Model and Properties of Exponentiated Generalized Odd Lomax Exponential Distribution

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Abstract

This study is based on the formulation of new probability model called Exponentiated Generalized Odd Lomax Exponential Distribution using Odd Lomax Generator and the exponential distribution. Some statistical properties like survival function, hazard rate function, quantile function and random deviate generation of the model is studied. Parameters of the model are estimated using Maximum likelihood, Least square and Cramer – von Mises method methods of estimation. Applicability of the model is studied taking a real COVID - 19 data set. For model comparison, different information criteria like Akanke, Bayesian, and corrected Akaike criteria are used and the goodness of fit of the proposed model is test using Kolmogrov - Smirnov, Cramer – von Mises and Anderson Darling method. To compare the suitability of the model, some already defined probability models are considered and compared on basis of different criteria. To study of the performance of MLEs, Monte - Carlo simulation is presented. All the calculations are performed using R programming language.

Keywords: *Covid-19, Information Criteria, Model Validation, Odd Lomax, Probability model.*

Introduction

Data analysis is one of the most important parts of the modern research (Qtt and Longnecker, 2015). Over all field of the research such as scientific, social, descriptive and exploratory etc, data analysis required. For analysis of the data during research, many of the tools and measurements are available in probability. Methods and tools used for analysis have their own applicability and superiority on different natured data. But not all the techniques available till now are equally applicable for all type of data. Many new data cannot be studied adequately and precisely using existing techniques of data analysis. So, new tools and techniques of the data analysis are essential in modern research (Seyedan & Mafakheri, 2020). One of the tools of data analysis is using probability models. Although there are various important and more useful probability models are available in literature, we need still need some new probability model to explain the hidden characteristic of the data more precisely that could not be

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explained by the existing models. Here, a new probability model Called Exponentiated Generalized Odd Lomax Exponential (EGOLE) model is formulated and it is studied that the model proposed here will be some more beneficial than some existing probability model in analysis of data.

This article is theoretical and pure scientific in nature. Here we have derived a new probability model. This model can help researchers for data analysis with more valid and precise results. Formulated model is better fitting the real life data compared to some well known probability models. Model may help for formulation of new probability models in future.

In literature we can find numerous techniques of getting new probability model. Some techniques are using family of distributions, adding some extra parameters and modifying the existing probability models. There are various exponentiated models such as the exponentiated generalized class of distributions (Cordeiro & Ortega, 2013), exponentiated Weibull distribution (Nadarajah, et al,, 2013) and Exponentiated distributions (Al-Hussaini & Ahsanullah, 2015) etc. A modified Weibull distribution by (Lai, et al., 2003), Beta modified distribution given by (Silva, et al., 2010) and a new modified Weibull distribution by (Almalki &Yuan, 2013) are modified probability models. Generalized Exponential Distribution is modified to give Modified Generalized Exponential distribution (Telee and Kumar, 2023). Modified Upside Down Bathtub-Shaped Hazard Function Distribution was introduced by (Chaudhary et al., 2023). Weibull-H class (Cordeiro,et al., 2017) and the exponential model's extension (Nadarajah & Haghighi, 2011) are used for formulating many new probability models. In recent years generalization is one of the important techniques of formulating new probability models. Weibull Generalized family of distribution (Bourguignon, et al., 2014), T - X family of distribution (Alzaatreh et al., 2013), and Lomax - G family (Coredeiro et al., 2019) etc are some family of distribution that helps in formulation of the new probability models. In this study, Odd Lomax-G family of the model is taken as the generator and exponential distribution (Gupta and Kundu, 2001) is taken as the base distribution to formulate the new probability model. Exponential distribution is a continuous probability distribution that explains the amount of time until some specific event happens. It is a process in which events happen continuously and independently at a constant average rate.

Model Formulation

In this study, odd Lomax generator (OLxG) is used for model formulation. The cumulative distribution function (CDF) and probability density function (PDF) of the OLxG for x > 0, α > 0, β > 0 are given as

$$F\left(x;\alpha,\beta,\lambda\right)=\alpha\beta^{\alpha}\int_{0}^{\frac{G(x;\lambda)}{1-G(x;\lambda)}}\left(\beta+t\right)^{-\alpha-1}dt=1-\beta^{\alpha}\left[\beta+\frac{G(x;\lambda)}{1-G(x;\lambda)}\right]^{-\alpha} \tag{1}$$

$$f\left(x;\alpha,\beta,\lambda\right)=\frac{\alpha\beta^{\alpha}g(x;\lambda)}{[1-G(x;\lambda)]^{2}}\left[\beta+\frac{G(x;\lambda)}{1-G(x;\lambda)}\right]^{-\alpha-1} \tag{2}$$

where, G(x; λ) and g(x; λ) are the CDF and PDF of the base distribution.

