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Investigating the Nature of Plasma Oscillations in Cold Plasma

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Abstract

Plasma oscillations are one of the important phenomena in plasma. The research in the field of plasma oscillation has a long history for more than six decades. The cold plasma is one in which electron temperature is very large but the neutral atoms are at room temperature. This study aims to obtain electron number density, electric field and velocity profile in cold plasma. Analytical method has been used to conduct this study. Fluid equations were solved to get the solution for number density, electric field and velocity of electron. To get the solution, first fluid equations are transferred to Lagrangian co-ordinates, where equations become linear and expression for respective parameter were obtained. A sinusoidal perturbation was taken as an initial condition and solution for number density, electric field and velocity were obtained. Value of perturbation (Δ) was varied and corresponding waveforms were achieved and analyzed. As the perturbation increases the wave amplitude also increased. It is found that as the perturbation value is $\Delta = 0.60$ there is wave breaking in number density profile. Unlike in the number density, no such wave breaking is obtained for electric field and velocity profile. However the amplitude of waveform of electric field and velocity profile found to increases with increase in perturbation level.

Keywords: Cold Plasma, plasma oscillation, wave breaking

Introduction

Plasma is a fourth state of matter after the gas. The plasma is also called as ionized gas. A plasma is a quasineutral gas of charged and neutral particles which exhibits collective behaviour (Chen, 2015). Plasmas can be characterized basically by two parameters viz.

electron number density and electronic temperature. Electron density of plasma varies in between $1\ electron\ cm^{-3}$ to $10^{25} electrons\ cm^{-3}$. From the temperature point of view temperature plasma are of two types i.e. thermal and non-thermal. In thermal or hot plasma, temperature inside them is of the order of millions ($10^{7^o}C - 10^{9^o}C$). In case of hot plasma, the temperature of electrons and heavy species (neutral particles and ions) remains same. The plasma produced at low pressure with small degree of ionization is called non-thermal or cold plasma. Cold plasmas are produced by electrical discharge method where electron is accelerated through the gas by applying high potential difference. In such plasmas the temperature of heavy species is around $25^0C - 100^0C$ but the electronic temperature is much higher between 5000^0C to $10^{5^0}\ C$ (Tabares and Junkar, 2021; Zhu et al., 2020). Cold plasma can be generated in laboratory by various methods few of them are; corona discharges, microwave plasma, radiofrequency plasma, and dielectric barrier discharge plasma. Cold plasma technology is a non-thermal technology having potential applications in industries. The popularity of cold plasma is because of its unique advantages over traditional technologies (Pankaj and Keeren, 2017).

Cold plasma is the one in which temperature of electron is very high (about 10000 K) but the neutral atoms are at room temperature. In case of cold plasma the density of electrons in the plasma is very low as compared to density of neutral atoms. When the electrons in the plasma are displaced from the uniform background of ions the electric field is set up in the plasma because of which there is oscillation of electron called the plasma oscillation. In above case electrons are pulled back by the electric field to restore the electrical neutrality of the plasma. The oscillation frequency of electrons is called as plasma frequency (Chen, 2015). An electrostatic plasma oscillation is a unique phenomenon in which electron oscillates about its equilibrium position with electron plasma frequency.

The concept of nonlinear electron oscillation in cold plasma was first introduced by Dawson. He gave an exact analysis of oscillations in plane, cylindrical and spherical symmetry (Dawson, 1959). He proposed that in a cold homogeneous plasma, where ions form uniform background, the increase in amplitude of perturbation leads to nonlinear oscillation of plasma. He postulated that below a critical amplitude, plane oscillations in a uniform plasma are stable. However, multistream flow or fine-scale mixing sets on the oscillation for larger amplitude. In contrast to plane oscillations, for cylindrical or spherical symmetry multistream flow always occurs. In the field of plasma oscillations. The breaking of large amplitude electron plasma oscillations is the important issue gaining an interest since 1959. The concept of wave breaking finds its usefulness from laboratory to astrophysical plasmas.

Tonks and Langmuir (1929) investigated the plasma oscillation experimentally and provided basic theoretical explanation. Dawson (1959) discussed the phenomenon of energy exchange between particles and wave. As soon as amplitude of electron plasma wave increases, different types of nonlinear effects appears. For the large amplitude the waveform starts to change its behavior and deviates from sinusoidal nature i.e. it steepens. When the steepening gets so extreme, the wave breaking happens. Likewise, phase mixing of plasma oscillation is another important phenomena which is a loss of coherence of plasma oscillations due to the fine scale mixing of various parts of oscillation whenever there is slight variation in local plasma frequency (Tajima and Dawson, 1979).

The research in the area of cold plasma growing day by day due to its wide application. The fundamental governing equations of nonlinear large amplitude plasma oscillations is well illustrated by Davidson and Schram (Davidson and Schram, 1968). To solve the fluid equations first they transform fluid equations in Lagranges' coordinates where the solution become linear and easily solvable. To get the exact solution they transformed the solution from Lagrangian coordinate system to Eulerian coordinate system.

Verma et al. (2011) found an exact general analytical solution describing the nonlinear evolution of large amplitude plasma oscillation initiated by perturbation in density in the form of fourier series. First they reproduced the result of Davidson and Schram, later generalized for triangular and square wave. They demonstrated wave breaking condition for the case when initial particle position is perturbed sinusoidally and also for triangular and rectangular initial condition. They also stated that the wave breaking limit is $a_1 \geq 0.5$. They verified analytical findings of space time evolution and wave breaking limits by PIC simulation.

