a_{n,k}\}$ will convert the space H into the appropriate space l∞ The sapce C of all convergent sequences and the space l∞ of all bounded sequences. First, we'll state a lemma as a result of Çolak, Et, & Malkovsky, (2004).

Lemma: - If a then matrix A converts a BK-space X_p in to ℓ_∞ BK-space this is a continuous and linear transformation. Each coordinate mapping in BK-space has a banach space available where $x \to x_k$ is continuous.
For example, $\ell_\infty(1\leq p = \infty)$, C and the space C_0 of all null sequences with uniform norms are all BK-space.
Proof: -
All the finite sequences are contained in the associate space H, if $x^{(n)} = \left\{x_j^{(n)}\right\} \in H$, with

$$\rho\left\{x^{(n)}\right\} \to 0 \text{ as } n \to \infty$$

Then,

$$\left|x_j^{(n)}\right| = \left|\sum_{k=1}^{\infty} x_j^{(n)} e^j\right| \leq \rho\left(x^{(n)}\right) \rho'\left(e^j\right) \to 0. \text{ As } n \to \infty$$

Where e^j is indeed the sequence 1 just at j_{th} position and zero everywhere else.
Cesaro Sequence space spaces and the matrix transformation.

Let $X_p(1 \leq \infty)$ and X_∞ respectively the spaces of all$x \in X$ with

$$\|X\|_p = \left(\sum_{n=1}^{\infty}\left|\frac{1}{n}\sum_{k=1}^{n} x_k\right|^p\right)^{\frac{1}{p}} \leq \infty$$

And $\|X\|_\infty = sup\left\{\left|\frac{1}{n}\sum_{k=1}^{n} X_k\right|^p ; k = 1,2,3, \dots\right\} < \infty$
The above norms, with the exception of p=1. By shiue (1970) and of Kamthan and Gupta (1981), the Cesaro sequence space defined as

$$Ces_p = \left\{a = \{a_n\}_{n=1}^{\infty}: \|a\|^p = \left(\sum_{n=1}^{\infty}\left(\frac{1}{n}\sum_{k=1}^{n}|a_k|\right)^p\right)^{\frac{1}{p}} < \infty\right\}$$

for$1 \leq p < \infty$ and

$$Ces_\infty = \left\{a = \{a_n\}\|a\|_\infty = sup_n\left(\frac{1}{n}\sum_{k=1}^{n}|a_k|\right) < \infty\right\}$$

we note that the space X_p , is distinct from P as defined above. Spaces in the Cesaro sequence $Ces_p(1 \leq p < \infty)$. In fact, $Ces_p \subset X_p(1 \leq p < \infty)$. and $Ces_p = X_p$. Mursaleen M [4], We can demonstrate this X_p are non-absolute Banach sequence spaces by using theorem. We will now present the results as a result of Lee [1]

Theorem: - Let y_q be the space of all y Є X such that

$|ky_k| \leq M,$ for all k=1,2, ……-----------------(1)

$\mu_q(y) = (\sum_{k=1}^{\infty}|k(y_k - y_{k+1})|^q)^{\frac{1}{q}} < \infty$ for $1 \leq q < \infty$……………………(2)

And

$\mu_q(y) = sup\{|k(y_k - y_{k+1})|: k = 1,2,3, \dots\} < \infty$

The matric transformation of X_p

Here we Find conditions that are both necessary and an infinite matrix is sufficient to transform the Cesaro sequence spaces X_p into the spaces ℓ_∞ of all bounded sequences and C of all convergent sequences, respectively.

Transforms of X_p in to ℓ_∞

Consider the following matrix transformation.

$$y_n = \sum_{k=1}^{\infty} a_{n,k}x_{k,}\ \ n = 1,2,3, \dots \dots$$

If the right-hand series is convergent. Now we'll prove a theorem based on Malkovsky and Parashar (1997).

Theorem: - A transformation of a matrix $A = (a_{n,k})$ maps the space $X_p(1 \leq p \leq \infty)$ in to the space ℓ_∞ *if and only if*

$$sup_{n\geq\infty} \left\| \{k(a_n - a_{n,k+1}\}_{k\geq1} \right\|_{\ell(q)} < \infty\text{……. …. (2)}$$

$$sup_{k\geq1}|ka_{n,k}| < \infty.\ \text{for every fixed n……….. (3)}$$

Where, 1/p+1/q=1.and$\|.\|_{\ell(q)}$ is ℓ_q norm.