The CDF and PDF of the exponential distribution taken as the base line distribution for formulation of the model are given as

$$G(x;\lambda) = 1 - e^{-\lambda x};\ \lambda > 0 \tag{3}$$

and $$g(x;\lambda) = \lambda e^{-\lambda x} \tag{4}$$

The CDF and PDF of the base function are applied in equations (1) and (2) resulting the Odd Lomax Exponential (Ogunsanya et al., 2019) distribution. Let θ > 0 is an additional parameter that is used as exponent to the CDF of the Odd Lomax Exponential model to get the proposed model EGOLE with CDF and PDF defined by

$$F(x;\alpha,\beta,\lambda,\theta) = \left[1 - \beta^{\alpha}\left(\beta + e^{\lambda x} - 1\right)^{-\alpha}\right]^{\theta};x > 0,(\alpha,\beta,\lambda,\theta) > 0 \text{ and,} \tag{5}$$

$$f(x;\alpha,\beta,\lambda,\theta) = \alpha\beta^{\alpha}\lambda\theta e^{\lambda x}\left(\beta + e^{\lambda x} - 1\right)^{-\alpha-1}\left[1 - \beta^{\alpha}\left(\beta + e^{\lambda x} - 1\right)^{-\alpha}\right]^{\theta-1} \tag{6}$$

$$;x > 0,(\alpha,\beta,\lambda,\theta) > 0$$

Special Cases

(i) Taking θ =1, we can get the Odd Lomax Exponential distribution with CDF as

$$F(x;\alpha,\beta,\lambda) = \left[1 - \beta^{\alpha}\left(\beta + e^{\lambda x} - 1\right)^{-\alpha}\right];x > 0,(\alpha,\beta,\lambda) > 0$$

(ii) Taking α =1,β = 1 and θ =1, we will get the Exponential distribution with CDF as

$$F(x;\lambda) = \left[1 - e^{-\lambda x}\right];x > 0,\text{and}\lambda > 0$$

(i). Taking α =1, and β =1 we will get the Generalized Exponential (Gupta and Kundu, 2001) distribution with CDF as

$$F(x;\lambda) = \left[1 - e^{-\lambda x}\right]^{\theta};x > 0,\text{and}\left(\lambda,\theta\right) > 0$$

Some Properties

Reliability function of the model

Reliability function of the probability model is the complementary function of the CDF and is

$$R(x;\alpha,\beta,\lambda,\theta) = 1 - F(x;\alpha,\beta,\lambda,\theta) = 1 - \left[1 - \beta^{\alpha}\left(\beta + e^{\lambda x} - 1\right)^{-\alpha}\right]^{\theta};x \ge 0,(\alpha,\beta,\lambda,\theta) > 0 \tag{7}$$

Hazard rate function

Hazard rate is the instantaneous failure rate. Hazard rate function h(x) can be defined as

$$h(x)=\frac{f(x)}{1-F(x)}=\alpha\beta^{\alpha}\lambda\theta e^{\lambda x}\left(\beta+e^{\lambda x}-1\right)^{-\alpha-1}\left[1-\beta^{\alpha}\left(\beta+e^{\lambda x}-1\right)^{-\alpha}\right]^{\theta-1}$$
$$\left[1-\left\{1-\beta^{\alpha}\left(\beta+e^{\lambda x}-1\right)^{-\alpha}\right\}^{\theta}\right] \tag{8}$$

The PDF and Hazard rate function of the EGOLE is displayed in figure 1. The shape of the pdf is of different shape depending on the values of parameters showing that model may me more flexible. Also the hazard rate plot is decreasing – increasing as well as of the bathtub shaped.

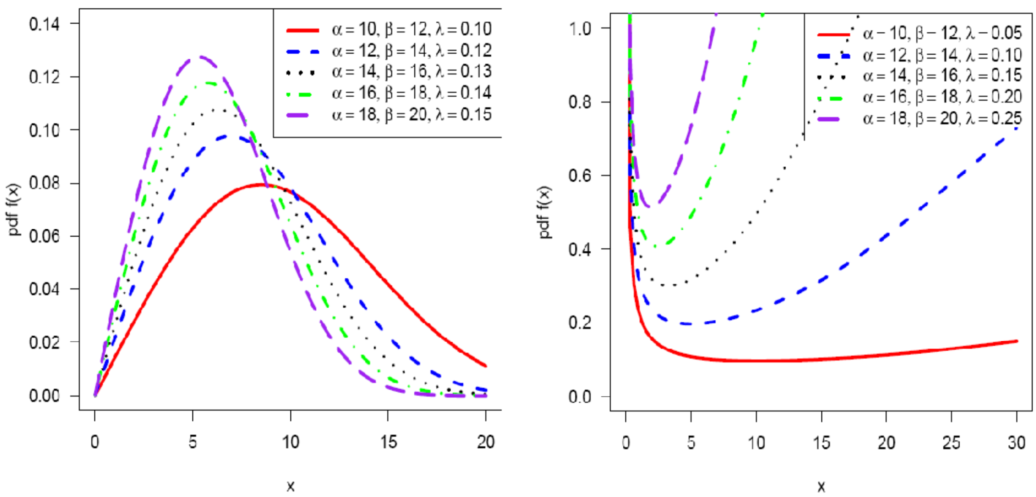


Figure 1: The PDF for constant θ = 2(Left) and HRF for constant θ = 0.2 (Right) of EGOLE