Akhiezer and Polovin (1956) investigated the nonlinear wave motions of an electron plasma carried by arbitrary electron velocities. They characterized the state of plasma by the particle density rather than by distribution function. They established a correspondence between the wave motion of plasma and non-relativistic particle in a certain potential field.

Mukherjee and Sengupta (2014) studied phase mixing and breaking of Akhiezer and Polovin wave subjected to small amplitude longitudinal perturbation. They analytically find the phase mixing time. Albritton and Koch (1975) investigated that, in the wave breaking region collective energy is converted into random energy. Lehmann et al. (2007) discussed and analyzed electrostatic wake-field generation behind a short laser pulse. They highlighted the oscillation of laser pulse. When laser pulse width becomes very small, localized electrostatic wavepackets were evolved. They considered Lagrangian formulation for analyzing wake-field breaking. They also studied various cases of wave-breaking and found that numerical simulation results are in close agreement with analytical results.

Verma et al. (2010) introduced the viscosity and resistivity in the cold plasma model leading to new nonlinear plasma oscillations which forms splitting of density peak. They numerically studied nonlinear plasma oscillations in cold, viscous, and resistive plasma, by assuming electron viscosity coefficient independent of density. The domain of nonlinear electrostatic oscillation is one of the interesting research field having major application application in the laser based experiments and simulations. From the simulation perspectives; Particle-In-Cell (PIC), the Vlasov simulation, the sheet simulation are widely practiced methods. For the analytical treatment of plasma oscillations Lagrangian mechanics is widely used approach. A wave-breaking occurs when coherent energy is translated into kinetic energy as a result of wave particle interaction. Whenever nearby fluid elements which are in coherent oscillations cross each other, a wave-breaking occurs (Verma, 2018).

Brodin and Stenflo (2014) found a new large-amplitude wave solutions in cold plasma in long-wavelength limit. They derived two basic equations which describes the evolution of electromagnetic and electrostatic fields in cold plasma. They mentioned good agreement between analytical and numerical results. Infeld and Rowlands (1989) found the exact solution for Langmuir wave. They investigated explosive pattern of electron density and hence in electric field gradient. They numerically studied large amplitude relativistic Langmuir wave and found high steepening of wave more than expected. They believed relativistic effects may cause such steepening in the waveform.

Verma (2017) studied cold plasma oscillations in wave breaking region using one dimensional PIC simulation and investigated that after wave breaking plasma gets heated however a fraction of initial energy always go on with the wave . The theoretical model developed by them is in good agreement with numerical simulation.

This study aims to obtain the waveforms for electron number density, electric field and velocity. We have chosen three different value of perturbation and respective waveforms for above parameters will be obtained. This study will also make comparative analysis of waveform for different values of perturbation. This study is based on analytical method. The analytical solutions for those three parameters will be used for analysis.

Methodology

We have used analytical approach to carry out this research study. The analytical treatement for this study has been based on the study by Davidson and Schram (1968). We started with basic fluid equations by considering very simplest case of unmagnetized cold plasma. We then converted fluid equations in Lagranges variables. The differential equation becomes linear in Lagranges coordinates and solution for electron number density, electric field and velocity are obtained in Lagranges coordinates. On the basis of solution obtained in Lagranges system, the graphs for respective parameters were plotted along with the variation in perturbation level.

Let us consider cold plasma in which ions form fixed neutralizing background. The fluid equations in one dimension are discussed below.

The equation of continuity in one dimension can be written as;

$$\frac{\partial n_e}{\partial t} + \frac{\partial (n_e v_e)}{\partial x} = 0 \qquad (1)$$

Where, n_e represents electron number density and v_e represents mean electron velocity.

The momentum equation is;

$$\frac{\partial v_e}{\partial t} + v_e \frac{\partial v_e}{\partial x} = \frac{-e}{m_e} E \qquad (2)$$

Where, E represents electric field intensity, $(-e)$ represents the charge of an electron and m_e represents the mass of an electron.

From Gauss law,

$$\frac{\partial E}{\partial x} = -4\pi e \left(n_e - n_0\right) \qquad (3)$$

Where, (n_0) represents the ions density. Also rate of change of electric field is;

$$\frac{\partial E}{\partial t} = 4\pi\, e\, n_e\, v_e \qquad (4)$$

To determine the exact solution let us transform above set of equations in Lagranges co-ordinate (x_0, τ) on the basis of (Davidson and Schram, 1968).

$$\tau \equiv t$$

$$x_0 \equiv x - \int_0^{\tau} v_e(x_0, \tau')\, d\tau' \qquad (5)$$

Differentiating equation (5) with respect to x_0;

$$\frac{\partial x_0}{\partial x_0} = \frac{\partial x}{\partial x_0} - \int_0^{\tau} \frac{\partial v_e(x_0,\tau')}{\partial x_0} \ d\tau'$$

$$1 + \int_0^{\tau} \frac{\partial v_e(x_0,\tau')}{\partial x_0} \ d\tau' = \frac{\partial x}{\partial x_0}$$

$$\frac{\partial}{\partial x} = \left[1 + \int_0^{\tau} \frac{\partial v_e(x_0,\tau')}{\partial x_0} \ d\tau'\right]^{-1} \frac{\partial}{\partial x_0} \qquad (6)$$

In Lagrangian variables the convective derivative is;

$$\frac{\partial}{\partial \tau} = \frac{\partial}{\partial t} + v_e \frac{\partial}{\partial x}$$

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial \tau} - v_e \frac{\partial}{\partial x}$$

Putting value of $\frac{\partial}{\partial x}$ from equation (6);

