https://doi.org/10.3126/batuk.v7i2.39503
THE BATUK : A Peer Reviewed Journal of Interdisciplinary Studies
Vol. 7, Issue No. 2, July 2021, Page: 67-76
ISSN 2392-4802

Determination of Break-Even Point through Consumer Preference Relation

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Abstract

Social choice theory beliefs about how the consumers function to chose their interested goods and services. Preference relation with affine indifference curves that has a concave representation has a linear utility representation. This study asks how individual preference relations might be combined to give a single ordering which captures the overall wishes of the group of individuals. There are certain properties that one would like such a utility rule, utility have thus become a more abstract concept that is not necessarily solely based on the satisfaction or pleasure received. Concept of cardinal utility is studied in three different situations Debreu (1958) gave quite different approach. This study maintains link between mathematical theory and financial concept to determine break-even point through the consumers' preference relation.

***Keywords**:* consumption preference, utility function, axioms, convex, break-even point.

Introduction

In utility theory, the utility function of an agent is a function that ranks all pairs of consumption bundles by order of preference (completeness) such that any set of three or more bundles forms a transitive relation. This means that for each bundle (x, y) there is a unique relation, U(x,y) representing the utility (satisfaction) relation associated with (x,y). The relation (x,y) → U(x,y) is called the utility function. The range of the function is a set of real numbers. The actual values of the function have no importance. Only the ranking of those values has content for the theory. More precisely, if $U(x, y) \geq U(x, y')$ then the bundle (x,y) is described as at least as good as the bundle (x', y'). If U(x, y)> $U(x', y)$, the bundle (x,y) is described as strictly preferred to the bundle (x',y') Every continuous convex preference relation on has a continuous utility representation U : $\mathbb{R}^{l}_{+} \rightarrow \mathbb{R}$. which is quasi-concave. It is natural to ask if the preference relation has another representation: one that is concave. The first study on this subject is de Finetti

(1949). There he already mentions the importance of this question in utility theory. Fenchel (1953) has a deeper study of this problem. He presents necessary and sufficient conditions. Later, Kannai (1977) deepens the study of Fenchel's[3]for the concave representation when there is one.

We begin it by presenting the classic theory preference relation (also called social choice theory), this discussion helps the study of choice problem. It is possible that the majority of the society prefers different alternatives. Consumer's preference depends on the transparent facts since the households are the only decision makers. The set of all preference relations defined on the space of commodity bundles is one of the central elements that determine economy. In order to investigate the varying families of economies and behavior of their economic characteristics it is essential to considers some factors.

Microeconomics is based on the decisions of individual agents. Each agent faces a choice problem, where a set of options is given and the individual chooses one. This assumes that individual are aware of all their options have a goal when they make the decision and are rational. The classical model of choice endows the decision-maker (DM) with a single preference relation that uses to select the best element from any set of alternatives. Suppose that we observe a DM's choice behavior on a finite set of alternatives X. Denote by P(X) the set of nonempty subsets of X. The DM's choice function $P(X) \in X$ the alternative $C(A) \rightarrow A$ that she chooses from each $A \in P(X)$. A rationalization of the DM's choice function consists of two components, a collection of selves U and an aggregator f that combines these. The DM's selves represent conflicting motivations or priorities.

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[1] The term preference relation is used to refer to orderings that describe human preferences for one thing over an- other. Economics and other social sciences, preference is the order that a person (an agent) gives to alternatives based on their relative utility, a process which results in an optimal ”choice” (whether real or theoretical).

[2]Manly three situations are stochastic objects of choice, stochastic act of choice and independent factors of the action set

[3]Condition VII - and obtains a formula see for example, Theorem .4

Notion

Although preferences are the conventional foundation of microeconomics, it is often convenient to represent preferences with a utility function and analyze human behavior indirectly with utility functions. Let X be the consumption set, the set of all mutually-exclusive baskets the consumer could conceivably consume. The consumer's utility function u: X → R ranks each package in the consumption set. If the consumer strictly prefers x to y or is indifferent between them, then u(x) ≥u(y):

Axioms

1. A set $\mathbb{S}$ is said to be a mixture set if for any a, b ∈ $\mathbb{S}$ and for any we can associate element, where we write as μa + (1 - μ) b, which is again in $\mathbb{S}$.
2. For any a, b, c ∈ S the sets { αa + (1 -α) b | ≥} and{ α | c | ≥ αa + (1 -α) b }are closed.
3. Weak axiom: Given A,B∈D and a,b∈ A ∩B,if a ∈C(A) and b ∈C(B) then a ∈C(B)
4. In the sequel, an order preserving function referred to be a utility function.

