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Earliest Arrival Flow with Partial Lane Reversals for Evacuation Planning

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Abstract

Contraflow evacuation planning strategy is very effective and widely accepted approach for the optimal use of available road network in evacuation management that increases the outward road capacities from the disastrous areas with lane (arc) reversals towards the safer places. It is highly applicable for shifting maximum number of evacuees from the disastrous areas to the safer places as quickly and efficiently as possible. We introduce the partial contraflow approach by reversing only necessary arc capacities to solve the earliest arrival contraflow problem with constant transit times and present efficient algorithms. We solve the earliest arrival partial contraflow problem in two terminal general network in pseudo-polynomial time complexity. On two terminal series parallel network, we solve the problem in strongly polynomial time complexity. Moreover, we present a fully polynomial approximation algorithm that solves the earliest arrival partial contraflow problem on two terminal general network in polynomial time. The unused arc capacities are very useful for the logistic and emergency supports to the evacuees at disastrous areas.

Keywords: Evacuation planning, transportation network, contraflow, earliest arrival flow problem.

1 Introduction

The challenges in evacuation planning have been vital because of rapid disasters and limited road capacity. Use of mathematical modeling for evacuation planning is a growing research area. Most of the research in evacuation planning is focused on the network optimization. For more details of mathematical models used in evacuation planning, we refer to the survey [1].

After disasters, the process of removing residence as quickly and efficiently as possible from the disastrous areas to the safer places is an evacuation planning problem. The disastrous areas and safer places are considered as sources S and sinks D, respectively, and the connection between these places are lanes or arcs. There may be number of intersections of street between the connections that are considered as nodes. Each lanes have limited capacities and fixed travel times. The flow is defined as the group of evacuees passing through the network as a homogeneous group. In such transportation network, the evacuation planer discourage people to move towards sources from sinks because of which the corresponding road lanes are unoccupied. However, the lanes outwards from sources become more congested due to large number of evacuees and vehicles on the streets.

The partial contraflow reconfiguration reverses the idle directions of empty lanes towards sinks satisfying the given constraints that increases the flow value, decrease the average evacuation time and save some lane capacity with excess capacities [7, 8]. The partial contraflow technique can be used by the emergency management team in urban areas to optimize different aspects of traffic flow in evacuation planning. If a decision maker wants to allow the maximum traffic flow at every time point from the beginning and listing unused capacities of the road segments which can be used for any other purpose during emergency management, finding the earliest arrival flow with partial lane reversals is very useful. Its main advantage is that the predetermined time horizon, i.e., the estimated time within which the evacuation process should be completed, is not necessary. An efficient method to estimate the evacuation time for shifting evacuees from the sources to sinks is still demanding. So, the problem of finding maximum flow from the beginning of time point with arc reversals is important in evacuation planning. This problem is the earliest arrival contraflow problem (EACFP).

From the series of literature on analytical contraflow approach [1, 6, 9], it is conformed that the complete contraflow configuration increases the flow value up to double for given time horizon. Similarly, evacuation time is minimized efficiently, if given flow value is to be transshipped from S to D. However, the unnecessary arc reversals are not prevented in the previous models. For example, if an arc has capacity 3 and we need to reverse only capacity 2, the complete contraflow models do not care it and reverse all 3 capacities. In this work, we add a new technique in previous model that enables to reverse only necessary capacities of arcs which is named as partial contraflow approach. With partial contraflow configuration, we solve the earliest arrival flow problem, i.e., flow is maximized at every time point $\theta, 0 \leq \theta \leq T$ with partial reversals of arc capacities.

This work is organized as follows. Section 2 sketches all the necessary notations and definitions with the partial contraflow model. Section 3 formulates the earliest arrival contraflow problems. The paper is concluded with Section 4.