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial \tau} - v_e(x_0,\tau)\left[1 + \int_0^{\tau} \frac{\partial v_e(x_0,\tau')}{\partial x_0} \ d\tau'\right]^{-1} \frac{\partial}{\partial x_0} \qquad (7)$$

In terms of Lagranges variables $(x_0, \ \tau)$ equation (2) becomes;

$$\left[\frac{\partial}{\partial t} + v_e \frac{\partial}{\partial x}\right] v_e(x_0,\tau) = \frac{-e}{m_e} \ E(x_0,\tau)$$

$$\frac{\partial v_e(x_0,\tau)}{\partial \tau} = \frac{-e}{m_e} \ E(x_0,\tau) \qquad (8)$$

Again from equation of continuity equation (1);

$$\frac{\partial}{\partial \tau}\left\{n_e(x_0,\tau) \left[1 + \int_0^{\tau} d\tau' \ \frac{\partial v_e(x_0,\tau')}{\partial x_0}\right]\right\} = 0 \qquad (9)$$

Multiplying equation (3) by v_e, we get;

$$v_e \frac{\partial E}{\partial x} = -4\pi e\, v_e (n_e - n_0) \qquad (10)$$

Adding equation (10) with equation (4);

$$\frac{\partial E}{\partial t} + v_e \frac{\partial E}{\partial x} = 4\pi\, e\, n_e\, v_e - 4\pi e\, v_e (n_e - n_0)$$

$$\left(\frac{\partial}{\partial t} + v_e \frac{\partial}{\partial x}\right) E = 4\,\pi\, e\, v_e n_0$$

$$\frac{\partial E(x_0, \tau)}{\partial \tau} = 4\,\pi\, e\, n_0\, v_e(x_0, \tau) \qquad (11)$$

Differentiating equation (8) on both sides with respect to τ;

$$\frac{\partial^2 v_e(x_0, \tau)}{\partial \tau^2} = \frac{-e}{m_e} \frac{\partial E(x_0, \tau)}{\partial \tau}$$

Putting value from equation (11);

$$\frac{\partial^2 v_e(x_0, \tau)}{\partial \tau^2} = \frac{-e}{m_e}\, 4\,\pi\, e\, n_0\, v_e(x_0, \tau)$$

$$\frac{\partial^2 v_e(x_0, \tau)}{\partial \tau^2} = \frac{-4\,\pi\, e^2 n_0}{m_e}\; v_e(x_0, \tau)$$

$$\frac{\partial^2 v_e(x_0, \tau)}{\partial \tau^2} + \omega_{pe}^2\; v_e(x_0, \tau) = 0 \qquad (12)$$

Where; $\omega_{pe}^2 = \frac{4\,\pi\, e^2\, n_0}{m_e}$, ω_{pe} represents the electron plasma frequency.

Equation (12) is the linear differential equation in $v_e(x_0, \tau)$. It's solution can be written as;

$$v_e(x_0, \tau) = A \cos(\omega_{pe}\tau) + B \sin(\omega_{pe}\tau) \qquad (13)$$

Using boundary condition; $v_e(x_0, 0) = V(x_0)$ for $\tau = 0$

$$A = V(x_0)$$

Differentiating equation (13);

$$\frac{\partial v_e(x_0, \tau)}{\partial \tau} = -A\, \omega_{pe} \sin(\omega_{pe}\tau) + B\omega_{pe} \cos(\omega_{pe}\tau)$$

$$\frac{-e}{m_e} E(x_0,\tau) = -A\,\omega_{pe} \sin(\omega_{pe}\tau) + B\omega_{pe} \cos(\omega_{pe}\tau)$$

Again using initial condition $\tau = 0$;

$$\frac{-e}{m_e} E(x_0,0) = B\omega_{pe}$$

$$B = \frac{-e\,E(x_0,0)}{m_e\,\omega_{pe}}$$

Putting values of A and B in equation (13);

$$v_e(x_0,\tau) = V(x_0)\cos(\omega_{pe}\tau) - \frac{e\,E(x_0,0)}{m_e\,\omega_{pe}}\sin(\omega_{pe}\tau)$$

$$v_e(x_0,\tau) = V(x_0)\cos(\omega_{pe}\tau) + \omega_{pe}\,X(x_0)\,sin(\omega_{pe}\tau) \qquad (14)$$

Where, $X(x_0) = -\frac{e\,E(x_0,0)}{m_e\,\omega_{pe}^2}$

Now electric field by using equation (8) is;

$$E(x_0,\tau) = -\frac{m_e}{e}\frac{\partial v_e(x_0,\tau)}{\partial\tau}$$

$$E(x_0,\tau) = -\frac{m_e}{e}\{-V(x_0)\,\omega_{pe}\;sin(\omega_{pe}\tau) + \omega_{pe}^2 X(x_0)\cos(\omega_{pe}\tau)\}$$

$$E(x_0,\tau) = \left(\frac{m_e}{e}\right)\omega_{pe}\,V(x_0)\,sin(\omega_{pe}\tau) - \left(\frac{m_e}{e}\right)\omega_{pe}^2\,X(x_0)\,\cos(\omega_{pe}\tau) \qquad (15)$$

