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Student's Engagement in Mathematics Learning on Cesaro Sequence Spaces on Matrix Transformations

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Abstract

This article explores the mathematics learning engagement among the Master's level students on Cesaro sequence spaces with matrix transformations at University level. It is a qualitative research in which the related theorem has been taken for the study to the sampled students of Master's level. It is found that students engaged in solving the problem through the related subject. It is found that the use of the non-absolute type of Cesaro sequence space given to transform the Cesaro sequence spaces, determines conditions that are both necessary and sufficient for a matrix that is infinite to exist in to the ℓ_∞ and the space C respectively. When the $x \in X$ sequences in which the series $\sum_{k=1}^{\infty} x_k$ Convergent then H is a non-absolute Banach sequence space and provides the requirements for an infinite matrix to transform the Cesaro sequence space X_p into the respective ℓ_∞ and all convergent sequences' space C.

Keywords: *Sequence space, Dual space, Transformation, Infinite matrix, Absolute type, Convergent*

Introduction

This article showed that the challenges the proof of theorems and solving the mathematical problem relate to the theorems. It is taken into account the sequence space available the entire real sequence $\{x_k\}$ the series for which $\sum_{k=1}^{\infty} x_k$ convergent. We will determine the H terms of Kothe theory, establish conditions that are both required and adequate for a transformation using a matrix to H should be translated into space $l\infty$, C of convergent sequences in general and of all bounded sequences. These findings will be used to determine conditions that are both necessary and sufficient to transform the Cesaro sequence spaces with an infinite matrix later in this note in to the $l\infty$ and the space C respectively.

Definition: - If a Sequence $X = \{x_k\}$ .Its absolute value too, belongs to a specific space $|X| = \{|x_k|\}$. Otherwise, it is said that the space is non-absolute. The non-absolute type of a sequence space has a number of undesirable characteristics.

Literature Review

In the beginning, Banach Hahn Mazur applied functional analysis to summability theory, and later, it was studied by many distinguished mathematicians, including Kojiman[12], Steinhaus, Schue[5], Mazur, Orlicz, Wilansky, Maddox[], and many others.

It is fundamental principle of Functional Analysis that investigators of spaces are often combined with spaces of operators known as dual spaces with specific properties from the perspective of dualistic theory, When we consider them equipped, the work of sequence spaces is much more fruitful with linear topology however in such case it is rather cumbersome to obtain to obtain their topological dual.Even if we are successful in finding these topological dual, we would like to deal with only those duel whose members are re-presentable as sequences Kelly,J.L[11] were first recognize the problem and to resolved it they introduced the notion of $\alpha -$ dual and $\beta -$ dual.

A linear sequence space with elements in another linear space is referred to as a sequence space. Summability is the research of linear transformations in sequence spaces. An early version of The theory of summability was proposed by Goottfried Wilhelm Leibniz $(1646 - 1716)$ to C.Wolf(1713) in which he attributed the sum $\frac{1}{2}$ to the oscillatory series:

$$1 - 1 + 1 - 1 + 1 - \ldots$$

Since $s = 1 - (1 - 1 + 1 - \ldots) = 1 - s$ or $s = \frac{1}{2}$

Similarly, squaring L.H.$(1 - 1 + 1 - 1 + \ldots)(1 - 1 + 1 - 1 + \ldots) = \frac{1}{4}$

i.e., $1 - (1.1 + 1.1) + (1.1 + 1.1 + 1.1) - \ldots = \frac{1}{4}$

Which gives $1 - 2 + 3 - 4 + \ldots = \frac{1}{4}$.

The expansion of the idea of the sum of a series, which is typically impacted by auxiliary series, is central to the theory of summability. In particular, in the above example the original oscillatory series $1 - 1 + 1 - 1 + 1 - \ldots$ is divergent but the new series $1 - 2 + 3 - 4 + \ldots$ is convergent.