Proof: - First, we demonstrate that the prerequisites exist. Suppose $A = (a_{n,k})$ maps X_p in to ℓ_q then the series $(A\,x)_{n.k} = \sum_{k=1}^{\infty} a_{n.k}\,x_k$ is convergent for every $x \in X_p$. Then the sequence $\{a_{n,k}\}_{k\geq1}$ is a component of y_q The condition (3) holds for every n, so

$\left\| \{k(a_n - a_{n,k+1}\}_{k\geq1} \right\|_{\ell(q)} < \infty$. since X_p and ℓ_∞ are BK-space and

That, $\|A\,x\|_{\ell(\infty)} \leq K\|x\|_p$ K is a real constant, and all $x \in X_p$ or

$$sup_{n\geq1}|(Ax)_n| \leq k\|S\|_{\ell_{(p)}}$$

For all $x \in X_p$ with $S = \{s_k\}$ where $S_k = \frac{1}{k}\sum_{i=}^{k} x_i$

It follows that

$$sup_{n\geq1}\left|\frac{\sum_{k=1}^{\infty} k(a_{n,k}-a_{n,k+1})S_k}{\|S\|_{\ell_{(p)}}}\right| \leq k$$

Hence, we have,

$sup_{n\geq 1}\left\|\{k(a_{n,k}-a_{n,k+1})\}_{k\geq 1}\right\|_{\ell(q)}\leq k$

Thus, the condition holds (2)

Conversely, suppose condition (2), (3) hold, then the sequence for every n = 1,2,3,... $\{a_{n,k}\}_{k\geq 1}$is a component of y_q as a result, each $x\in X_p$ and

We have Holder inequality,

$$sup_{n\geq 1}|(Ax)_n| = sup\left|\sum_{k=1}^{\infty} a_{n,k}x_k\right|$$

$$= sup\left|\sum_{k=1}^{\infty} k(a_{n,k}-a_{n,k+1})S_k\right|$$

$$= sup\left\|\{k(a_{n,k}-a_{n,k+1}\}_{k\geq 1}\right\|_{\ell(q)}\|S_k\| < \infty$$

Which shows that $Ax\in\ell_\infty$ and $A=(a_{n,k})$ maps $X_p$in to ℓ_∞. This completes the proof of the theorem.

Matrix transformation of X_p in to C

Consider the following matrix transformation.

$$Y_n=\sum_{k=1}^{\infty} a_{n,k}x_k,\ \ n=1,2,...$$

Theorem: - A matrix transformation $A=(a_{n,k})$ maps X_p in to the space C if and only if

$sup_{n\geq 1}\left\|\{k(a_{n,k}-a_{n,k+1})\}_{k\geq 1}\right\|_{\ell(q)}\leq\infty$ … … … … … ………….(4)

$sup_{k\geq 1}|ka_{n,k}|<\infty.$ for every fixed, n…………………………………… (5)

$\lim_{n\to\infty} k(a_{n,k}-a_{n,k+1}=\delta_k$ for every fixed k …………………….....(6)

Where 1/p+1/q=1

Results

From the above work we find the following results

THEOREM:- The associate space H' of the space H coincide with the conjugate space (Banach dual) H^* of the space H algebraically and Isometrically.

Proof:- For any $y\in H'$, then we see that $T_y(x)=\sum_{k=1}^{\infty} x_k y_k$ defines a linear continuous functional on H with norm $\| T_y \|=\rho'(y)$.

Conversely, if $t\in H^*$, let e^k denote the sequence with 1 in the k_{th} coordinate and zero elsewhere. For any $x\in H$, let also $x^N=\{x_1,x_2,...,x_N,0,...\}$. Then we have $x^N=\sum_{k=1}^{N} x_k e^k$ and $\rho(x-x^N)\to 0$ as $N\to\infty$. Since T is continuous, we have

ILAM Vol. 19, Issue 1, August 2023

$$T(x) = \lim_{N\to\infty} T(x^N)$$
$$= \lim_{N\to\infty} \sum_{k=1}^{N} x_k T(e^k)$$
$$= \sum_{k=1}^{\infty} x_k T(e^k)$$

Which is convergent for all $x \in H$. This implies that the sequence $\{T(e^k)\}$ is an element in H' by above proven theorem and

$$\| T \| = \sup\{|\sum_{k=1}^{\infty} x_k T(e^k)| : \rho(x) \le 1\}$$
$$= \rho'(T(e^k)).$$

This shows that every $T \in H^*$ can be represented by an element $\{T(e^k)\}$ in H'. Thus if we identify each $T \in H^*$ with $\{T(e^k)\}$ in H', we see that $H' = H^*$ algebraically and isometrically.