From equation (9);

$$\frac{\partial}{\partial\tau}\left\{n_e(x_0,\tau)\left[1 + \int_0^\tau d\tau'\left(\frac{\partial V(x_0)}{\partial x_0}\cos(\omega_{pe}\tau') + \omega_{pe}\frac{\partial X(x_0)}{\partial x_0}\sin(\omega_{pe}\tau')\right)\right]\right\} = 0$$

$$\frac{\partial}{\partial\tau}\left\{n_e(x_0,\tau)\left[1 + \frac{\partial V(x_0)}{\partial x_0}\frac{\sin(\omega_{pe}\tau)}{\omega_{pe}} - \omega_{pe}\frac{\partial X(x_0)}{\partial x_0}\frac{\cos(\omega_{pe}\tau)}{\omega_{pe}} + \omega_{pe}\frac{\partial X(x_0)}{\partial x_0}\frac{1}{\omega_{pe}}\right]\right\} = 0$$

$$\frac{\partial}{\partial\tau}\left\{n_e(x_0,\tau)\left[1 + \frac{\partial V(x_0)}{\partial x_0}\frac{\sin(\omega_{pe}\tau)}{\omega_{pe}} - \frac{\partial X(x_0)}{\partial x_0}(1 - \cos(\omega_{pe}\tau))\right]\right\} = 0$$

Integrating on both sides;

$$n_e(x_0,\tau)\left[1+\frac{\partial V(x_0)}{\partial x_0}\frac{\sin(\omega_{pe}\tau)}{\omega_{pe}}-\frac{\partial X(x_0)}{\partial x_0}\left(1-\cos(\omega_{pe}\tau)\right)\right]=n_e(x_0,0)$$

$$n_e(x_0,\tau)=\frac{n_e(x_0,0)}{\left[1+\frac{\partial V(x_0)}{\partial x_0}\frac{\sin(\omega_{pe}\tau)}{\omega_{pe}}-\frac{\partial X(x_0)}{\partial x_0}\left(1-\cos(\omega_{pe}\tau)\right)\right]} \quad (16)$$

The transformation relation (5) can be written in the form of $V(x_0)$ and $X(x_0)$ as;

$$t=\tau$$

$$x=x_0+\frac{V(x_0)}{\omega_{pe}}\sin(\omega_{pe}\tau)+X(x_0)\left(1-\cos(\omega_{pe}\tau)\right) \quad (17)$$

The initial condition for density is ; $n_e(x_0,0)\geq 0$. Let us also suppose that;

$$n_e(x_0,0)=n_0\left(1+\Delta\cos(kx_0)\right)$$

And zero mean electron velocity ; $v_e(x_0,0)=V(x_0)=0$

After solving equation (14)-(16) becomes;

$$v_e(x_0,\tau)=\frac{\omega_{pe}}{\tau}\Delta\sin(kx_0)\sin(\omega_{pe}\tau) \quad (18)$$

$$E(x_0,\tau)=\frac{-m_e}{e}\,\omega_{pe}^2\,\frac{\Delta}{k}\sin(kx_0)\cos(\omega_{pe}\tau) \quad (19)$$

$$n_e(x_0,\tau)=n_0\frac{1+\Delta\cos(kx_0)}{1+\Delta\cos(kx_0)\ \left(1-\cos(\omega_{pe}\tau)\right)} \quad (20)$$

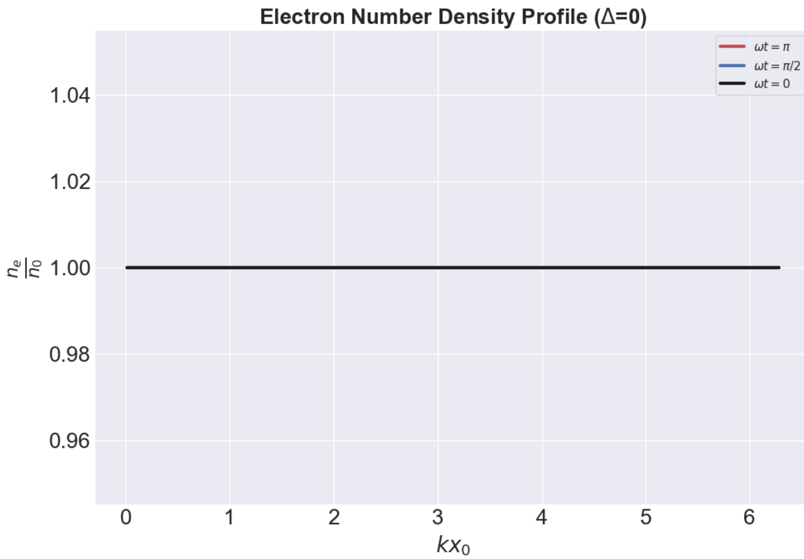
Results

Number Density Profile

The electron density profiles for different values of $\omega_{pe}\tau$ and Δ are shown in the figures below. The density is shown as a function of kx_0 for different values of the perturbation (Δ). The graphs for number density were plotted on the basis of analytical expression equation (20).