2 Model of contraflow configuration

During evacuation process, a flow has to travel from disastrous areas (sources) to safe areas (sinks or destinations) using a road network in which set of n nodes (sources, sinks, intersection of streets) V and set of m arcs (streets) A are included. Let S, D be the set of terminals (sources and sinks) respectively. If we have single source and single sink, s and d, respectively represent them. We represent each arc e by a pair (u, v). Each arc has integer capacity b, i.e., maximum amount of flow units moving along each arc and transit time τ, i.e., the time needed to travel an arc. Let T be the estimated time horizon within which the whole evacuation process has to be completed. Set of time horizon is denoted by $\mathbf{T} = \{0, 1, \ldots, T\}$. Let use assume that $B_v = \{e \mid e = (u, v) \in A\}$ and $A_v = \{e \mid e = (v, u) \in A\}$. Collecting all data, the evacuation network is represented as $\mathcal{N} = (V, A, b, \tau, S, D, T)$.

Let non negative functions $y : A \rightarrow R^+$ and $x : A \times T \rightarrow R^+$ represent the static and dynamic flow, respectively. A dynamic s-d flow x for given time T satisfies the flow conservation and capacity constraints (1-3). The inequality flow conservation constraint (2) allows to wait flow at intermediate nodes, however, the equality constraint (replace the inequality in (2) by equality) forces that flow entering an intermediate node must leave it again immediately.

$$\sum_{\sigma=\tau_e}^{T} \sum_{e \in B_v} x_e(\sigma - \tau_e) \quad = \quad \sum_{\sigma=0}^{T} \sum_{e \in A_v} x_e(\sigma), \ \forall\, v \notin \{s, d\} \tag{1}$$

$$\sum_{\sigma=\tau_e}^{\theta} \sum_{e \in B_v} x_e(\sigma - \tau_e) \quad \geq \quad \sum_{\sigma=0}^{\theta} \sum_{e \in A_v} x_e(\sigma), \ \forall\, v \notin \{s, d\}, \ \theta \in \mathbf{T} \tag{2}$$

$$b_e(\theta) \geq x_e(\theta) \quad \geq \quad 0, \ \forall\, e \in A, \ \theta \in \mathbf{T} \tag{3}$$

The earliest arrival flow problem maximizes $val(x, \theta)$ in (4) satisfying the constraints (1-3) for all $\theta \in \mathbf{T}$.

$$\max val(x, \theta) = \sum_{\sigma=0}^{\theta} \sum_{e \in A_s} x_e(\sigma) \quad = \quad \sum_{\sigma=\tau_e}^{\theta} \sum_{e \in B_d} x_e(\sigma - \tau_e) \tag{4}$$

Let the reversal of an arc $e = (v, w)$ be $e' = (w, v)$. We assume that all arcs towards a source s, i.e., (v, s) are empty in an emergency evacuation and all arcs outwards from source, i.e., (s, v) may be congested due to large number of evacuees in the streets. To design the algorithm for (partial) contraflow, with a point of view that all arcs towards the sources from sinks can be reversed, we make use of what is known as auxiliary network $\overline{\mathcal{N}} = (V, E, \overline{b}, \overline{\tau}, S, D, T)$ of the evacuation network $\mathcal{N} = (V, A, b, \tau, S, D, T)$ as follows. The arc set $\bar{E}$ contains all arcs $\bar{e}$ that is formed from arc set A by reversing all empty arcs towards sinks. The transit time and capacities of the auxiliary network are, respectively,

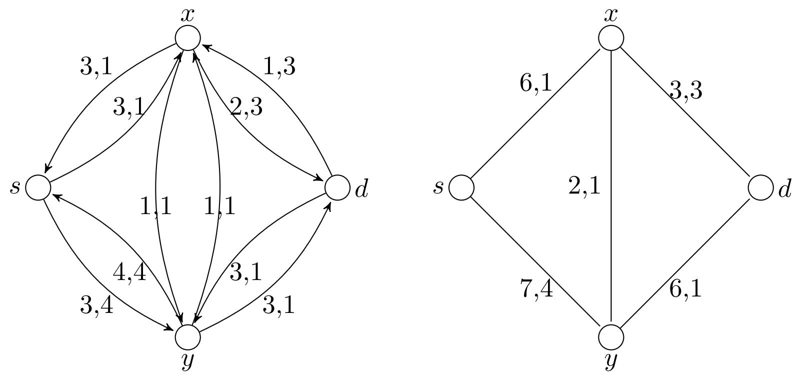
$$\tau_{\overline{e}} = \tau_e = \tau_{e'} \text{ and } b_{\overline{e}} = b_e + b_{e'}$$

where an edge $\overline{e} \in E$ in $\overline{\mathcal{N}}$ if $e \vee e' \in A$ in $\mathcal{N}$. The remaining graph structure and data are unaltered.