Cumulative hazard rate function

The cumulative hazard rate function H(x) is given as

$$H(x)=-\ln R(x)=-\ln\left[1-\left\{1-\beta^{\alpha}\left(\beta+e^{\lambda x}-1\right)^{-\alpha}\right\}^{\theta}\right] \tag{9}$$

Quantile Function

Quantile function Q of the model is an alternative of the distribution function that helps more study different characteristic such as central tendency, dispersion and moments etc function of quantile Q function of the model are given as

$$Q(u) = \left(\frac{1}{\lambda}\right)\log\left[1-\beta+\beta\left(1-u^{1/\theta}\right)^{(-1/\alpha)}\right]; \quad 0 \le \text{u} \le 1 \tag{10}$$

where u follows uniform distribution.

Median of the model can be defined taking u = 1/ 2 . That is

$$\text{Median}=\left(\frac{1}{\lambda}\right)\log\left[1-\beta+\beta\left(1-(0.5)^{1/\theta}\right)^{(-1/\alpha)}\right]$$

Random deviate generation

Random deviate generation of the model is given by

$$x = \left(\frac{1}{\lambda}\right)\log\left[1-\beta+\beta\left(1-u^{1/\theta}\right)^{(-1/\alpha)}\right]; \quad 0 \le \text{u} \le 1 \tag{11}$$

Asymptotic properties of the Model

An asymptotic property of the formulated distribution is obtained comparing $\lim_{x\to 0} f(x) = \lim_{x\to\infty} f(x)$ .If model satisfies the asymptotic properties then there will be unique modal value. Taking limiting at end points,

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \alpha\beta^{\alpha}\lambda\theta e^{\lambda x}\left(\beta+e^{\lambda x}-1\right)^{-\alpha-1}\left[1-\beta^{\alpha}\left(\beta+e^{\lambda x}-1\right)^{-\alpha}\right]^{\theta-1} = 0 \tag{12}$$

$$\lim_{x\to \infty} f(x) = \lim_{x\to \infty} \alpha\beta^{\alpha}\lambda\theta e^{\lambda x}\left(\beta+e^{\lambda x}-1\right)^{-\alpha-1}\left[1-\beta^{\alpha}\left(\beta+e^{\lambda x}-1\right)^{-\alpha}\right]^{\theta-1} = 0 \tag{13}$$

Here, $\lim_{x\to 0} f(x) = \lim_{x\to\infty} f(x)$ so modal value of the proposed model will exist.

Skewness and kurtosis

Skewness describes about the consistency of the data. Here we have used Bowley's coefficient of skewness (Al-saiary et al., 2019) based on quantiles as,

$$\text{SK}(\text{B}) = \frac{\text{Q}(0.75)+\text{Q}(0.25)\text{-}2{*}\text{Q}(0.50)}{\text{Q}(0.75)\text{-Q}(0.25)}$$

Coefficient of Octiles Kurtosis by (Moors, 1998) and (Al-saiary et al., 2019) can be calculated using relation,

$$\text{K}_\text{u} = \frac{\text{Q}\,(0.875)\text{ - Q}(0.625) + \text{Q}\,(0.375)\text{ - Q}\,(0.125)}{\text{Q}\,(0.75)\text{ - Q}(0.25)}$$

Estimation of parameters techniques

Parameters can be estimated applying different methods. We have applied following methods.

Estimation using Maximum Likelihood (MLE)

Defining the log likelihood function for the proposed model in (14) .Let $\underline{x} = (x_1,\ldots,x_n)$ be a random sample of size 'n' from MEIE then the log likelihood function can be written as,

$$\ell(\alpha,\beta,\lambda,\theta \mid \underline{x}) = n\log(\alpha\lambda\theta) + n\alpha\log\beta + (\theta-1)\sum_{i=1}^{n}\log\left[1-\beta^{\alpha}\left(\beta+e^{\lambda x_i}-1\right)^{-\alpha}\right] + \alpha\sum_{i=1}^{n}x_i - (1+\alpha)\sum_{i=1}^{n}\log\left[\left(\beta+e^{\lambda x_i}-1\right)\right] \tag{14}$$

After differentiating (14) with respect to α, β, λ and θ, we can get the first order and second order partial derivatives of log likelihood function.

