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Lexicographic Earliest Arrival Contraflow Evacuation Problem on UPL-TTSP Network

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Abstract

An evacuation plan proposes a way out for maximum utilization of an existing transportation system so that the safest tour and most efficient evacuation time of all expected evacuees could be ensured. This paper studies the auto-based earliest arrival contraflow problem modeled on network with capacitated intermediate vertices. The vertices serve as transshipment vertices as well as temporary shelters of given priority order. It also proposes an efficient solution procedure to the problem for the uniform path length two terminal series parallel (UPL-TTSP) network.

Keywords: Network contraflow, Capacitated vertices, Evacuation planning problem, Disaster management

1. Introduction

Increasing number of disasters due to different kind of natural and human caused hazards have drawn attention of stakeholders for preparation of optimal evacuation plans. Evacuation planning problems can be modeled as network optimization flow problems. Since it is usually not known when disaster will actually happen, it is desirable to organize an evacuation in such a way that as many evacuees as possible are sent to the sink as early as possible. Moreover, it is not always possible to send all the evacuees into the sink due to network capacity constraint and/or time constraint, but can be held at relatively safe intermediate spots besides sending to the sink. One of the potential ways to accelerate the process of achieving this objective is to apply notion of network contraflow in evacuation model with non-conservation flow constraints.

Maximum dynamic flow (MDF) problem, introduced by Ford and Fulkerson [11, 12] is a central problem in network flow theory and in mathematical evacuation planning problem that attempts to determine maximum flow of single commodity from a single source to a single sink over specified time horizon. The problem with arc direction reversal capability (contraflow), known as maximum dynamic contraflow (MDCF) problem, has been studied and proposed polynomial time solution algorithm in [18]. The MDCF problem with continuous time setting has been studied in [15]. Authors in [5] consider not necessarily equal transit time on anti-parallel arcs and study MDCF problem for general network for discrete as well as continuous time setting.

An extension of MDF problem is earliest arrival flow (EAF) problem. An EAF problem aims to optimize the objective of MDF problem for every time step within pre-specified time horizon. Existence of solution to earliest arrival flow problem was proved by Gale [13]. The problem has been studied in different contexts since then, see [2], [3], [14], [17], [20], [21], etc. To the best of our knowledge no polynomial time exact algorithm for earliest arrival flow problem on general network is known till now. Ruzika et al. [19] considered maximum dynamic flow problem on special class of networks known as two terminal series parallel (TTSP) network. They developed a polynomial time MDF algorithm for TTSP networks and showed that this also solves earliest arrival flow problem for these networks. Authors also pointed out that on series-parallel networks it is always possible to find an earliest arrival flow as a temporally repeated flow. However, the converse is not necessarily true. Earliest arrival contraflow (EACF) problem has been studied and proposed a polynomial time solution algorithm, based on the idea in [19], in [10] for TTSP network. Authors in [5] consider not necessarily equal transit time on anti-parallel arcs to study EACF problem for this class of networks. Network flow problem with non-conservation flow constraints is studied in the context of evacuation planning problem in [9] (cf. [4], [6], [16]). By considering the multinetwork the contraflow evacuation problems with capability of holding evacuees at intermediate spots have been studied in [8]. Authors have proposed polynomial time solution algorithm for static case and pseudo-polynomial time solution algorithm for dynamic case.

This paper studies the earliest arrival contraflow problem modeled on the network that allows holding of flow units at intermediate vertices. It discusses solution procedure for the problem modeled for uniform path length two terminal series parallel (UPL-TTSP) network. The problem is formulated in Section 2. Section 3 discusses its solution procedure. Section 4 concludes the paper.

2. Problem Formulation

Consider a network $N = (V, A, u(a), \tau(a), k(v), s, d)$ with vertex set V and arc set A, both to be finite, such that $n: = |V|$ and $m: = |A|$. Vertices s and d represent the source and the sink, respectively. Here, $u: A \to \mathbb{N}_0 = \mathbb{N} \cup \{0\}$ represent the arc capacity function which bounds the number of flow units on each arc $a \in A$ at each time step $t \in \mathcal{T} := \{0, 1, \dots, T\}$ where T is given time horizon. Moreover, the transit time function $\tau: A \to \mathbb{N}$ specifies the time needed by a flow unit to traverse an arc. We assume a terminal set $S: = \{v_1, \dots, v_r\} \subset V$ with prioritized from higher to lower priority, i.e., $d = v_1 \succcurlyeq \cdots \succcurlyeq v_r$, to be given. Similarly, the vertex capacity function $k: S \to \{0, \infty\}$ delimits the total number of flow units, which may be held in each of the vertices $v_i \in S$.