Figure 1

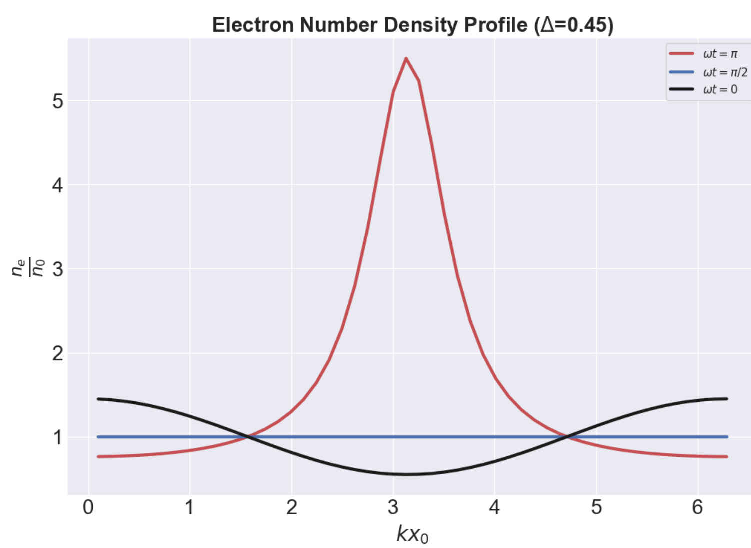
Normalized plasma density n_e/n_0 *as a function of* kx_0 *at* $\omega_{pe}\tau = \pi, \frac{\pi}{2}$ & 0 *with* $\Delta = 0$



In Figure 1, we plot the electron density profile for $\omega_{pe}\tau = \pi, \pi/2\ and\ 0$ with $\Delta = 0$. As there is no perturbation there is no fluctation in the density. In this electron and ion density remains same.

Figure 2

Normalized plasma density n_e/n_0 *as a function of* kx_0 *at* $\omega_{pe}\tau = \pi, \frac{\pi}{2}$ & 0 *with* $\Delta = 0.45$



In Figure 2 we plot the electron density profile for $\omega_{pe}\tau = \pi, \frac{\pi}{2}$ & 0 with $\Delta = 0.45$. As there is certain amount of perturbation in the number density, electons fluctuates continuously. For $\omega_{pe}\tau = \pi$ (indicated by red line) electrons density varies rapidly and takes the maximum value of $5.5\ n_0$. For $\omega_{pe}\tau = \frac{\pi}{2}$ (indicated by blue line) there is no fluctuation in the density so that $n_e = n_0$. For $\omega_{pe}\tau = 0$ (indicated by black line) there is slight fluctuation in the number density but not as like for the case of $\omega_{pe}\tau = \pi$.

Figure 3

Normalized plasma density n_e/n_0 *as a function of* kx_0 *at* $\omega_{pe}\tau = \pi, \frac{\pi}{2}$ & 0 *with* $\Delta = 0.60$

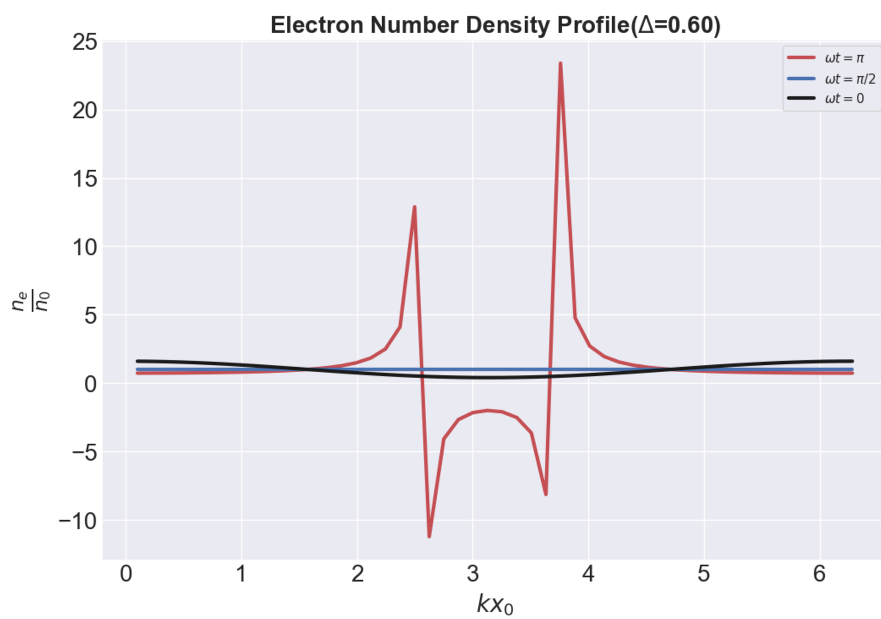


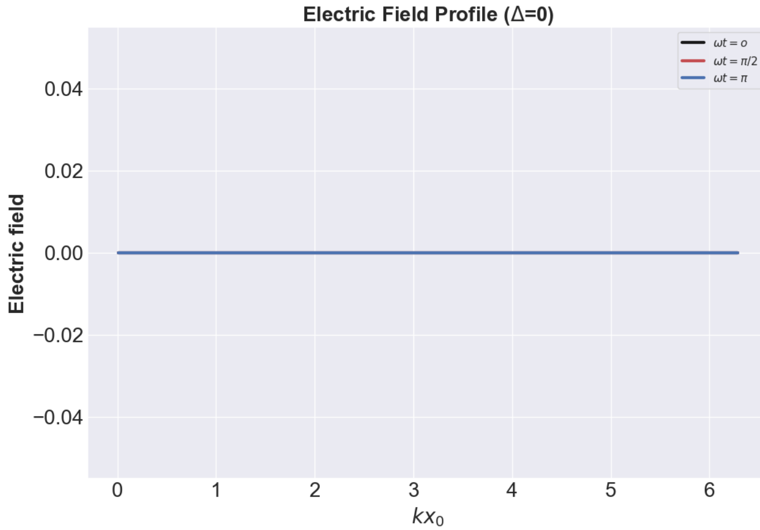
Figure 3 shows the variation of plasma density when perturbation value is $\Delta = 0.60$. For $\omega_{pe}\tau = \pi$ (indicated by red line) there is rapid enhancement in the plasma density. Figure shows the wave breakng. Which also agrees with the previous research done by (Davidson & Schram, 1968). They indicated that $\Delta = 0.5$ is the wave breaking limit.