Solving above first order derivatives to zero, parameters of the proposed model can be estimated. Solution of above equation is not possible so computer programming can be used. Let $\underline{\hat{\Theta}} = (\hat{\alpha},\hat{\beta},\hat{\lambda},\hat{\theta})$ and $\underline{\Theta} = (\alpha,\beta,\lambda,\theta)$, are estimated constants and parameter vector respectively then resulting asymptotic normality will be, $\left(\underline{\hat{\Theta}}-\underline{\Theta}\right) \to N_3\left[0,\left(I\left(\underline{\Theta}\right)\right)^{-1}\right]$. The Fisher's information matrix $I\left(\underline{\Theta}\right)$ can be given by

$$I(\underline{\Theta}) = -\begin{bmatrix} E\left(\frac{\partial^2 l}{\partial\alpha^2}\right) & E\left(\frac{\partial^2 l}{\partial\alpha\partial\beta}\right) & E\left(\frac{\partial^2 l}{\partial\alpha\partial\lambda}\right) & E\left(\frac{\partial^2 l}{\partial\alpha\partial\theta}\right) \\ E\left(\frac{\partial^2 l}{\partial\beta.\partial\alpha}\right) & E\left(\frac{\partial^2 l}{\partial\beta^2}\right) & E\left(\frac{\partial^2 l}{\partial\beta.\partial\lambda}\right) & E\left(\frac{\partial^2 l}{\partial\beta\partial\theta}\right) \\ E\left(\frac{\partial^2 l}{\partial\lambda.\partial\alpha}\right) & E\left(\frac{\partial^2 l}{\partial\lambda\partial\beta}\right) & E\left(\frac{\partial^2 l}{\partial\lambda^2}\right) & E\left(\frac{\partial^2 l}{\partial\lambda\partial\theta}\right) \\ E\left(\frac{\partial^2 l}{\partial\theta.\partial\alpha}\right) & E\left(\frac{\partial^2 l}{\partial\theta\partial\beta}\right) & E\left(\frac{\partial^2 l}{\partial\theta\partial\lambda}\right) & E\left(\frac{\partial^2 l}{\partial\theta^2}\right) \end{bmatrix}$$

Asymptotic variance $\left(I\left(\underline{\Theta}\right)\right)^{-1}$ since $\underline{\Theta}$ cannot be obtained. If $O(\underline{\hat{\Theta}})$ be the observed fisher information matrix. Estimate $O(\underline{\hat{\Theta}})$ of $I\left(\underline{\Theta}\right)$ is defined below where H is hessian matrix.

$$O(\underline{\hat{\Theta}}) = -\begin{pmatrix} \left(\frac{\partial^2 l}{\partial\alpha^2}\right) & \left(\frac{\partial^2 l}{\partial\alpha\partial\beta}\right) & \left(\frac{\partial^2 l}{\partial\alpha\partial\lambda}\right) & \left(\frac{\partial^2 l}{\partial\alpha\partial\theta}\right) \\ \left(\frac{\partial^2 l}{\partial\beta.\partial\alpha}\right) & \left(\frac{\partial^2 l}{\partial\beta^2}\right) & \left(\frac{\partial^2 l}{\partial\beta.\partial\lambda}\right) & \left(\frac{\partial^2 l}{\partial\beta\partial\theta}\right) \\ \left(\frac{\partial^2 l}{\partial\lambda.\partial\alpha}\right) & \left(\frac{\partial^2 l}{\partial\lambda\partial\beta}\right) & \left(\frac{\partial^2 l}{\partial\lambda^2}\right) & \left(\frac{\partial^2 l}{\partial\lambda\partial\theta}\right) \\ \left(\frac{\partial^2 l}{\partial\theta.\partial\alpha}\right) & \left(\frac{\partial^2 l}{\partial\theta\partial\beta}\right) & \left(\frac{\partial^2 l}{\partial\theta\partial\lambda}\right) & \left(\frac{\partial^2 l}{\partial\theta^2}\right) \end{pmatrix} = -H\left(\underline{\Theta}\right)\Big|_{\left(\underline{\Theta}=\underline{\hat{\Theta}}\right)} \tag{15}$$

We have defined Variance covariance matrix as,

$$\left[-H\left(\underline{\Theta}\right)_{|(\Theta=\hat{\Theta})}\right]^{-1}=\begin{pmatrix} Var(\hat{\alpha}) & Cov(\hat{\alpha},\hat{\beta}) & Cov(\hat{\alpha},\hat{\lambda}) & Cov(\hat{\alpha},\hat{\theta}) \\ Cov(\hat{\beta},\hat{\alpha}) & Var(\hat{\beta}) & Cov(\hat{\beta},\hat{\lambda}) & Cov(\hat{\beta},\hat{\theta}) \\ Cov(\hat{\lambda},\hat{\alpha}) & Cov(\hat{\lambda},\hat{\beta}) & Var(\hat{\lambda}) & Cov(\hat{\lambda},\hat{\theta}) \\ Cov(\hat{\theta},\hat{\alpha}) & Cov(\hat{\theta},\hat{\beta}) & Cov(\hat{\theta},\hat{\lambda}) & Var(\hat{\theta}) \end{pmatrix} \tag{16}$$