The notion of convergence of an infinite series was first resolved satisfactory by the french mathematician A.L Cauchy,Frobenious in 1980 introduced a generalized method of summability by arithmetic means by Ernesto Cesaro in 1890 as the (C, K) method of summability. Towards the 19^{th} century, Mathematicians were influenced by problems like those in summability theory to analysis general theory of sequence and transformation on them..

Summability theory was used to further investigate sequence space. The theory of identifying limits, also known as summability theory is based on functional analysis, function theory, topology, and functional analysis.

The cesaro refers to (also known as cesaro average) the order $\{X_n\}$ are the terms of sequence $\{c_n\}$ where $c_n = \sum_{i=1}^{n} x_n$ is consisted of the first n elements, with the arithmetic mean $\{x_n\}$.

This idea bears Ernesto Cesaro's name. Convergent sequences are preserved along with their limits by the Cesaro Means operation by Cesaro Summability view in divergent sequences theory. If the series is referred to as Cesaro summable if the Cesaro means sequence is convergent. There are numerous instances where the Cesaro Means Sequence converges, but not the original Sequence for example, sequence $\{x_n\} = \{(-1)^n\}$ which is Cesaro summable.

Let ω denotes the set of all sequences of all real or complex numbers and $\ell\infty, c_1, c_0$ denote the spaces of all bounded, convergent and null sequences $x = (x_k)$ with the usual norm $\| x \|_\infty = sup|x_k|$, where $k \in \mathbb{N} = 1,2, \ldots$ the set of positive integers. Also by bs , cs, ℓ_1 and ℓ_p ; we denote the spaces of all bounded convergent, absolutely summable, and p- absolutely summable sequences respectively.

Let, $\bar{X} = (x_k) = (x_k)_{k=1}^{\infty}$ and ω will denote the difference- classes of all sequences $\bar{X} = (x_k), k \geq 0$ over the field $\mathbb{C}$ of complex number.

1. $c_0 = \{\bar{X} = (x_k) \in \omega\text{: } |x_k| \to 0 \text{ as } k \to 0)\}$

   $\to$ (The space of null sequence).

2. $c_1 = \{\bar{X} = (x_k) \in \omega \; \exists \; \ell \in \mathbb{C}$, s.t. $|x_k - \ell| \to 0 \text{ as } k \to 0\}$ $\to$ (The space of convergent sequences).

3. $\ell_\infty = \{\bar{X} = (x_k) \in \omega;\ \sup_k |x_k| < \infty\}$ $\to$ (The space of bounded sequence).

4. $\ell_p = \{\bar{X} = (x_k) \in \omega\text{: } \sum_{k=1}^{\infty} |x_k|^p < \infty\}, 0 < p < \infty.$ $\to$ (the space of absolutely p- summable sequence).

Methodology

The research methodology for this study consisted of theorems. The aim of this study was to answer the mathematics problems through their meaningful results. And try to find out the new results that what are the motivational factors that help considering the context within which this work was situated.

The study focused on some proof of the theorems and analysis in solving the mathematics problems using theorems. Some theorems were taken from the Master's level about the cesaro sequence space on matrix transformation. we have defined the sequence space with koth dual space. And proved some related theorem in different forms of Cesaro sequence space can be proved with their matrix transformation. Some theorems and lemmas are proved as following

Theorem: - The Space H is a non-absolute Banach sequence.

Proof:

let $\{x^{(i)}\}$ be a Cauchy sequence in H, such that we have ϵ > 0, $\left\{\rho(x^{(i)} - x^{(j)}\right\}$ < ∈ for all $i, j \geq n_0(\in)$

We write $x^{(i)} = \{x^{(j)}\}$.