Electric Field Profile

The electric field profiles for different values of $\omega_{pe}\tau$ and Δ are shown in Figure 4-6. The electric field is shown as a function of kx_0. These curves are drawn on the basis of analytical expression of electric field equation (19).

Figure 4

Electric field profile as a function of kx_0 *at* $\omega_{pe}\tau = 0, \frac{\pi}{2}, \pi$ *with* $\Delta = 0$



When there is no perturbation $n_e = n_0$ at all time. Since there is equal concentration of electrons and ions there is no net electric field. Hence electric field intensity becomes zero.

Figure 5

Electric field profile as a function of kx_0 *at* $\omega_{pe}\tau = 0, \frac{\pi}{2}, \pi$ *with* $\Delta = 0.45$

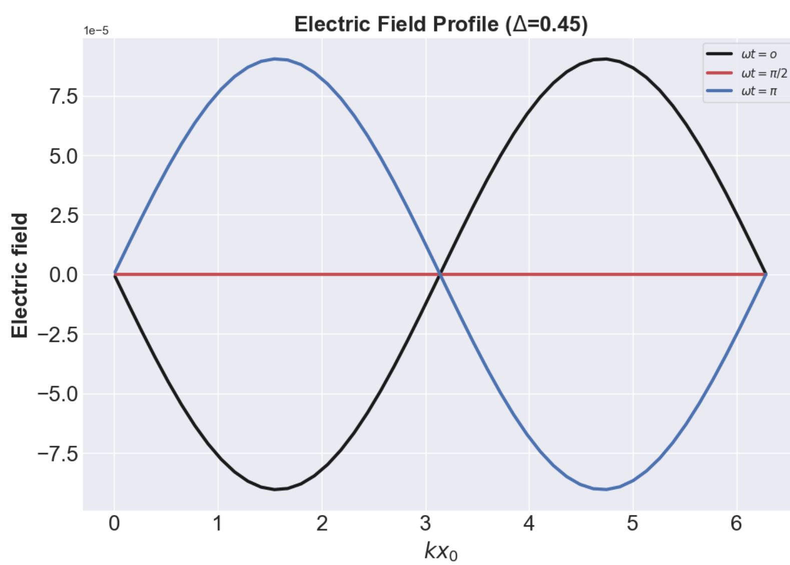
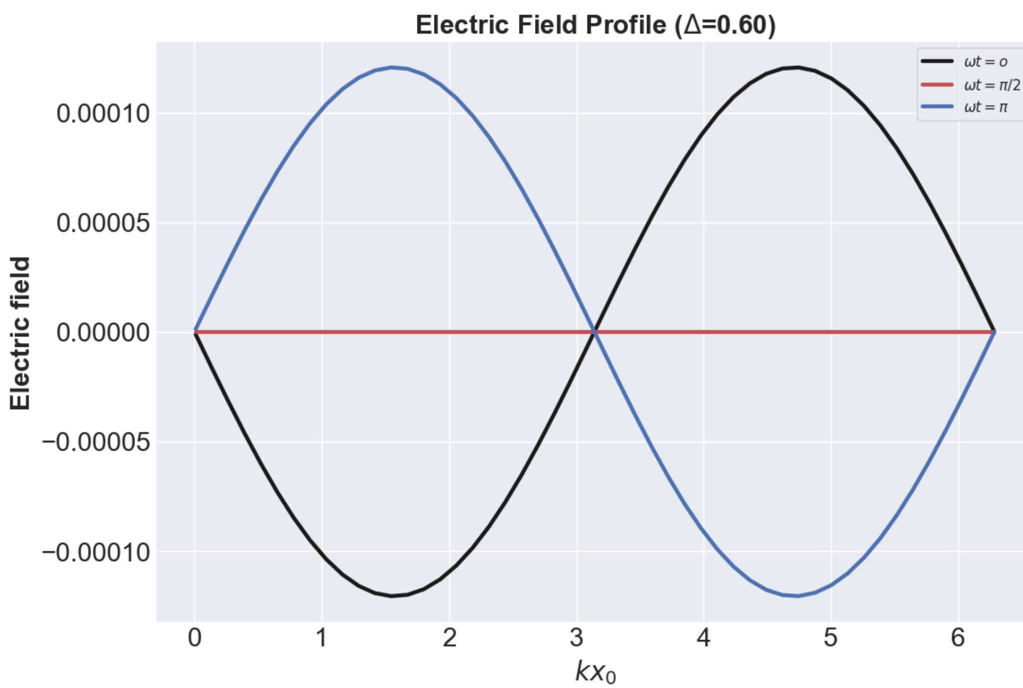


Figure 5 represents the electric field profile for perturbation $(\Delta = 0.45)$. From the graph it is clear that waveform for $\omega_{pe}t = \pi$ and $\omega_{pe}t = 0$ have the same magnitude but are in opposite phase. Whereas at $\omega_{pe}t = \pi/2$ the electric field becomes zero which can be clear from the analytical expression of electric field.