Example 1. *Let us consider an evacuation network as shown in Figure 1(a) in which each node represents a city or a region and each arc represents the road segment between them. Nodes s and d are modeled as a source and a sink nodes, respectively. Nodes x and y are intermediate nodes. Each arc has capacity and transit time. For example, an arc between nodes s and x has capacity 3 and transit time 1. If we assume that a time unit is 2 minutes, it takes 2 minutes for evacuees to travel from s to x and a maximum of 3 evacuees can simultaneously travel through the arc. Contraflow reconfiguration of the evacuation network is represented in Figure 1(b) where arc capacities of both directions are added to form new arc capacity but the transit time is same. So that maximum 6 unit flow can travel along arc (s, x) in 2 minutes.*

3 Earliest arrival contraflow

Authors in [7] investigated the dynamic contraflow problems with partial lane reversals and presented efficients algorithms to solve it on number of particular networks. Based on their finding, we describe the importance of partial contraflow configuration with following example by computing the maximum dynamic partial contraflow solution.



(a) (capacity, transit time) (b)

Figure 1: (a) Evacuation network (b) Reconfigured network

Example 2. *Figure 2(a) represents the maximum static contraflow solution with complete contraflow solution, from which we can compute the solution of maximum dynamic contraflow problem by solving the temporally repeated flow for time horizon $T = 5$ as in Equation 5*

$$val(x.T) = (T+1)val(y) - \sum_{e \in E} \tau_{\bar{e}} y_{\bar{e}}. \tag{5}$$

where y is the maximum static flow of value $val(y)$. From three paths $P_1 = s - x - y - d$, $P_2 = s - x - d$ and $P_3 = s - y - d$ with flow 2, 3 and 4, respectively give the maximum static flow $y = 9$. For given time horizon $T = 5$, the maximum dynamic contraflow $val(x,T) = 16$ is obtained with complete contraflow configuration. In addition to the maximum static contraflow obtained in Figure 2(a), we calculate residual capacity of arcs $s_{cap} = b_{\bar{e}} - x_{\bar{e}}$ in Figure2(b) as follows. The maximum static contraflows along path $s - x - y - d$ through arcs (s,x),(x,y) and (y,d) is 2, along path $s - x - d$ with arcs (s,x) and (x,d) is 3, and along path $s - y - d$ with arcs (s,y) and (y,d) is 4. By subtracting the total static flows along an arc from the capacities of respective arcs, we compute the residual capacities as $(s,x) = 6 - 5 = 1$ and $(s,y) = 7 - 4 = 3$. These residual capacities are saved for all time $\theta \in T$ in all the temporally repeated paths using these arcs (s,x) and (s,y). The saved arc capacities are allowed to flow along opposite direction (x,s) and (y,d) whenever required for other purposes.

In this section, we introduce the abstract earliest arrival partial contraflow problem (EAPCFP) (cf. Problem 1) which seeks to maximize the flow value at each point of time with path reversal capability by saving the unnecessary reversals of element capacities. On two-terminal evacuation network, we present an efficient algorithm to solve it.

Problem 1. *For given network $\mathcal{N} = (V, A, b, \tau, S, D)$ with integer input , the EAPCFP is to find the earliest arrival flow from S-D flow for all time θ, $0 \leq \theta \leq T$ with partial reversals of arc capacities.*

In general the *S-D* EACFP is NP-hard even with the case of complete contraflow. From this, we directly say that the *S-D* EAPCFP is NP-hard. However, the EACFP can be solved efficiently in different particular networks. We use different temporally repeated flow