Definitions and Statement of Results

Definition 1. Preference relation on X = R_+^n consumption set is a subset of X. When (x, y) is an element of this set, we say that x is preferred to y and denoted by x y. Assume that a consumer's test is characterized by a relation called a consumer's preference on the set. $R_+^{`}$ a subset of R^n with the following three basic properties;

1. Reflexive: for any x ∈ $\mathbb{R}^l_+$ x ≥x
2. Completeness: ∀ x,y ∈ $\mathbb{R}^l_+$ x ≥ y ⋁ y ≥ x
3. Transitivity: ∀ x,y,z ∈ $\mathbb{R}^l_+$ x ≥y ∧ y ≥z ⇒ x ≥ z

Definition 2. Preference relation ≿ is rational if it possesses the following properties;

Completeness: For all x,y∈ X, we have that x ≿y or y ≿x or both

Transitivity: For all x,y,z∈ X ,if x ≿y and y ≿z ,then x ≿z

Definition 3. Let (X, ⪯) be related set, then a subset A of X is said to be decreasing if , for every x ∈ X and y ∈ A, x ,y implies that x ∈ A and a real valued function u is said to be

1. Increasing- if u(x) ≤ u(y) for all x,y ∈X such that x⪯ y .
2. order-preserving -if it is increasing and u(x) < u(y) for all x, y ⪯ X such that x≺ y.

Definition 4. A utility function, u, quasi-concave if for all y, the set {x | u(x) ≥u(y)} is convex.

Definition 5. We define the order dimension of a set $(X,\succeq)$, denoted $\dim(X,\succeq)$, as the minimum number of linear extensions of the intersection of which is equal to, provided that this number is finite, and as ∞, otherwise.

It is easy to see that $\dim(X , \succeq) = 1$ if and only if $(X , \succeq)$ is a chain on the other hand, the dimension of any anti-chain is equal to 2 in particular, for the vector dominance (strong Pareto) ordering $\geq$ on the Euclidean space $\mathbb{R}^n$, $n \epsilon \mathbb{N}$, we have $\dim(\mathbb{R}^n, \geq)=n$

Definition 6. A relation$\succeq$ is said to be partial order if it is an anti-symmetric preorder.

Definition 7. The strict preference is defined by $x\succ y \Leftrightarrow x \succeq y$ and y not successor and equal x.

The preference relation $\succeq$is a linear order if it is a complete partial order. We say that $(X,\succeq)$ is a preordered set whenever X is any nonempty set and is preorder on X.

Definition 8. A preference relation $\succeq$, is continuous if whenever $x_0\succ y_0$ there are ϵ −balls around x and y such that for all $x \in B\epsilon(x_0)$ and $y \in B\epsilon(y_0)$ we have $x \succ y$.

Definition 9. A preference relation $\succeq$, is continuous if whenever $x_0\succ y_0$ there are balls around x and y such that for all $x \in B (x_0)$ and $y \in B (y_0)$ we have $x \succ y$.

Definition 10. A preference relation, is continuous if $G(\succeq)$ is a closed set in $X \times X$ that is, if $\{(x_n, y_n)\}\subseteqq G(\succeq)$ is a sequence that converges in $X \times X$ to x^*, y^* then $x, y \in G(\succeq)$.

If X is a finite set of nonempty sets, then one can construct a choice function for X by picking one element from each member of X: This requires only finitely many choices. In general choice function by C(A) is studied if there exists preference.

Proposition 1. suppose we observe a choice function, C , on a domain, D , of X that contains all the subsets of size 2 and 3. Suppose that for all A, B ∈ D with $A \subseteq B$ and $C(B)A, C(B) = C(A)$. Then, we may attach a set of preferences such that all choices maximize this set of preferences.

Proof: Let a, b ∈ X and $C(\{a, b\})$ and element of size 2. Let us define $a \geq b$ if $C(\{a, b\}) = a$.

Then,

1. If $C(\{a, b\}) = a$ then $a \geq b$
2. If $C(\{ b, a\}) = a$ then $b \geq a$.

This shows that there are two options and one of the two options must be chosen. Thus, $\succeq$ is complete. Furthermore for transitivity from above definition:

1. If C({ a, b}) = a then a $\succeq$ b.
2. If C({b, c}) = b then b$\succeq c$
3. If C({c, a}) = c then c $\succeq a$.

Clearly, from above definitions three options are must considered C ({a, b, c} and all three sets are contained, no element are chosen and we reach in a contradiction of the assumptions.

Again, suppose that this preference relation does not map to the choice function then $\exists$ an element $x \in X$ such that $x \in A$ but C(A) $y \succeq x$.

From above assumption we have

C({ x, C(A)})$\neq$ C(A) which contradicts {x, C(A)} $\subseteq$ A .

Thus we may attach a set of preference such that all choices maximize the set of preferences.

Example 1 Consider the bundles (4, 1), (2.3) and (2, 2). Suppose that (2, 3) $\succ$(4, 1) ~ (2, 2) Assign to these bundles any numbers that preserve the preferences ordering: if the utility level is defined by

$u(x_1, x_2) = x_1 \times x_2$.... (1)

By using definition the levels of utility are u(2, 3) = 6, U (2, 2) = 4 and u (4, 1) = 4 . Thus, u(2, 2) = u(4, 1) = 4 and u(2, 3) = 6

Theorem 1. C Satisfies the weak axiom if and only if there exists preferences $\succeq$, such that C(A) = x|x $\succeq a \forall a \in A$}.