Here,100(1-γ) % C.I. for α, β , λ and θ are,

$$\hat{\alpha}\pm Z_{\cdot\gamma/2}\sqrt{Var(\hat{\alpha})}\ ,\ \hat{\beta}\pm Z_{\gamma/2}\sqrt{Var(\hat{\beta})}, \hat{\lambda}\pm Z_{\gamma/2}\sqrt{Var(\hat{\lambda})},\ \text{and}\ \hat{\theta}\pm Z_{\gamma/2}\sqrt{Var(\hat{\theta})}$$

Estimation using Least-Square (LSE)

Let $X_{(1)} < X_{(2)} < \ldots < X_{(n)}$ is ordered random variables and a random sample $\{X_1, X_2, \ldots, X_n\}$ of size n is taken from a distribution function F (.). We define a function A using $F(X_{(i)})$ as CDF of ordered statistics by equation (5)

$$A\left(x;\alpha,\beta,\lambda,\theta\right)=\sum_{i=1}^{n}\left[F(X_{(i)})-\frac{i}{n+1}\right]^2 \tag{17}$$

Minimizing function (4.4), parameters of EGOLE are obtained by calculating the partial derivatives.

$$D\left(X;\alpha,\beta,\lambda,\theta\right)=\sum_{i=1}^{n}w_i\left[F(X_{(i)})-\frac{i}{n+1}\right] \quad \text{Where, } w_i=\frac{1}{Var(X_{(i)})}=\frac{(n+1)^2(n+2)}{i(n-i+1)}$$

Using the CDF of the order statistics and weight w_i in above expression with respect to α, β, λ and θ

$$D\left(X;\alpha,\beta,\lambda,\theta\right)=\sum_{i=1}^{n}\frac{(n+1)^2(n+2)}{i(n-i+1)}\left[F(X_{(i)})-\frac{i}{n+1}\right]^2 \tag{18}$$

Cramer-Von-Mises (CVM) method

Using this method, parameters α, β, λ and θ can be estimated by minimizing the function (19)

$$Z\left(X;\alpha,\beta,\lambda,\theta\right)=\frac{1}{12n}+\sum_{i=1}^{n}\left[F\left(x_{i:n}\mid\alpha,\beta,\lambda,\theta\right)-\frac{2i-1}{2n}\right]^2 \tag{19}$$

Finding first order partial derivatives of (19) with respect to α, β, λ and θ, we can get the first and second order partial derivatives of function Z and solving $\frac{\partial Z}{\partial\alpha}=0$, $\frac{\partial Z}{\partial\beta}=0$, $\frac{\partial Z}{\partial\lambda}=0$ and $\frac{\partial Z}{\partial\theta}=0$, CVM estimates can be obtained.

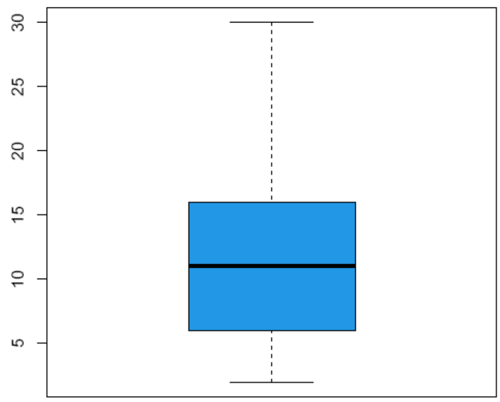
Data set for applying the model

This section of the research contains the application of the formulated model where we have considered a new real data set based on the Covid-19 data of the Nepal. The set of data considered here is total number of deaths due to Covid-19 recorded per day at ending of the December 2020. It is the data of first wave of the Covid-19 pandemic in Nepal from 23 January to 24 December (Government of Nepal Ministry of Health and Population, 2020).

2, 2, 2, 2, 3, 2, 2, 3, 2, 3, 3, 4, 2, 5, 5, 2, 4, 4, 8, 4, 4, 3, 2, 3, 7, 6, 6, 11, 9, 3, 8, 7, 11, 8, 12, 12, 14, 7, 12, 11, 6, 14, 9, 9, 11, 6, 6, 5, 5, 14, 9, 15, 11, 8, 4, 7 , 11, 10, 16, 2, 7, 17, 6, 8, 10, 4, 10, 7, 11, 11, 8, 7, 19, 9, 15, 12, 10, 14, 22, 9, 18, 12, 19, 21, 12, 12, 18, 8, 26, 21, 17, 13, 5, 15, 14, 11,13, 6, 16, 12, 11, 7, 3, 5, 5, 17, 16, 17, 23, 24, 20, 30, 18, 18, 17, 21, 18, 26, 15, 13, 13, 6, 9, 12, 17, 22, 7, 16, 16, 24, 28, 23, 22, 23, 19, 25, 29, 21, 9, 13, 16, 10, 17, 20, 23, 14, 12, 11, 15, 9, 18, 17, 14.