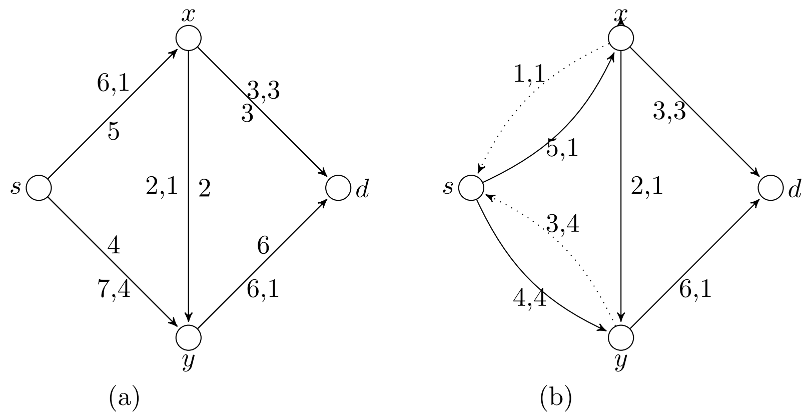


Figure 2: (a) Complete contraflow (b) Partial Contraflow for maximum flow

algorithms as well as successive shortest path algorithms developed for different particular networks that give the EAPCF solutions. For example, on two terminal general network and series parallel network, the EAPCFP will be investigated. We modify the EACF algorithm presented by authors in [10, 9] by reversing only partial arc capacities and present Algorithm 1.

Algorithm 1. *EACF algorithm with partial lane reversals*

1. ***Input:*** *Given an evacuation network* $\mathcal{N} = (V, A, b, \tau, s, d)$.
2. *Obtain auxiliary network* $\overline{\mathcal{N}} = (V, E, \bar{b}, \bar{\tau}, s, d)$.
3. *On the abstract network* $\overline{\mathcal{N}}$, *compute the earliest arrival flow.*
4. *Calculate* $s_{cap}(\bar{e}, \theta) = b_{\bar{e}} - x_{\bar{e}}(\theta)$. *The capacity of the arc* $\bar{e}$ *at time* θ *is saved if* $s_{cap}(\bar{e}, \theta) > 0$, $\theta = 1, \ldots, T$.
5. ***Output:*** *An EACF with saved capacities, if any, of arc for the evacuation network* $\mathcal{N}$.

The earliest arrival flow, in Step 3 of our algorithm, is calculated by using different existing algorithms. For the two terminal general network, one can apply the successive shortest path computation as in [12] or [4] that gives the earliest arrival solution using successive shortest path computations in corresponding time expanded network. However, we use the non-standard chain decomposition introduced in [3] that also gives the earliest arrival flow solution on the auxiliary network. Recall that the earliest arrival flow continues the already obtained flows in earlier steps to forthcoming flows in forward steps, the final solution may change the direction of arcs and obeys the backward flow laws in its processing. If the arc reversals is made at any time whenever necessary, the EAPCFP can be solved using these algorithms. Due to the successive shortest path computations, its time complexity is pseudo-polynomial time. As Pyakurel and Dhamala [10] and [9] solved the earliest arrival contraflow problem with complete contraflow configuration of evacuation network in discrete time setting and continuous time setting, respectively, here we modify their algorithm for the partial contraflow configuration.

Example 3. *As the shortest distances paths at all successive time points are necessary for the earliest arrival contraflow, the maximum dynamic partial contraflow solution from Figure 2(b) also gives the earliest partial contraflow. This can be represented as in Figure 3.*

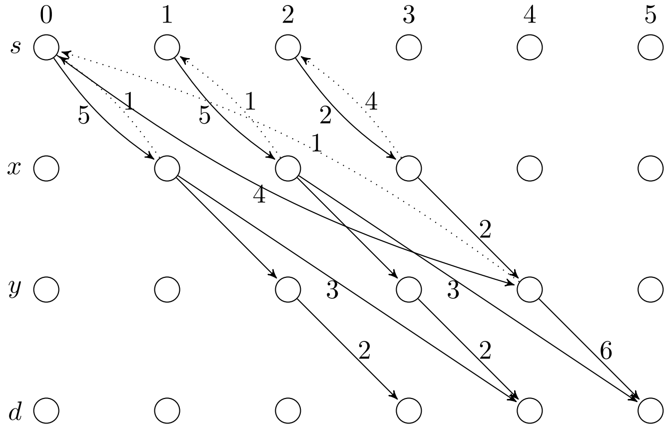


Figure 3: Earliest arrival partial contraflow solution of Figure 2(b)

From Figure 3, we can say that the earliest arrival partial contraflow can be computed with temporally repeated flow on general two terminal network, however, it is not true always. We consider Figure 4(a) in which the temporally repeated flow property will be violated while computing the earliest arrival partial contraflow solution.