The nonnegative variables $f(a,t)$ defined by $f: A \times \mathcal{T} \rightarrow \mathbb{N}_0$ is the number of flow units entering arc a at time step t that specify the flow over time in the network N. This flow unit should be bounded by the capacity of arc, i.e., $f(a,t)$ satisfies the capacity constraints for all $a \in A$ and for all $t \in \mathcal{T}$. That is,

$$0 \leq f(a,t) \leq u(a) \qquad \forall\, a \in A \text{ and } t \in \mathcal{T}. \tag{1}$$

Moreover, $f(a,t)$ has to be equal to zero for all $t > T - \tau(a)$ for all $a \in A$. The excess flow at vertex $v \in V$ at time $t \in \mathcal{T}$, denoted by $ex_f(v,t)$, is defined as

$$0 \leq ex_f(v,t) := \sum_{a \in \delta^{-(v)}} \sum_{\xi=0}^{t-\tau(a)} f(a,\xi) - \sum_{a \in \delta^{+(v)}} \sum_{\xi=0}^{t} f(a,\xi) \tag{2}$$

where $\delta^{-(v)} := \{ a \in A: a = (w,v) \text{ for some vertex } w \in V\}$ and $\delta^{+(v)} := \{ a \in A: a = (v,w) \text{ for some vertex } w \in V\}$ denote the set of arcs entering and leaving vertex $v \in V$, respectively.

Further, we need to ensure that the excess flow at each vertex $v \in S$ over time horizon T is to be bounded by the capacity (v) , i.e.,

$$ex_f(v,T) \leq k(v) \text{ for all } v \in S. \tag{3}$$

Consequently, the total flow of evacuees leaving the source s equals the total flow of the evacuees held at vertices $v \in S$ over the time horizon T, i.e.,

$$\sum_{a \in \delta^{+(s)}} \sum_{\xi=0}^{T} f(a,\xi) - \sum_{a \in \delta^{-(s)}} \sum_{\xi=0}^{T} f(a,\xi) = \sum_{v \in S} ex_f(v,T). \tag{4}$$

With these settings, *lexicographic earliest arrival contraflow (LexEACF) problem* on N aims to maximize the flow units sent to the terminals on S in given priority order at each time step $t \in \mathcal{T}$, if reversal of arc direction is allowed. An arc $a = (v,w) \in A$ in which the flow could travel from vertex v to vertex w is replaced by the arc (w,v) for contraflow purpose.

3. Solution Discussion

The solution procedure for the lexicographic earliest arrival contraflow problem for a UPL-TTSP network N is discussed here. A directed dynamic network is a uniform path length (UPL) network for which the sum of the transit times on arcs on any possible path from the source s to the vertex v, for all $v \in V$, is equal. A two terminal series-parallel network $N = (V, A)$ is a directed network with a single source s and a single sink d which has a single arc (s, d) or is obtained from two series parallel networks $N_1$and N_2 by one of the two operations: Parallel Composition and Series Composition. The first suggests to merge source vertices s_1 of $N_1$and s_2 of N_2 to form the source vertex s of N and merge sink vertices d_1 of $N_1$and d_2 of N_2 to form the sink vertex d of N. The second suggests to merge the sink vertex d_1 of $N_1$with the source vertex s_2 of $N_2$to form the network N with source vertex s_1 and sink vertex d_2.

To solve the LexEACF problem, if arcs reversibility is allowed only once at time zero, the input network is modified to its corresponding auxiliary network by the technique given in [18] and a maximum dynamic flow is computed on it by using the technique given in [6]. For the modification of input network $N = (V, A, u(a), \tau(a), s, d, T)$, it is transformed into its auxiliary network $\widetilde{N} = (V, \tilde{A}, u(\tilde{a}), \tau(\tilde{a}), k(v), s, d, T)$ where arc set $\tilde{A}$ contains undirected arc (v, w) , if $(v, w) \in A$ and/or $(w, v) \in A$ such that capacity $u(\widetilde{v, w}) := u(v, w) + u(w, v)$ and transit time $\tau(\widetilde{v, w}) := \tau(v, w)$ if $(v, w) \in A$, otherwise, $\tau(\widetilde{v, w}) := \tau(w, v)$. During the dynamic flow computation, a lexicographic dynamic flow is computed on so-formed auxiliary network $\widetilde{N}$ by using the Lexicographically maximum dynamic flow (LexMDF) algorithm of [7]. The maximum dynamic $s - v_i$ flow on $\widetilde{N}$ is obtained iteratively by using temporally repeated flows (TRFs) on it. The TRF is a dynamic flow that can be generated by repeating all possible source to sink path flows starting at time zero and then adopting temporal repetition as far as possible.

