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Analysis of Two-layered Blood Flow through Artery with Mild Stenosis

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Abstract

Employing the Navier-Stokes equations, and considering the blood flow in an artery in an axi-symmetric form, the blood flow dynamics in cylindrical artery are analyzed by evaluating pressure, pressure drop against the wall, shear stress on the wall. Plasma and core layers are considered in the blood flow in cylindrical artery with mild stenosis. It is analyzed the volumetric rate of blood flow in plasma and core layers of the cylindrical shape artery. It is also observed the pressure gradient on the surface of the stenosis in both layers. The dynamics of blood are analyzed by evaluating the ratio of shear stresses of the surface of stenosis with the radius of the artery.

Keywords: cylindrical artery, plasma and core layer, pressure drop, shear stress

Introduction

Stenosis in blood vessels, especially in arteries involves narrowing of the inner surfaces. It is the main cause of well-known serious diseases such as atherosclerosis. The study of blood flow in a stenotic artery is useful for the understanding of circulatory disorders (Ku, 1997; Phaijoo, 2013; Pokhrel et al., 2020). Circulatory disorders are known to be responsible for over seventy five percent of all deaths and atherosclerosis or stenosis is one of the frequently occurring cardiovascular diseases (Srivastava et al., 2010). It is an abnormal and unnatural growth that develops at various locations of the cardiovascular system under diseased conditions and causes serious circulatory disorders.

Blood behaves as a Newtonian fluid when it flows through arteries with a larger diameter at a high shear rate, whereas it exhibits a non-Newtonian fluid while flowing through arteries with smaller diameter at a low shear rate (Eldesoky, 2012; Jain et al., 2010;). One of the major causes of the deaths in the world is due to heart diseases, and

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the most commonly heard names among the same are ischemia, atherosclerosis, and angina pectoris. Ischemia is the deficiency of oxygen in a part of the body, usually temporary. It can be due to a constriction (stenosis) or obstruction in the blood vessel supplying the blood in that part (Phaijoo, 2013; Pokhrel et al. 2020;Pralhad and Schultz 2004). The mathematical investigation of blood flow in the human circulatory system is one of the major challenges from the past few decades to many years to come. The development of more effective and accurate numerical simulation techniques could provide a better understanding of the hemodynamical abnormalities due to stenosis (Phaijoo, 2013). Blood flow under atherosclerosis which together with flow pulsatility can be the cause of some periodic turbulences (Varghese & Frankel, 2003). Turbulence in blood flow might affect some physiological processes such as the flow resistance, high shear stress on the blood vessel wall, tensile stress in endothelial cell membrane, change in blood rheology due to deformability of red blood cells, the surface cell loss as well as internal cell motion due to pressure and shear stress, and the human circulatory system is a closed cardiovascular type flowing in the network of arteries, veins, and capillaries (Brinkman, 1949; Fung 1993; Jasit, 2016; Varghese and Frankel, 2003).

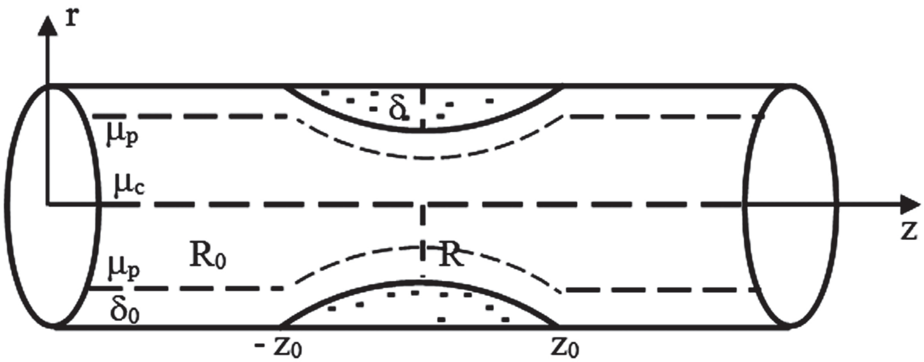
Many research activities are found to be focused on the study of blood flow in human arteries with stenosis. Varghese and Frankel (2003) developed the pulsatile turbulent flow in a stenotic vessel using the Reynolds-averaged Navier-Stokes equation approach. Srivastava et al. (2010) studied the increased impedance and other flow characteristics during artery catheterization with a composite stenosis assuming that the flowing blood behaves like a Newtonian fluid. Jain et al. (2010 solved the equation of motion of blood flow analytically by deriving the expressions for axial velocity, volumetric flow rate, pressure gradient, resistance to blood flow, and shear stress. Stenosis can lead to serious circulatory disorders, affecting many hydrodynamic factors such as resistance to flow, wall shear stress, and apparent viscosity (Pokhrel et al., 2020).