Figure 4(a) represents an evacuation scenario in which each arc contains integer capacity and transit time. With contraflow configuration, we construct its auxiliary network and choose the direction of arc randomly. First we choose the direction as in Figure 4(b). In the static network, the maximum static flow is as shown in Table 1. Using the non-standard chain

Table 1: Optimal maximum static flow in auxiliary network, Figure 4(b)

Arcs	(s,x)	(s,y)	(x,u)	(x,d)	(u,y)	(y,d)	(d,s)
Flow(y)	7	1	6	1	6	7	8

decomposition technique of [3], the earliest arrival flow can be obtained from Figure 4(b) as follows. Let us fix the set of chain flows with $\Gamma = \{P_1, P_2, P_3\}$ with $P_1 = \{s, y, v, x, d\}$, $P_2 = \{s, x, v, y, d\}$ and $P_3 = \{s, x, u, y, d\}$. Note that Γ is not standard chain decomposition because $P_2 \in \Gamma$ uses arc (v, x) and (y, v) that are in opposite direction which do not lie in E. In Figure 4(c), we can see that, chain P_2 starts using arc (y, v) in direction (v, y) at time $t = 3$ and stops using it at time $t = 4$. From other side, the chain flow P_1 starts using arc (y, v) at time $t = 1$ and stops using it at time $t = 4$. Moreover, we can save the unused arc capacity at each time point, for example, the capacities of arcs (y, v) and (v, x) are saved by 4 units at time zero and 5 units at each time point from 1 to 4. Thus, total 14 units of flow value can be computed in time step 7 which is the earliest arrival partial contraflow solution. However, it is directly dependent with time horizon. Thus, its complexity is pseudo-polynomial.