Figure 6

Electric field profile as a function of kx_0 *at* $\omega_{pe}\tau = 0, \frac{\pi}{2}, \pi$ *with* $\Delta = 0.60$



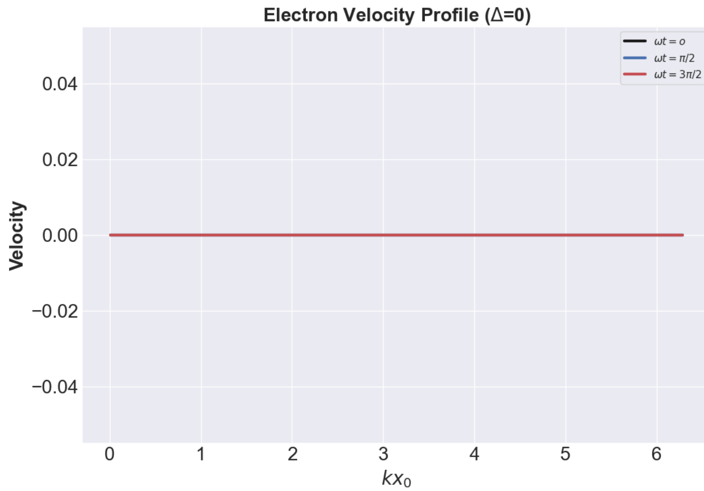
For the perturbation value$(\Delta = 0.60)$, the waveform is similar like that of at $(\Delta) = 0.45$ however the magnitude of electric field increases because of the increase in the perturbation level.

Velocity Profile

The mean velocity of electron are shown in the Figure 7-9. The graphs for the electron velocity are plotted on the basis of analytical expression equation (18).

Figure 7

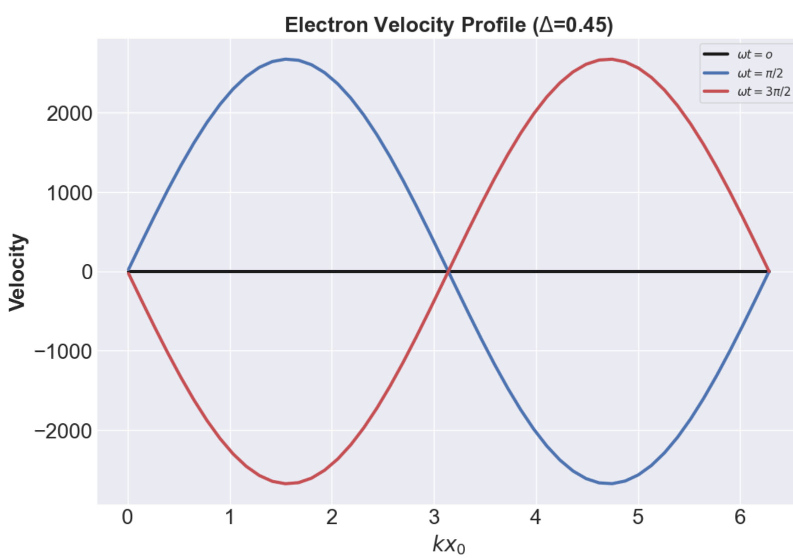
Velocity profile as a function of kx_0 *at* $\omega_{pe}\tau = 0, \frac{\pi}{2}, \frac{3\pi}{2}$ *with* $\Delta = 0$



The Figure 7 represents the case when there is no perturbation i.e. plasma is in equilibrium state. As the electron and ion density being equal, the electrons net velocity becomes zero.

Figure 8

Velocity profile as a function of kx_0 *at* $\omega_{pe}\tau = 0, \frac{\pi}{2}, \frac{3\pi}{2}$ *with* $\Delta = 0.45$



From Figure 8, the velocity profile also seems sinusoidal in nature. However, phase of wave form changes as the $\omega\tau$ varies.

Figure 9

Velocity profile as a function of kx_0 at $\omega_{pe}\tau = 0, \frac{\pi}{2}, \frac{3\pi}{2}$ with $\Delta = 0.60$

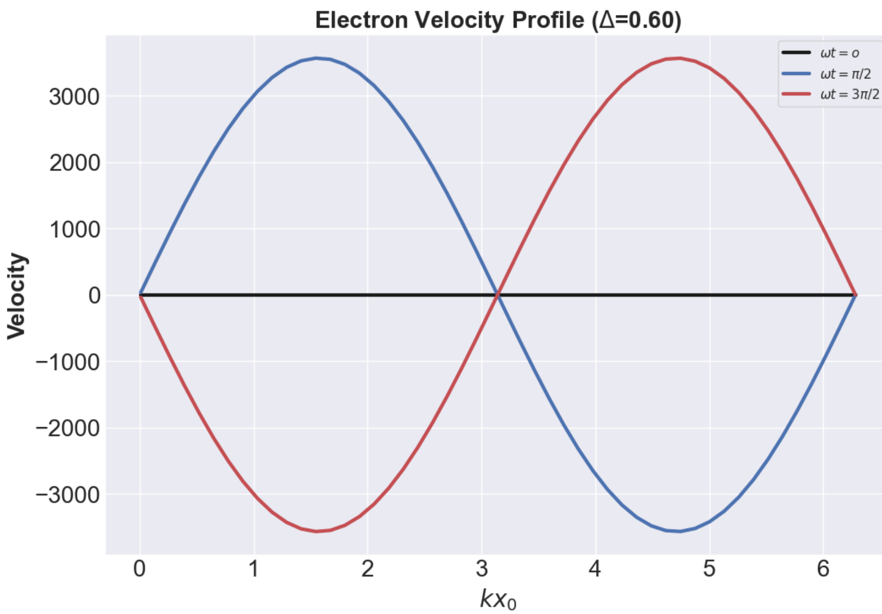


Figure 9 shows the velocity profile for perturbation level $\Delta = 0.60$. If we compare the velocity profiles for $\Delta = 0.45$ & 0.60 both of them shows the same nature only differs in magnitude. The magnitude of velocity is large for $\Delta = 0.60$ because of the high perturbation amplitude.