In this paper, the blood flow dynamics in the cylindrical artery with mild stenosis considering two-layered blood flow are analyzed. A plasma layer near the walls consists only the plasma and a core layer consists red cells. It is analyzed the volumetric rate of blood flow in plasma and core layers of the cylindrical shape artery. It is also observed the pressure gradient on the surface of the stenosis in both layers. The dynamics of blood are analyzed by evaluating the ratio of shear stresses of the surface of stenosis with the radius of the artery.

Model Equations

Let three components v^r, v^θ and v^z be the of velocities along the radius vector, perpendicular to the radius vector, and parallel to the z-axis in cylindrical polar coordinates (r, θ, z). Let r be the radius, p be the pressure on the cylindrical shape artery respectively, then the continuity equation is given (Kappur, 1985) as :

Figure 1. A cylindrical artery of two-layered flow with stenosis.



$$\frac{1}{r}\frac{\partial}{\partial r}(r v^r) + \frac{\partial}{\partial z}(v^z) = 0, \qquad (1)$$

Navier- Stokes equation in the radial and z-axis is given (Kappur, 1985) as:

$$\rho\left(\frac{\partial v^r}{\partial t} + v^r\frac{\partial v^r}{\partial r} + v^z\frac{\partial v^r}{\partial z}\right) = -\frac{\partial p}{\partial r} + \mu\left(\frac{\partial^2 v^r}{\partial r^2} + \frac{\partial^2 v^r}{\partial z^2} + \frac{1}{r}\frac{\partial v^r}{\partial r} - \frac{v^r}{r^2}\right), \qquad (2)$$

$$\rho\left(\frac{\partial v^z}{\partial t} + v^r\frac{\partial v^z}{\partial r} + v^z\frac{\partial v^z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 v^z}{\partial r^2} + \frac{\partial^2 v^z}{\partial z^2} + \frac{1}{r}\frac{\partial v^z}{\partial r}\right). \qquad (3)$$

Let μ_p and μ_c be the viscosity coefficients in plasma and core layer in the cylindrical artery. In the case of axi-symmetric $v^\theta = 0$, and v^r, v^z, and p are independent of θ. Let ρ be constant density for the steady flow, then the velocity component parallel to the z- axis is $v^z = v$, and $v^r = 0$, $v^\theta = 0$, and the equations (1) - (3) reduce to

$$v^z = v(r), \quad 0 = -\frac{\partial p}{\partial r}, \qquad \ldots(4)$$

$$0 = -\frac{\partial p}{\partial z} + \mu_p\left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r}\right), \ldots(5)$$

and $$0 = -\frac{\partial p}{\partial z} + \mu_c\left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r}\right). \ldots \quad (6)$$

Denoting $P(z) = - \partial p / \partial z$, the equation (5) reduces to

$$-P(z)\frac{r}{\mu_p} = \frac{\partial}{\partial r}\left(\sqrt{\frac{\partial v}{\partial r}}\right). \quad \ldots(7)$$

The boundary condition is the no-slip condition on the stenosis surface (Kapur, 1985) as :

$$v = \begin{cases} 0 & \text{at } r = R(z), \quad -z_0 \leq z \leq z_0 \\ 0 & \text{at } r = R_0, \quad |z| \geq z_0. \end{cases} \quad \ldots(8)$$

Integrating (7) with respect to r, taking z as constant gives

$$r\frac{\partial v}{\partial r} = -P(z)\frac{r^2}{4\mu_p} + C(z),$$ where C(z) is the constant of integration.