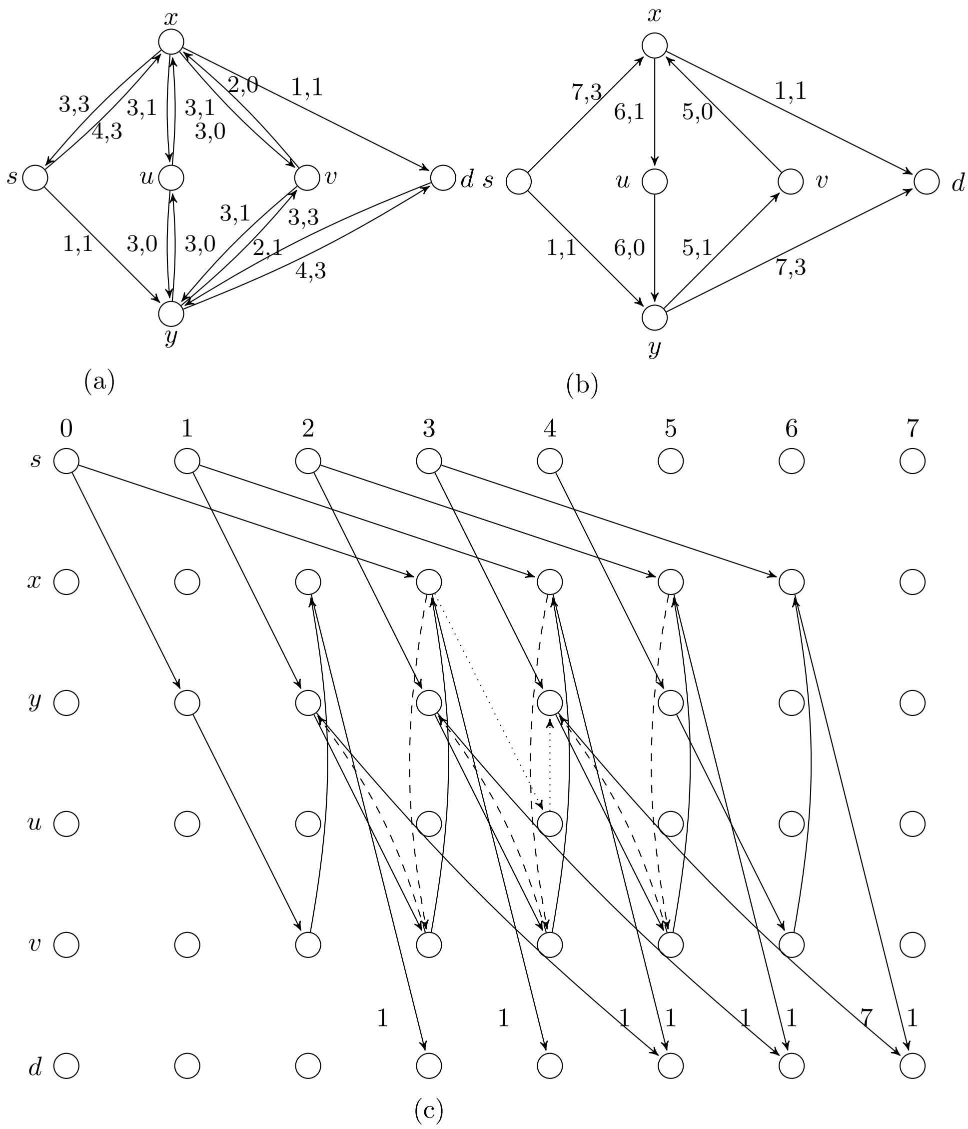


Figure 4: EAPCF solution on general network

Theorem 1. *On s-d network, the EAPCFP can be solved in pseudo-polynomial time complexity by reversing only partial arc capacities at any time.*

Saving unused arc capacities in Step 4 does not violate the optimal solution obtained from Algorithm 1. Thus, the optimality proof of Theorem 1 is as in [10, 9]. Moreover, the complexity of Algorithm 1 is dominated by the complexity of Step 3. As the earliest arrival flow computation in Step 3 using algorithms of [4, 12] have pseudo-polynomial, the complexity of our algorithm is pseudo-polynomial on two terminal general network with arc reversals at any time.

By reversing the arc direction at time zero, we solved the EAPCFP on two terminal series-parallel network in strongly polynomial time complexity. A single arc $e = (v, w)$ with tail node v and head node w is series-parallel. Let G_1 and G_2 be two series-parallel graphs with nodes s_1 and d_1, and s_2 and d_2, respectively. Then, the graph $S(G_1, G_2)$ obtained by identifying d_1 as s_2 in the series combination is a series-parallel graph with s_1 and d_2 as its terminals. The graph $P(G_1, G_2)$ obtained by identifying s_1 as s_2 and also d_1 as d_2 in the parallel combination is a series-parallel graph with $s_1(= s_2)$ and $d_1(= d_2)$ as its terminals.

First we apply Algorithm 1 on two terminal series parallel network. With the temporally repeated minimum cost circulation flow (MCCF) algorithm of [11], we compute the EAPCF solution in strongly polynomial time with partial reversals of arc capacities. A MCCF solution has minimum cost if and only if the corresponding residual network does not contain a cycle with negative cost. The main advantage in series-parallel graphs is that every cycle in the residual network has nonnegative cycle length. In the auxiliary network $\overline{\mathcal{N}}$,, the maximum dynamic partial contraflow problem is solved using the MCCF algorithm of [11]. As the obtained temporally repeated flow satisfies the earliest arrival flow property, i.e., a cumulative amount of flows reaching (leaving) the sink (source) in every considered time point and all preceding time points of the considered one have to be maximal, the optimal solution to the EAPCFP is similar to the solution of maximum dynamic partial contraflow problem.
Example 4. *Figure 5(a) represents a series-parallel contraflow network, Figure 5(b) is the auxiliary network and Figure 5(c) gives the temporally repeated flow solution for the earliest arrival contraflow problem.*
Theorem 2. *On s-d series parallel network, the EAPCF solution can be computed in* $O(nm + mlogm)$ *time complexity by reversing only partial arc capacities at time zero.*

As there is not any polynomial time algorithm to solve the EAPCFP on two-terminal general network with arc reversals at time zero, we investigate its approximation solution. Authors in [5, 6] presented a polynomial time approximation algorithms with complete contraflow configuration at time zero in both discrete and continuous time settings that obtain a flow value within $(1 - \epsilon)$ of optimal earliest arrival contraflow on two-terminal general network.
Problem 2. *Let* $\mathcal{N} = (V, A, b, \tau, s, d, T)$ *be a two-terminal network. Let* $\epsilon > 0$ *be given. The problem is to find an approximation solution for EAPCFP from source s to sink d within a factor of* $(1 - \epsilon)$*, for* $\epsilon > 0$ *in time* T *if the direction of the arcs can be reversed at time zero by saving unused arc capacities.*