For a uniform path length (UPL) network $N = (V, A, u(a), \tau(a), k(v), s, d, T)$, with prioritized vertices $v_i \in V$ sorted as $d = v_1 \succcurlyeq \cdots \succcurlyeq v_r$, the procedure for solving LexMDF problem has following steps: The main idea of the solution procedure of the problem is to find $s - v_i$ paths, for all $v_i \in V: k(v_i) > 0$, at all possible time steps $t \in \mathcal{T}$ with corresponding flow value and send as many units of flow as possible along the paths as long as possible. Such paths can be found by decomposing the flow on solving the *lexicographic minimum cost flow (LexMinCF) problem* on N, see [7]. It gives an extended set $\Gamma_{v_i}^E$ that contains all minimum cost $s - v_i$ paths that exist at any time $t \in \mathcal{T}$ on the residual network of N with respect to the optimal flow $f(v_{i-1})$ at previous immediate prioritized vertex v_{i-1}. Flow units of corresponding values are pushed along each path as long as possible. Moreover, the flow is pushed along the paths in $\Gamma_{v_i}^E$ with the strategy of saving unused paths for the use of next less prioritized vertex v_{i+1} without violating the optimality at v_i. This is assured by selecting the path with highest $F_t\ (\gamma_{v_i})$, the time step at which the flow along γ_{v_i}stops to get repeated, among the paths $\gamma_{v_i} \in \Gamma_{v_i}^E$ with highest $I_t\ (\gamma_{v_i})$, the time step at which the flow along γ_{v_i} starts to get repeated, at the first and so on. This procedure yields an optimal solution to the LexMDF problem on UPL network N in polynomial time.

Here the LexMDF algorithm is applied for UPL-TTSP network after reducing it into its auxiliary network that modifies the LexEAF algorithm proposed in [7] and solves the LexEACF problem. The modified procedure is given in Algorithm 1 that solves LexEACF problem when arc reversibility on N is allowed only once at time zero.

Algorithm 1. Lexicographic Earliest Arrival Contraflow Algorithm

1. Given a UPL-TTSP network $N = (V, A, u(a), \tau(a), k(v), s, d, T)$, $S := \{v_1, \dots, v_r\} \subset V$ with $d = v_1 \succcurlyeq \cdots \succcurlyeq v_r$ and integer inputs.
2. Transform N into $\widetilde{N} = (V, \tilde{A}, u(\tilde{a}), \tau(\tilde{a}), k(v), s, d, T)$ as in [18].
3. Compute LexMDF on $\widetilde{N}$ using Algorithm in [7]
4. Perform flow decomposition into path and cycle flows of maximum flow obtained from step-3 and remove all cycle flows.
5. Arc $(w, v) \in A$ is reversed if and only if the flow along arc $(v, w) \in A$ is greater than $u(v, w)$ or if there is non-negative flow along arc $\notin A$.
6. Obtain LexEACF solution for UPL-TTSP network N.

We make the following claim which turns out to be important in proving the optimality of solution computed by Algorithm 1.

Claim 1. UPL-TTSP network N, after transforming into its auxiliary network $\widetilde{N}$, remains UPL-TTSP network.

Theorem 1. Given a UPL-TTSP network $N = (V, A, u(a), \tau(a), k(v), s, d, T)$, source s and terminal set $S := \{v_1, \dots, v_r\} \subset V$ with $d = v_1 \succcurlyeq \cdots \succcurlyeq v_r$. Then, Algorithm 1 computes a lexicographic earliest arrival contraflow on N in polynomial time.

Proof: The construction of auxiliary network $\widetilde{N}$ of input network N is well defined and the transformed network remains a UPL-TTSP network due to Claim 1. The algorithm (Step 3) pushes flow of value $f(\gamma_{v_i})$ along each path on extended set $\Gamma^E_{v_i}$ for each possible time step $t \in \{0, 1, 2, \dots, T - \tau(\gamma_{v_i})\}$ from the source vertex s to each of the destination vertex $v_i \in S$ in given priority order [6]. Therefore, a maximum flow at each $v_i \in S$ is obtained at the termination of algorithm, [12]. Moreover, the network N being a two terminal series parallel in structure, this flow has an earliest arrival property [19].

The time complexity of Algorithm 1 depends on time complexity of the solution procedure on the reduced network $\widetilde{N}$. Also, it is dominated by solving a LexMDF problem on $\tilde{N}$ since the flow decomposition in each iteration and network transformation can be done only in $O(mn)$ ([1]) and $O(m)$ time, respectively. The LexMDF problem can be solved in strongly polynomial time, [7].

4. Conclusion

Earliest arrival flow problems are of much interest to evacuators because these problems ensure that the number of evacuated persons is maximum at each time point within given time horizon T. Evacuation models with intermediate temporary shelters could be extra benefit while implementing them and contraflow approach seems to be a crucial tool to speed up the overall evacuation process during disasters. This paper proposed a network contraflow evacuation model that allows holding of evacuees at intermediate spots despite sending them to the safe destination. It also proposed polynomial time solution algorithm to the lexicographic earliest arrival contraflow problem for UPL-TTSP network.

Searching of solutions to the problem at which the arc reversibility is permitted at any time point within the specified time horizon would be further research in the field considered here. Problem modeled for multinetwork would also be interesting that captures the situation with multiple lanes connecting two places with unequal transit time on them.

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