Applying $\partial v / \partial r = 0$ at $r = 0$ gives $C(z) = 0$, to get

$$\frac{\partial v}{\partial r} = -P(z)\frac{r}{4\mu_p}.$$

$$v(r) = -\frac{P}{4\mu_p}\left[\frac{r^2}{2}\right] + D(z),$$

where D(z) is another constant of integration. Applying $v = 0$ at $r = R$ gives $D(z) = PR^2 / 4\mu$, and so the velocity distribution along the plasma layer will be

$$v_p = \frac{P}{4\mu_p}(R^2 - r^2), \;|\; R - \delta_0 \leq r \leq R,$$

where δ_0 is thickness of plasma layer. The velocity distribution along the core layer is given by (Kapur, 1985):

$$v_c = \frac{P}{2\mu_p}\int_r^R r\,dr + \frac{P}{\mu_p}\int_{R-\delta_0}^{R} r\,dr + \frac{P}{\mu_c}\int_{R}^{R-\delta_0} r\,dr.$$

After integration, to Get

$$v_c(r, z) = \frac{P}{4\mu_p}(R^2 - r^2) + \frac{P}{\mu_c}[R^2 - (R - \delta_0)^2]\left(\frac{\mu_c}{\mu_p} - 1\right), \quad 0 \leq r \leq R - \delta_0.$$

the radius of the cylinder R is the function of z on the surface of cylindrical pipe is given by (Kapur, 1985):

$$\frac{R}{R_0} = 1 - \frac{\delta}{2R_0}\left(1 + \cos\frac{\pi z}{z_0}\right), \quad \ldots(9)$$

where δ be radius of stenosis, and at $z = z_0$, the velocity is maximum along the axis, and it vanishes on the surface of the artery. The flux through the cylindrical artery in the core layer can be obtained as (Kapur, 1985):

$$Q_c = \int_0^R 2\pi r v_c \, dr = \int_0^{R(z)} 2\pi. \left[\frac{P\pi}{4\mu_p}(r R^2 - r^3)\, dr + \frac{P\pi}{\mu_c}[R^2 - (R-\delta_0)^2]\left(\frac{\mu_c}{\mu_p} - 1\right)\right] dr$$

$$= 2\pi. \left[\frac{P\pi}{4\mu_p}(r R^2 - r^3)\, dr + \frac{P\pi}{\mu_c}[R^2 - (R-\delta_0)^2]\left(\frac{\mu_c}{\mu_p} - 1\right)\right]_0^R$$

$$= \frac{P\pi R^4}{8\mu_p} + \frac{P\pi}{\mu_c}[R^4 - (R-\delta_0)^2 R^2]\left(\frac{\mu_c}{\mu_p} - 1\right).$$

The flux through the cylindrical artery in the plasma layer can be obtained as (Kapur, 1985):

$$Q_p = \int_{R-\delta}^{R} 2\pi r v_p \, dr = \frac{P\pi}{2\mu_p}\int_{R-\delta}^{R} (R^2 r - r^3)\, dr = \frac{P\pi}{2\mu_p}\left[\frac{R^2 r^2}{2} - \frac{r^4}{4}\right]_R^{R-\delta}$$

$$= \frac{P\pi}{2\mu_p}\left[\frac{R^2(R-\delta)^2}{2} - \frac{(R-\delta)^4}{4} - \frac{R^4}{2} + \frac{R^4}{4}\right] = \frac{P\pi}{2\mu_p}[(R-\delta)^2 - R^2]^2.$$

The flux through the cylindrical artery in plasma and core layers can be obtained as (Kapur, 1985):

$$Q = \frac{P\pi R^4}{8\mu_p} + \frac{P\pi}{\mu_c}[R^4 - (R-\delta_0)^2 R^2]\left(\frac{\mu_c}{\mu_p} - 1\right) + \frac{P\pi}{2\mu_p}[(R-\delta)^2 - R^2]^2. \qquad \ldots(10)$$

The pressure drop across the length of the stenosis with plasma and core layers is denoted by (Kapur, 1985):

$$\Delta P = \frac{8\mu_p Q}{\pi R_0^4}\int_{-z_0}^{z_0} \frac{dz}{\left(1 - \frac{\delta}{2R_0}\left(1 + \cos\frac{\pi z}{z_0}\right)\right)^4 - \left(1 - \frac{\delta}{2R_0}\left(1 + \cos\frac{\pi z}{z_0}\right) - \frac{\delta_0}{R_0}\right)^4\left(1 - \frac{\mu_p}{\mu_c}\right)}.$$

In the case of δ = 0, i.e., there is no stenosis, the pressure drop across the stenosis with plasma and core layers is

$$(\Delta P)_p = \frac{8\mu_p Q}{\pi R_0^4}\;\frac{2z_0}{1 - \left(1 - \frac{\delta_0}{R_0}\right)^4\left(1 - \frac{\mu_p}{\mu_c}\right)}. \qquad \ldots(11)$$