Then for fixed k, $\{x_k^i\}$ is convergent if $\lim_{i\to\infty} x_k^{(i)} = x_k$,

then

$$\left|\sum_{k=1}^{m}\left(x_k^{(i)} - x_k\right)\right| < \in \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(1)}$$

For $i \geq n_0(\in)$ and all m= 1,2,3…

Hence, we have $\rho(x^{(i)} - x^{(j)} \leq \in$ for all $i \geq n_0(\in)$.

As a result, H is final.

Theorem: - The Space H is separable.

Proof:- For every x∈H, Such that $x = \{x_k\}$. Let $x^N = \left\{x_{1,}x_2, x_3 \ldots . x_N, 0, ..\right\}$. Then it's clear $\rho\{x - x^N\} \to 0$ as N→ ∞. If A is a dense real-number system subset that can be counted, then H is a countable dense subset of A. The associate space of H will be determined in the following steps. Assume V is the totality of all spaces, y ∈X as a result,

$\sum_{k=1}^{\infty}|(y_k - y_{k+1})| < \infty$ ……………. (ii)

Lemma: - If a then matrix A converts a BK-space X_p in to ℓ_∞ BK-space this is a continuous and linear transformation. Each coordinate mapping in BK-space has a banach space available where $x \to x_k$ is continuous.

For example, $\ell_\infty(1 \leq p = \infty)$, C and the space C_0 of all null sequences with uniform norms are all BK-space.

Proof: -

All the finite sequences are contained in the associate space H,

if $x^{(n)} = \left\{x_j^{(n)}\right\} \in H$, with $\rho\left\{x^{(n)}\right\} \to 0$ as $n \to \infty$

Then,

$$\left|x_j^{(n)}\right| = \left|\sum_{k=1}^{\infty} x_j^{(n)} e^j\right| \leq \rho\left(x^{(n)}\right) \rho'\left(e^j\right) \to 0. \text{ As } n \to \infty$$

Where e^j is indeed the sequence 1 just at j_{th} position and zero everywhere else.

Cesaro Sequence space spaces and the matrix transformation.

Let $X_p(1 \leq \infty)$ and X_∞ respectively the spaces of all$x \in X$ with

$$\|X\|_p = \left(\sum_{n=1}^{\infty}\left|\frac{1}{n}\sum_{k=1}^{n} x_k\right|^p\right)^{\frac{1}{p}} \leq \infty$$

And $\|X\|_\infty = sup\left\{\left|\frac{1}{n}\sum_{k=1}^{n} X_k\right|^p ; k = 1,2,3, \dots\right\} < \infty$

The above norms, with the exception of p=1. By shiue and Kamthan P.K the cesaro sequence space defined as

$$Ces_p = \left\{a = \{a_n\}_{n=1}^{\infty} : \|a\|^p = \left(\sum_{n=1}^{\infty}\left(\frac{1}{n}\sum_{k=1}^{n}|a_k|\right)^p\right)^{\frac{1}{p}} < \infty\right\}$$

for$1 \leq p < \infty$ and

$$Ces_\infty = \left\{a = \{a_n\} : \|a\|_\infty = sup_n\left(\frac{1}{n}\sum_{k=1}^{n}|a_k|\right) < \infty\right\}$$

we note that the space X_p , is distinct from P as defined above. Spaces in the Cesaro sequence $Ces_p(1 \leq p < \infty)$. In fact $Ces_p \subset X_p(1 \leq p < \infty)$Aand $Ces_p = X_p$. Mursaleen M [4], We can demonstrate this X_p are non-absolute Banach sequence spaces by using theorem. We will now present the results as a result of Lee [3].

Theorem: - Let y_q be the space of all y Є X such that

$|ky_k| \leq M,$ for all k=1,2, ……-----------------(1)

$\mu_q(y) = (\sum_{k=1}^{\infty}|k(y_k - y_{k+1})|^q)^{\frac{1}{q}} < \infty$ for $1 \leq q < \infty$……………………(2)

And

$\mu_q(y) = sup\{|k(y_k - y_{k+1})| : k = 1,2,3, \dots\} < \infty$

The matric transformation of X_p

Here we Find conditions that are both necessary and an infinite matrix is sufficient to transform the Cesaro sequence spaces X_p into the spaces ℓ_∞ of all bounded sequences and C of all convergent sequences, respectively.