Discussion

We have plotted graphs for electron number density, electric field and electron velocity as shown in Figures 1-9. We solved fluid equation and obtained their solutions in Lagrangian variables. We have used analytical solutions of respective parameters in Lagrangian coordinates.

The number density profile is shown in Figure 1-3. When $\Delta = 0$, electron density is equal to background ion density. The electron density remains equal to ion density forever and no any variation due to kx_0.When $\Delta = 0.45$, the electron density abruptly takes maximum value at $\omega\tau = \pi$. This result aligns with finding of (Davidson & Schram, 1968). However, when $\Delta = 0.60$, the wave breaking occurs in number density as shown in Figure

3. As suggested by Davidson and Schram (1968), wave breaking occurs if $\Delta \geq 0.50$, our results also is in good agreement with Davidson. Our findings on number density are also in accordance with Verma et al. (2011).

The electric field profile is shown in Figure 4-6. There is no wave breaking type phenomenon as like in number density. With the increase in perturbation level, the magnitude of electric field found to be increased. This is in good agreement with Dawson (1959). For no external perturbation, net electric field is found to be zero.

The mean electron velocity is shown in Figure 7-9. In case of velocity, we did not find any wave breaking. The perturbation level highly affected the magnitude of velocity. For no perturbation, net velocity of electron is found to be zero. As the perturbation increased the magnitude of velocity also enhanced.

Conclusion

This study deals about the plasma oscillations in cold plasma. Plasma oscillation is very important phenomena in the area of laser and several other applied field. This study demonstrates the analytical expression for electron number density, electron mean velocity and electric field in Lagrangian coordinate system when sinusoidal perturbation is applied to the system. We have studied plasma oscillations for perturbation value $\Delta = 0, 0.45, and\ 0.60$. At perturbation $\Delta = 0$, electron density is equal with background ion density. At perturbation $\Delta = 0.45$, electron density abruptly rises or bunching of electron for $\omega\tau = \pi$. It is found that at $\Delta = 0.60$, there is wavebreaking in the number density profile. Past researches mentioned $\Delta \geq 0.5$ is the wave breaking limit, our study also in coherence with past researchers as we also found wave breaking at $\Delta = 0.60$. Electric field and velocity profile found as sinusoidal in nature and their magnitude found to be increased with the increase in perturbation value. Further increase in value of Δ may leads to steepening of the wave as suggested by Prior research by Davidson and Schram. This study indicates that initial perturbation highly affects the waveform of number density, electric field and mean electron velocity profile. This study can be more strengthened by verifying these finding from analytical study with the numerical simulation study. Along with this relativistic correction can also be introduced for more realistic solutions.

References

Akhiezer, A. I., & Polovin, R. V. (1956). Theory of wave motion of an electron plasma. *Soviet Phys. JETP*, *3*.

Albritton, J., & Koch, P. (1975). Cold plasma wavebreaking: Production of energetic electrons. *The Physics of Fluids*, *18*(9), 1136-1139.

Chen, F. F. (2015). *Introduction to plasma physics and controlled fusion*. New York: Plenum press.

Davidson, R. C., & Schram, P. P. J. M. (1968). Nonlinear oscillations in a cold plasma. *Nuclear Fusion*, *8*(3), 183.

Dawson, J. M. (1959). Nonlinear electron oscillations in a cold plasma. *Physical Review*, *113*(2), 383.

Ginzburg, V. L. (1970). The propagation of electromagnetic waves in plasmas. *International Series of Monographs in Electromagnetic Waves*.

Infeld, E., & Rowlands, G. (1989). Relativistic bursts. *Physical review letters*, *62*(10), 1122.

Langmuir, I. (1929). Tonks., L. Ion oscillations in a warm plasma. *Phys. Rev*, *33*, 195-210.

Lehmann, G., Laedke, E. W., & Spatschek, K. H. (2007). Localized wake-field excitation and relativistic wave-breaking. *Physics of Plasmas*, *14*(10).

Mukherjee, A., & Sengupta, S. (2014). Analytical estimate of phase mixing time of longitudinal Akhiezer-Polovin waves. *Physics of Plasmas*, *21*(11), 112104.

Pankaj, S. K., & Keener, K. M. (2017). Cold plasma: Background, applications and current trends. *Current Opinion in Food Science*, *16*, 49-52.

Tabares, F. L., & Junkar, I. (2021). Cold plasma systems and their application in surface treatments for medicine. *Molecules*, *26*(7), 1903.

Tajima, T., & Dawson, J. M. (1979). Laser electron accelerator. *Physical review letters*, *43*(4), 267.

Verma, P. S. (2017). Generation of low frequency plasma waves after the wave-breaking. *Physics Letters A*, *381*(48), 4005-4010.

Verma, P. S. (2018). A novel numerical scheme for nonlinear electron plasma oscillations. *Physics of Plasmas, 25(3)*

Verma, P. S., Sengupta, S., & Kaw, P. K. (2011). Nonlinear evolution of an arbitrary density perturbation in a cold homogeneous unmagnetized plasma. *Physics of Plasmas*, *18*(1), 012301.

Verma, P. S., Soni, J. K., Sengupta, S., & Kaw, P. K. (2010). Nonlinear oscillations in a cold dissipative plasma. *Physics of Plasmas*, *17*(4).

Zhu, Y., Li, C., Cui, H., & Lin, L. (2020). Feasibility of cold plasma for the control of biofilms in food industry. *Trends in Food Science & Technology*, *99*, 142-151.