The shear stress on the stenosis surface $\tau = -\mu_p(\partial v_p/\partial r = P(z)R(z)/2$ gives

$$\tau = \frac{4\mu_p Q}{R^3} \frac{1}{1-\left(1-\frac{\delta_0}{R}\right)^4\left(1-\frac{\mu_p}{\mu_c}\right)}.$$

In the case of $\delta = 0$, i.e., there is no stenosis . The shear stress across the stenosis length is

$$(\tau)_p = \frac{4\mu_p Q}{R_0^{\ 3}} \frac{1}{1-\left(1-\frac{\delta_0}{R}\right)^4\left(1-\frac{\mu_p}{\mu_c}\right)}.$$

The ratio of the shear stenosis length is

$$\frac{\tau}{\tau_p} = \frac{R_0^{\ 3}}{R^3(z)} \frac{1-\left(1-\frac{\delta_0}{R_0}\right)^4\left(1-\frac{\mu_p}{\mu_c}\right)}{1-\left(1-\frac{\delta_0}{R}\right)^4\left(1-\frac{\mu_p}{\mu_c}\right)}. \qquad \ldots(12)$$

Results and Discussion

Volumetric rate of blood flow in plasma and core layers on stenosis

Figure 2 shows that the volumetric flow rate of blood in a plasma and core layers of cylindrical shape artery with mild stenosis for different values of coefficient of viscosity in a plasma layer. Jain et al. (2010) showed that R decreases, the coefficient of viscosity decreases and R increases the coefficient of viscosity increases. For the fixed coefficient of viscosity in core layer $\mu_c = 0.020$ gram mm^{-1} s^{-1}, the maximum value of viscosity in plasma layer, $\mu_p = 0.060$ gram mm^{-1} s^{-1}, the flow rate is the least and for the least value of viscosity in plasma layer, $\mu_p = 0.030$ gram mm^{-1} s^{-1}, the flow rate is maximum. Higher viscous the fluid behaves, less rapidly it can flow. Mean flow rate is 1052.1 mm^3 / s, approximately 10.521 ml /s. This shows that the higher value of μ_p, less steeper the curves become, and the effects of μ_p approaches to decline and curves become closer and closer due to stenosis. The artery is completely blocked, and there is no flow at all when R becomes zero.

Figure 2 : Volumetric flow rate in plasma and core layers with variation of viscosity coefficients μ_p as given by the equation (10) .