To solve Problem 2, we present a fully polynomial approximation algorithm, Algorithm 2. In Step 3, we solve the approximate earliest arrival flow problem using the algorithm of [3].
Algorithm 2. *Approximate EAPCF algorithm*

1. ***Input:*** *Given an evacuation network* $\mathcal{N} = (V, A, b, \tau, s, d)$.
2. *Obtain auxiliary network* $\overline{\mathcal{N}} = (V, E, \bar{b}, \bar{\tau}, s, d)$.
3. *In* $\overline{\mathcal{N}}$ *solve the earliest arrival flow problem on fully polynomial time approximate algorithm of [3].*
4. *Calculate* $s_{cap}(\overline{e}, \theta) = b_{\overline{e}} - x_{\overline{e}}(\theta)$. *The capacity of the arc* $\overline{e}$ *at time* θ *is saved if* $s_{cap}(\overline{e}, \theta) > 0$, $\theta = 1, \ldots, T$.
5. *Obtain* $(1 - \epsilon)$*-approximation solution of EAPCFP for the network* $\mathcal{N}$.

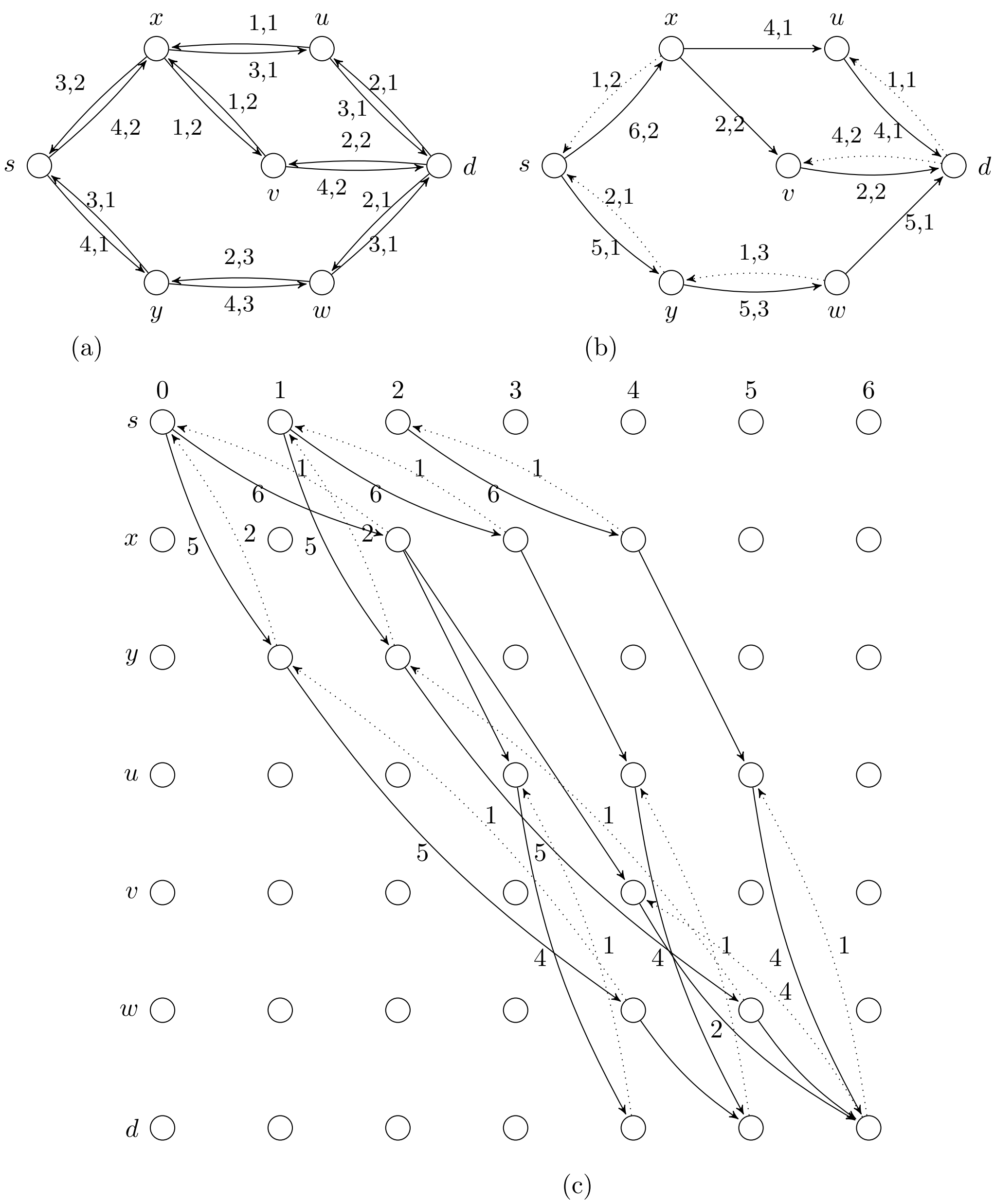


Figure 5: EAPCF on two terminal series parallel graph

The fully polynomial approximate algorithm of [3] works as follows. Set the initial flow $x = 0$, scaling factor $\Delta = 1$, $\bar{b}(v, w) = b(v, w)$, $\epsilon > 0$, and $\Gamma = \phi$. The algorithm combines capacity scaling with the shortest augmenting paths algorithm. In their dynamic networks, all capacities are evenly divisible by $\Delta = 1$. Initially the static flow is obtained using shortest augmenting paths algorithm until the flow value exceeds $\frac{m}{\epsilon}$ where the residual capacities are rounded down to the nearest even number. The flow computed from each augmentation is added to a set of chain flows Γ that will give the final dynamic flow. Thus, each successive phase uses the residual networks obtained from previous phase that is divisible by Δ, multiple of 2, augments flow along shortest paths until the flow value of the new augmentation exceeds $\Delta\frac{m}{\epsilon}$, adds the augmenting chain flows on Γ, and finally rounds the residual capacities down so that they are evenly divisible by 2Δ. This process continues until there is no augmenting path of length less than or equal to T.