Descriptive data analysis

Descriptive data analysis introduced by (Tukey, 1977) gives initial calculation to know the pattern and summary of the data. Figure 2 contains the TTT and Box plot of the data.



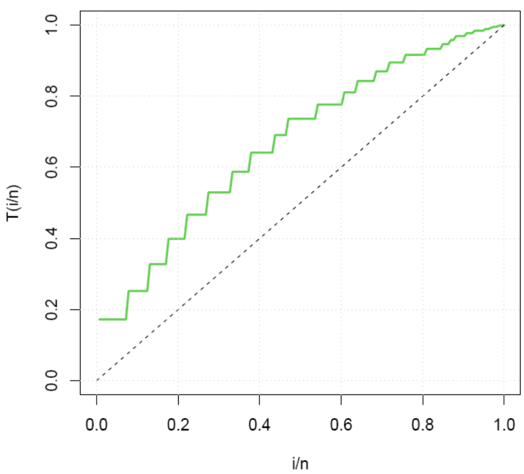


Figure 2. Box plot Left side) and TTT plot (Right side)

Box plot shows that the data is positively skewed and the TTT plot of the data is concave indicating that increasing the hazard rate shape.

Table 1

Summary Statistics

Min	Q_1	Median	Mean	Q_3	Sd	Skewness	Kurtosis	Max
2.00	6.00	11.00	11.61	16.00	6.75913	0.5083276	2.547717	30.00

Data is not normal and has negative skewness.

Parameter estimation

Estimation of parameters using analytical method is impossible due to the non linearity of the partial derivatives. For the estimation, optim () function of R software is used (R Core Team, 2019). MLE of the parameters are displayed in table 2.

Table 2

Estimated Parameters Using MLE, LSE and, CVME

Methods	Alpha	Beta	Lambda	Theta
MLE	54.800	38.711	0.053	1.682
LSE	54.802	38.709	0.050	1.512
CVME	54.801	38.710	0.051	1.532

Model Validation:

For model validation P-P plot and Q-Q plot of the distribution are plotted and are displayed in figure 3.

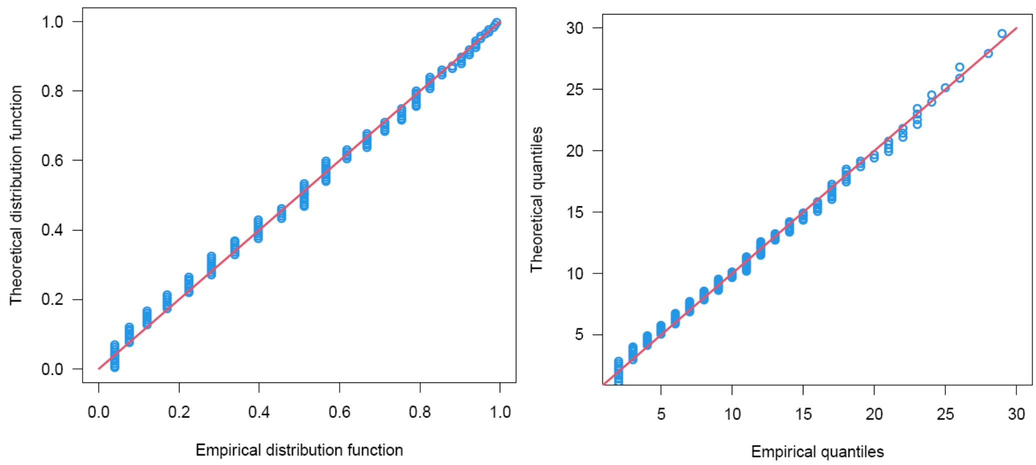


Figure 3. P-P plot (Left) and Q-Q plot of the model

Table 3 represents the Kolmogorov-Smirnov (KS), Cramer-Von Mises (W) statistics and, Anderson-Darling (A^2) for the proposed model respective to parameter based on different methods of estimation. Calculation shows that CVME method of estimation has lower test statistics with higher p values.

Table 3

KS, W and A^2 for different methods .