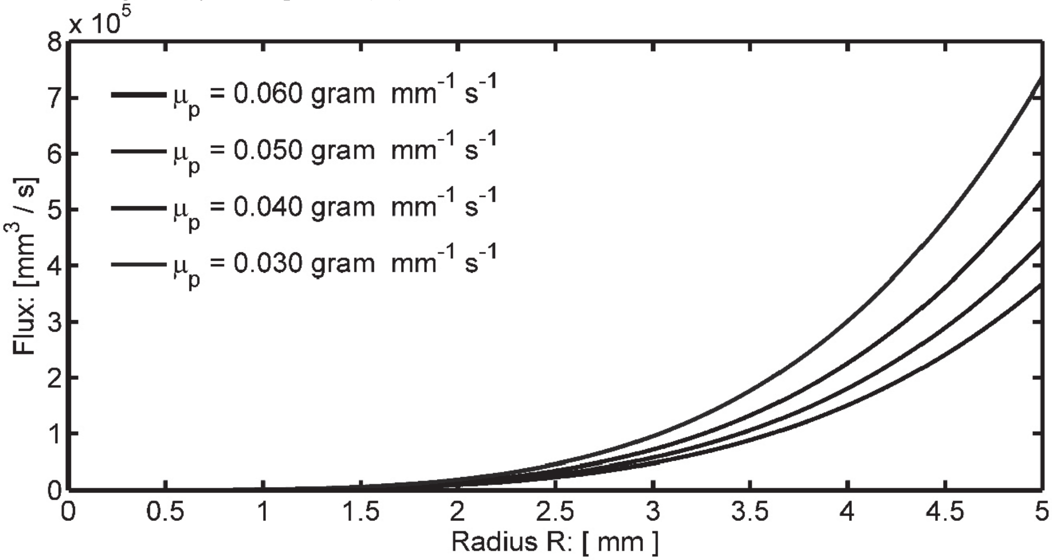
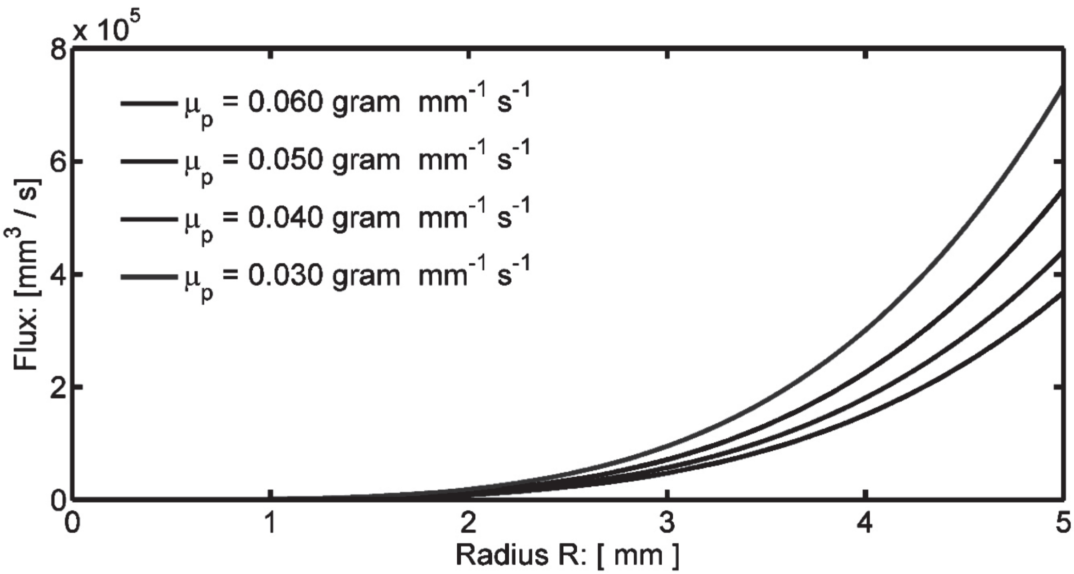


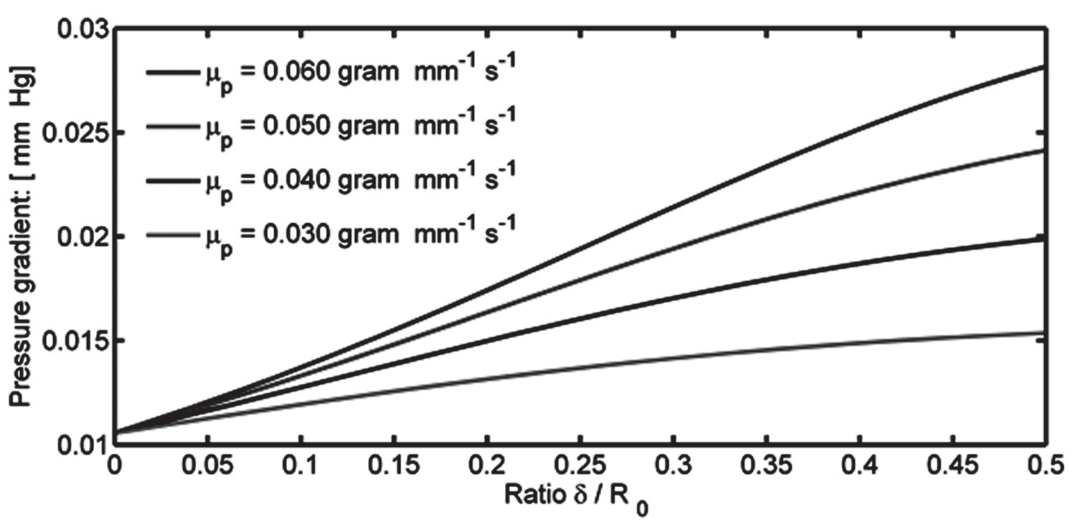
Figure 2 : Volumetric flow rate of blood flow in plasma and core layers with variation of viscosity coefficients μ_p as given by the equation (10).