Theorem:- The associate space X' of X_p is the space $y_q$with the norm μ_q where,

1/p +1/q =1.

Lemma:- The space H is a BK-space.

Proof

Since the associate space H' contains all the finite sequences, if $x^{(n)} = \{x_k^{(n)}\} \in H$, with $\rho(x^{(n)}) \to 0$as$n \to \infty$,

then $|x_j^{(n)}| = |\sum_{k=1}^{\infty} x_k^{(n)} e^j| \le \rho(x^{(n)})\rho'(e^j) \to 0asn \to \infty$

where e^j is the sequence 1 at the j_{th} place and zero elsewhere.

Theorem: $\delta_k = 0$ if C is replaced by the space C_0 of all null space for all k.

Conclusion

In this article the following conclusion can be used to investigate the characteristics of various existing sequence spaces studied in functional analysis for further generalization and unification. It can be used as a basis for developing ideas in every aspect of human knowledge in future work.

The non-absolute type of Cesaro sequence space, a matrix transformation $A = (a_{n,k})$ maps X_p to the space ℓ_∞ and C by satisfying

$\left\|\{k(a_n - a_{n,k+1}\}_{k\ge1}\right\|_{\ell(q)} < \infty$ and X_p and ℓ_∞ are BK-space and $(A\,x)_{n.k} = \sum_{k=1}^{\infty} a_{n.k}\, x_k$ is convergent for every $x \in X_p$ also a transformation of a matrix $A = (a_{n,k})$ maps X_p in to C_0 replaced C by C_0 for δ_k= 0 for all k.

1. The matrix A converts a BK-space X_p in to ℓ_∞ BK-space, this is a continuous and linear transformation.
2. Each coordinate mapping in BK-space is a banach space is available where $x \to x_k$ is continuous.

3. The conditions that an infinite matrix must meet in order to transform the Cesaro sequence space X_p into the respective ℓ_∞ of and all convergent sequences' space C.
4. Some theorems are developed with matrix transformation.

Reference

Zaanen, A. C. (1967). *Integration* (2nd ed.). North-Holland.

Colak, R. & Some, M. E. (2004). *Topics of sequence spaces*. Lecture Notes in mathematics, Firat Univ. Press.

Kelly, J. L. (1955). *General topology*. Van Nostrand.

Kizmaz, H. (1981). *On certain sequence space.* Canadian Math. Bull 24

Malkowsky, E. & Parashar, S. D. (1997). Matrix transformations in bounded and convergent difference sequence spaces of order m. *Analysis*, *17*, 87-97

Et, M. and Çolak, R. (1995) On some generalized difference sequence spaces. *Soochow Journal of Mathematics*, *21*, 377-386.

Mishra, K. *(1993),* Matrix maps involving specific sequence spaces. *Indian Journal of Pure and Applied Mathematics*, *24*(2), 125–132.

Khatib, M., & Qamaruddin, M. A. (1997). On difference Cesearo sequence spaces of non– absolute type, *Bull. Cal. Math. Soc*. *89*, 337–342.

Ng, P. N., & Lee, P. Y. (1978). Cesáro sequences spaces of non-absolute type. *Comment Math Prace Mat*, *20*, 429-433.

Orhan, C. (1983). Casaro differance sequence spaces and related matrix transformations. *Communications Faculty of Sciences University of Ankara Series A1 Mathematics and Statistics, 32*, https://doi.org/10.1501/Commua1_0000000222

Kamthan, P. K. & Gupta, G. (1981). *Sequence space and series*. Dekker Inc.

Shiue, J. S. (1970). On the Cesaro sequence spaces. *Tamkang Journal of Math. 1*, 19-25.

Tripathy, B. C. & Esi, A. (2006). A new type of difference sequence spaces. *International Journal of Science and Technology, 1*(1), 11-14.