Methods	K.S(p-value)	W(p.-value)	A^2(p-.value)
MLE	0.0493 (0.8508)	0.0604(0.8121)	0.5260(0.7202)
LSE	0.0503 (0.8344)	0.0432(0.9169)	0.4522(0.7956)
CVME	0.0488 (0.8587)	0.0427(0.9195)	0.4369(0.8113)

Akaike information criterion (AIC), Corrected Akaike information criterion (CAIC), Bayesian information criterion (BIC) and Hannan-Quinn information criterion (HQIC) corresponding to different methods of estimation are computed. The findings are displayed below (Table 4).

Table 4

Log-likelihood values and information criteria values for model .

Model	L.L	AI.C	BIC	CAIC	HQIC
MLE	-496.4853	1000.971	1013.092	1001.241	999.4326
LSE	-497.1998	1002.4	1014.521	1002.67	1000.862
CVM	-497.0181	1002.036	1014.158	1002.306	1000.498

Model Comparison

For model comparison, six previously defined probability models are considered. Model considered are: Exponentiated Od.d Lomax Exponential (E.OLE) distribution (Dhungana and Kumar , 2022), Marshall - Olkin power generalized Weibull (MOPGW) (Afify et al., 2020), Generalized odd inverted exponentiated exponential (GOIEE) (Djibrila. 2019), Odd Lomax exponential (OLE) (Corederio et al., 2019), Exponential half logistic exponential (EHLE) (Almarashi et al., 2018) and Marshall –Olkin logistic exponential (MOLE) (Monsoor et al., 2019).

Figure 4, displays the density fit of proposed model and model considered as well the empirical cdf versus the fitted cdf of the model.

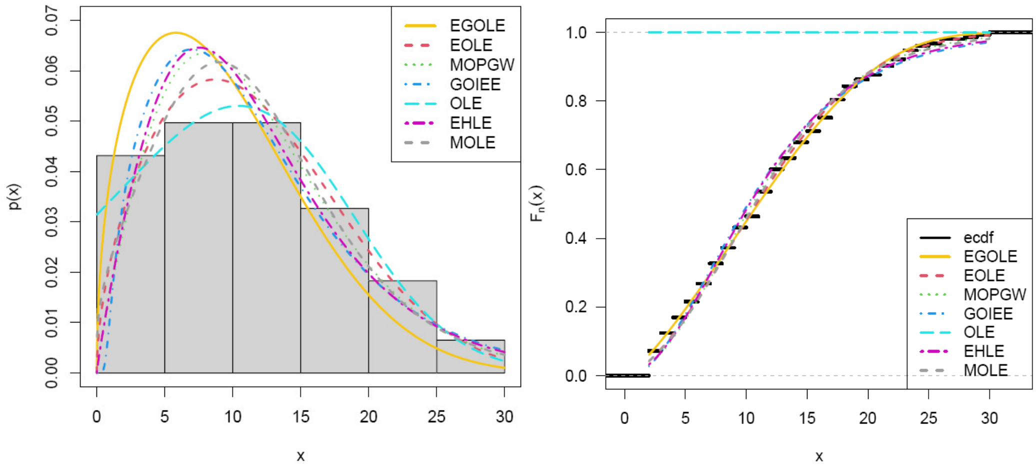


Figure 4. The pdf plot of models (Left) and ecdf versus the fitted cdf of the models (Right)

Table 5 displays the parameter estimate by MLE and their standard error of estimates for models taken in consideration

Table 5

Estimatated parameters by using MLE

Model	Alpha	Beta	Theta	Lambda	delta	Gamma
EGOLE	54.8003	38.7112	1.6817	0.0532	-	-
EOLE	0.0267 (0.0096)		1.5457 (0.0096)	28.6035 (4.9964)	7.5254 (5.296)	
MOPGW	0.7836 (0.2434)	1.9197 (0.2191)		0.0105 (0.0037)		
GOIEE			0.1572 (0.0137)	0.6142 (0.1901)		4.3124 (2.0892)
OLE	3.6466 (0.9066)	14.6636 (4.1751)	0.1264 (0.0187)			
EHLE	0.0954 (0.4054)	1.9055 (0.2216)		1.7502 (7.4483)		
MOLE	1.2874 (0.2403)		5.5873 (3.0400)	0.1468 (0.0449)		

AIC, BIC, CAIC, BIC and HQIC corresponding to different methods of estimation are computed for the entire considered model showing that model is good corresponding to the competitive distributions. The findings are displayed in Table 6.

Table 6

Log-likelihood (LL), AIC, BIC, CAIC, and HQIC

Model	LL	AIC	BIC	CAIC	HQIC
EGOLE	-496.4853	1000.971	1013.092	1001.241	999.4326
EOLE	-496.8049	1001.609	1013.732	1001.879	1006.534
MOPGW	-497.9371	1001.874	1010.966	1002.035	1005.567
GOIEE	-498.4237	1002.847	1011.939	1003.008	1006.540
OLE	-499.1686	1005.169	1014.260	1005.330	1008.862
EHLE	-499.7089	1005.418	1014.509	1005.579	1009.111
MOLE	-499.7126	1005.425	1014.517	1005.586	1009.118

To study of the performance of MLEs, Monte-Carlo simulation is presented. For study tool bias is used. Here, 1000 times repetition is done for generating 20 samples of size n= (50, 100, 150, 200, 250, 300, 350, 400, 450, 500, 550, 600, 650, 700, 750, 800, 850, 900, 950, 1000) taking the parameter set as α=5, β = 4, θ= 2, λ=3. Table 7 contains the average values, and biases.