Results: From the above work we find the following results

THEOREM:- The associate space H' of the space H coincide with the conjugate space (Banach dual) H^* of the space H algebraically and Isometrically.

proof:- For any $y \in H'$, then we see that $T_y(x) = \sum_{k=1}^{\infty} x_k y_k$ defines a linear continuous functional on H with norm $\| T_y \| = \rho'(y)$.

conversely, if $t \in H^*$, let e^k denote the sequence with 1 in the k_{th} coordinate and zero elsewhere. For any $x \in H$, let also $x^N = \{x_1, x_2, \dots, x_N, 0, \dots\}$.

then we have $x^N = \sum_{k=1}^{N} x_k e^k$ and $\rho(x - x^N) \to 0$ as $N \to \infty$. Since T is continuous, we have

$$T(x) = \lim_{N\to\infty} T(x^N)$$
$$= \lim_{N\to\infty} \sum_{k=1}^{N} x_k T(e^k)$$

$$= \sum_{k=1}^{\infty} x_k T(e^k)$$

which is convergent for all $x \in H$. This implies that the sequence $\{T(e^k)\}$ is an element in H' by above proven theorem and

$$\parallel T \parallel = \sup\{|\sum_{k=1}^{\infty} x_k T(e^k)| : \rho(x) \leq 1\}$$
$$= \rho'(T(e^k)).$$

This shows that every $T \in H^*$ can be represented by an element $\{T(e^k)\}$ in H'. Thus if we identify each $T \in H^*$ with $\{T(e^k)\}$ in H', we see that $H' = H^*$ algebraically and isometrically.

Theorem: - The associate space X' of X_p is the space y_q with the norm μ_q where 1/p +1/q =1.

lemma:- The space H is a BK-space.

proof:-

Since the associate space H' contains all the finite sequences,

if $x^{(n)} = \{x_k^{(n)}\} \in H$, with $\rho\left(x^{(n)}\right) \to 0$ as $n \to \infty$,

then

$$|x_j^{(n)}| = |\sum_{k=1}^{\infty} x_k^{(n)} e^j| \leq \rho(x^{(n)})\rho'(e^j) \to 0 \text{as} n \to \infty$$

where e^j is the sequence 1 at the j_{th} place and zero elsewhere.

Theorem: $\delta_k = 0$ if C is replaced by the space C_0 of all null space for all k.

Conclusion: In this article the following that can be concluded to investigate the characteristics of various existing sequence spaces studied in functional analysis for further generalization and unification. It can be used as a basis for developing ideas in every aspects of human knowledge in future work.

- the non-absolute type of Cesaro sequence space, a matrix transformation $A = \left(a_{n,k}\right)$ maps X_p to the space ℓ_∞ and C by satisfying $\left\|\{k(a_n - a_{n,k+1}\}_{k\geq 1}\right\|_{\ell(q)} < \infty$ and X_p and ℓ_∞ are BK-space and $(A\,x)_{n.k} = \sum_{k=1}^{\infty} a_{n.k}\, x_k$ is convergent forever y $x \in X_p$ also a

transformation of a matrix $A = \left(a_{n,k}\right)$ maps X_p in to C_0 replaced C by C_0 for $\delta_k = 0$ for all k.

- the matrix A converts a BK-space X_p in to ℓ_∞ BK-space this is a continuous and linear transformation.
- Each coordinate mapping in BK-space is a banach space is available where $x \to x_k$ is continuous.
- the conditions that an infinite matrix must meet in order to transform the Cesaro sequence space X_p into the respective ℓ_∞ of and all convergent sequences' space C.
- some theorems are developed with matrix transformation.

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