Theorem 3. *Algorithm 2 computes $(1-\epsilon)$-approximation for EAPCFP on two-terminal arbitrary networks in $O(m\epsilon^{-1}(m+nlogn)logU)$ time, where U is the maximum capacity of the network .*

Proof. Algorithm 2 is feasible as all steps are feasible. First, the auxiliary network $\overline{\mathcal{N}}$ is constructed from given network by reversing the direction of arcs at time zero. On $\overline{\mathcal{N}}$, $(1-\epsilon)$-approximate earliest arrival flow is computed using algorithm of [3]. According to [5, 6], thus obtained $(1-\epsilon)$-approximate earliest arrival flow on $\overline{\mathcal{N}}$ is equivalent to the $(1-\epsilon)$-approximation for EAPCFP on original network.

Moreover, the fully polynomial time approximation algorithm for earliest arrival flow of [3] requires $O(m\epsilon^{-1}(m+n\log n)\log U)$ time. Thus the complexity of Algorithm 2 requires the similar complexity to compute the approximate EAPCF solution. □

Algorithms 1 and 2 solve the EAPCFP in discrete time setting. Using natural transformation of [2], these algorithms can be converted into continuous time. Thus, the EAPCFP can be solved in continuous time with the same complexity as in discrete time setting.

4 Conclusions

In literature, the evacuation planning problem with complete contraflow configuration of the evacuation network has been studied. Here, partial contraflow configuration approach is introduced by reversing only partial arc capacities. The earliest arrival partial contraflow problem is solved with efficient algorithms. The problem is solved in strongly polynomial time in two terminal series-parallel network and in pseudo-polynomial time in two terminal general network. An approximate solution to the earliest arrival partial contraflow problem has been obtained on two terminal general network.

To the best of our knowledge, the problem we introduced is for the first time in the partial contraflow approach. Moreover, we are interested to extend the partial contraflow model and algorithm to solve other dynamic network flow problems with constant as well as variable transit times and make them more relevant in applications.

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