Table 7

Mean estimates and mean bias

n	Estimates				Bias				MSE			
	α=50	**β = 30**	**λ=0.5**	**θ= 2**	**α=50**	**β = 30**	**λ=0.5**	**θ= 2**	**α=50**	**β = 30**	**λ=0.5**	**θ= 2**
50	46.0830	35.1220	0.5808	2.1973	-3.9170	5.1220	0.0808	0.1973	15.343	26.235	0.0065	0.0389
100	47.8639	33.1543	0.5431	2.0788	-2.1361	3.1543	0.0431	0.0788	4.5631	9.9498	0.0019	0.0062
150	48.5069	31.8799	0.5276	2.0528	-1.4931	1.8799	0.0276	0.0528	2.2293	3.5342	0.0008	0.0027
200	48.9284	31.5900	0.5189	2.0401	-1.0716	1.5900	0.0189	0.0401	1.1482	2.5281	0.0004	0.0016
250	48.9267	31.8339	0.5206	2.0193	-1.0733	1.8339	0.0206	0.0193	1.1519	3.3631	0.0004	0.0004
300	49.2134	31.4369	0.5150	2.0200	-0.7866	1.4369	0.0150	0.0200	0.6188	2.0646	0.0002	0.0004
350	49.3053	31.1518	0.5120	2.0169	-0.6947	1.1518	0.0120	0.0169	0.4825	1.3267	0.0001	0.0002
400	49.3895	30.9071	0.5110	2.0127	-0.6105	0.9071	0.0110	0.0127	0.3727	0.8228	0.0001	0.0002
450	49.4298	30.9234	0.5096	2.0099	-0.5702	0.9234	0.0096	0.0099	0.3251	0.8526	0.0001	0.0001
500	49.5449	30.7989	0.5079	2.0077	-0.4551	0.6382	0.0001	0.0001	0.2071	0.6382	0.0001	0.0001
550	49.5860	30.5828	0.5072	2.0072	-0.4140	0.5828	0.0072	0.0072	0.1714	0.3396	0.0001	0.0001
600	49.7017	30.4962	0.5042	2.0087	-0.2983	0.4962	0.0042	0.0087	0.0890	0.2462	0.0000	0.0001
650	49.7298	30.4093	0.5034	2.0098	-0.2702	0.4093	0.0034	0.0098	0.0730	0.1675	0.0000	0.0001
700	49.7972	30.3493	0.5027	2.0114	-0.2028	0.3493	0.0027	0.0114	0.0411	0.1220	0.0000	0.0001
750	49.7521	30.3628	0.5031	2.0096	-0.2479	0.3628	0.0031	0.0096	0.0614	0.1316	0.0000	0.0001
800	49.7216	30.3413	0.5042	2.0084	-0.2784	0.3413	0.0042	0.0084	0.0775	0.1165	0.0000	0.0001
850	49.7461	30.3235	0.5038	2.0077	-0.2539	0.3235	0.0038	0.0077	0.0644	0.1040	0.0000	0.0000
900	49.7888	30.2620	0.5024	2.0079	-0.2112	0.2620	0.0024	0.0079	0.0446	0.0686	0.0000	0.0001
950	49.8278	30.2947	0.5026	2.0079	-0.1722	0.2947	0.0026	0.0079	0.0296	0.0860	0.0000	0.0001
1000	49.8084	30.3152	0.5031	2.0067	-0.1916	0.3152	0.0031	0.0067	0.0367	0.0993	0.0000	0.0000

Conclusion:

Here, we have introduced four parameter continuous distribution using Odd Lomax Generator and exponential distribution. Some statistical properties like survival, hazard rate function and random deviate generation are mentioned here. Parameters of the model are estimated using maximum Likelihood estimation, least square estimation method and Cramer-Von Mises methods. Applicability of the model is tested using a real data set. For model comparison we have considered six already published models. For comparison of the model we have obtained different information criteria values showing that model fits to the real data set well compared to competing models. For testing the model validation, different information criteria are used and the goodness of fit of the proposed model is test using Kolmogrov - Smirnov, Cramer – von Mises and Anderson Darling method. To study of the performance of MLEs, Monte-Carlo simulation is presented. R programming is used for data analysis and graphics. Main finding is the New Probability model that is better compared to some well known existing models. These existing models are being used in researches of different fields.

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