Pressure gradient in plasma and core layers across the stenosis

Figure 3 shows that the pressure gradient of blood flow in a plasma and core layers of cylindrical shape artery with mild stenosis for different values of coefficient of viscosity in a plasma layer. For the fixed coefficient of viscosity in core layer $\mu_c = 0.020$ gram mm $^{-1}$ s $^{-1}$, the maximum value of viscosity in plasma layer, $\mu_p = 0.060$ gram mm^{-1} s^{-1}, the pressure gradient is the maximum against the ratio of thickness of stenosis with radius of the artery (i.e., δ / R_0), and for the least value of viscosity in plasma layer, $\mu_p = 0.030$ gram mm $^{-1}$ s $^{-1}$, the pressure gradient is minimum. Mean pressure gradient is 0.0166 mm Hg . Higher viscous the fluid becomes that higher the pressure gradient accelerate the blood on the artery against the ratio of thickness of stenosis with the radius of artery. This shows that the less value of μ_p, higher steeper the curves become. More viscous in plasma layer than in core layer become higher pressure gradient against the ratio of thickness of stenosis with radius of the artery.

Figure 3 : Pressure gradient of the surface of stenosis in plasma and core layers with variation of viscosity coefficients μ_p as given by the equation (11).



Shear stress on the stenosis surface in plasma and core layers

Figure 4 : Ratios of the shear stresses in plasma and core layers with variation of viscosity coefficients μ_p as given by the equation (12).

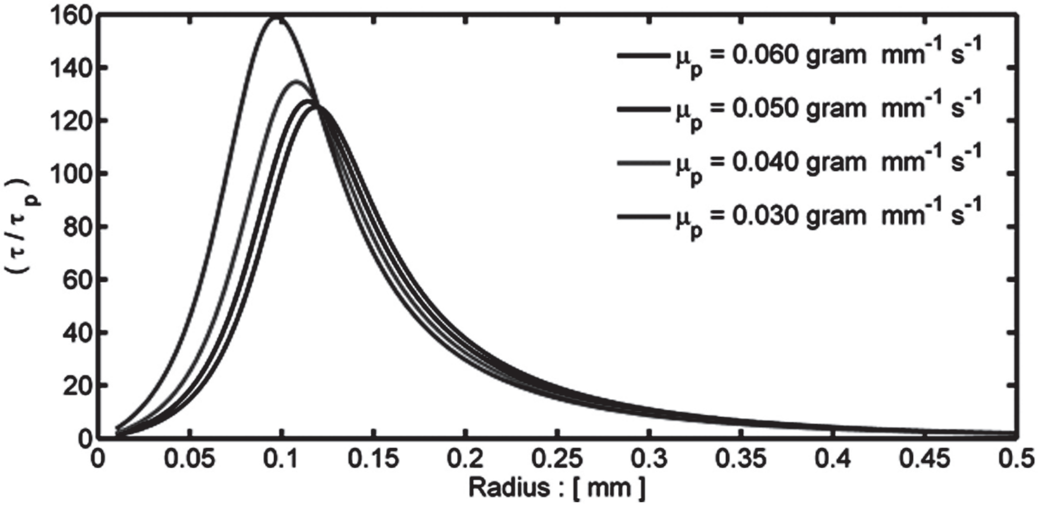


Figure 4 shows that the ratio of shear stresses increases exponential with bell-shaped curve against the radius of the artery in a plasma and core layers with mild stenosis for different values of coefficient of viscosity in a plasma layer. For the fixed coefficient of viscosity in core layer μ_c = 0.020 gram mm^{-1} s^{-1}, the least value of viscosity in plasma layer, μ_p = 0.030 gram mm^{-1} s^{-1}, the stresses on the surface of stenosis has the maximum ratio 158.98 against the radius of artery at R = 0.097 mm, and the maximum value of viscosity in plasma layer, μ_p = 0.060 gram mm^{-1} s^{-1}, the stresses on the surface of stenosis has minimum ratio 125 against the radius of artery at R = 0.12 mm. When the radius of the artery R = 0. 5 mm, the ratio of shear stresses approaches to zero.

Conclusion

Here, I presented the Navier-Stokes equations in cylindrical form for the two-layered blood flow of the artery, and analyzed the volumetric flow rate of blood flow in plasma and core layers in a cylindrical shape artery with mild stenosis regarding various coefficients of plasma viscosity. It was observed that the pressure gradient of the blood flow increase along with the increase in the ratio of thickness of stenosis with radius of artery. Meanwhile in case of increment of radius of the artery, the shear stresses ratio of the surface on the stenosis increases up to certain ratio and decreases to zero when the radius of artery approaches to R = 0.5 mm in plasma and core layers.

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