Proof. $\Leftarrow$ Let X be a set and D be the domain of choice function C and A, B $\in$ D. Suppose that there exists preference, $\succeq$, such that C(A) = x|x $\succeq a \forall a \in A$}. And

C(B) = x|x $\succeq b \forall b \in A$}.

Let a $\in C(A)$and b$\in C(B)$ this implies that a~b, that is a $\in C(B)$

Thus, 'a' is maximized in the set B and hence C satisfies the weak axiom.

$\Rightarrow$ Suppose that C satisfies the weak axiom. Let A, B $\in$ D and a, b $\in$ A $\cap$ B. If a$\in$ C(A) and b$\in C(B)$ then a $\in C(B)$.

Now, define a $\succeq b$if a $\in$C({a,b}). We show that this relation is preferences, $\succeq$.

Now, by using above definition at first we shoe that this relation is complete;

1. a $\succeq b$ if a $\in$C({a,b}).

2. b $\succeq a$ if b $\in$C({b, a}).

Since C({b, a}) $\neq \emptyset$ and being a $\succeq$ b, b$\succeq$a or both. Thus, $\succeq$ satisfies the completenessaxiom. Again, for transitivity Axiom;

1. a $\succeq b$ if a $\in$C({a,b}).
2. b $\succeq c$ if b $\in$C({b, c}).

But a $\not\succeq$ C .then we must have C ({a, b, c}) = $\emptyset$, which is impossible.

Suppose that this is not the correct preference relation. Let us assume that there exists a maximal element C $\succeq$ (B) such that C(B) = C $\succeq$ (B) .

If there is some x $\in$ C (B) with x $\notin C \succeq (B)$, then there is some y $\in$ $C \succeq (B)$with y $\succeq$x. Clearly, C ({x, y}) = {x}.

Since C satisfies the weak axiom so that C ({x, y}) = {y} contradicts the definition of weak axiom.

Again, suppose that there is some x $\in C \succeq (B)$ with $x \neq C(B)$. since$C \succeq (B)$is the maximal element in the set B. Now, choose any y $\in$ C(B) . By definition x $\notin$ C({x, y}). Thus, C ({x, y}) = {y}, it means that y $\succ$ x. This contradicts x $\in$ $C \succeq (B)$.

Hence, We must have $C \succeq (B)$= C(B).

Every convex function is quasi- convex. A Concave function can be quasi- convex function. For example x$\rightarrow$ $logx$ is concave, and it is quasi - convex. Any monotonic function is both quasi- convex and quasi- concave. More generally, a function which decreases up to a point and increases from that point on is quasi- convex.

A convex set is a set of points such that, given any two points lie in that set, this means that the set is connected which is extended to the quasi- concave set by the following proposition.

Claim 1. A preference relation is convex if and only if its corresponding utility function is quasi - concave.

Proof. Let a preference relation, $\succeq$, on X, satisfies convexity if for all y $\in$ X and the set As Good (y) = {z $\in$ X|z $\succeq$ y} is convex.

Let u be a utility function then for all y $\in$ X the set {x|u(x) $\geq$ u(y)} is convex. Thus by definition of quasi concave the utility function u is quasi - concave.

Conversely: Suppose that u be a utility function and it be a quasi - concave function. Then for all y$\in$ X the set

$\{x|u(x) \geq u(y)\}$

Thus the above set is convex and hence preference relation $\succeq$, on X, satisfies convexity. Thus, preference relation $\succeq$, on, X is convex.

When defining a preference relation by using a utility function that has an intuitive meaning that carries with it additional information. More precisely stated, ordinal of preferences is implicitly defined by the following result.

Proposition 2 If a utility function u(x) represents the preference relation, $\succsim$, any monotonic strictly increasing transformatyion of u(x), f(u(x)) also represents the same preference.

Proof. Suppose u represents some particular preference relation, $\succsim$, tehn by definition,

$u(x) \geq u(y) \Leftrightarrow x \succsim y$

But if f(u) is a monotonic strictly increasing transformation of u , then

$u(x) \geq u(y) \Leftrightarrow f\big(u(x)\big) = f(u(y))$

Then $f(u(x)) \geq f(u(y)) \Leftrightarrow x \succsim y$

And the function f(u) represents the preference relation , $\succsim$, in the same way as the function u.

If a preference relation[4] has a utility representation[4] , then it has an infinite number of such representation, as the following simple claim shows this fact;

Function v(x) = f(u(x)) represents $\succsim$ as well.

Theorem 2 (Continuity Theorem) If a, b, c $\in \mathbb{S}$ and a $\succsim$ b $\succsim$ c, then there exists a μ such that b $\sim \mu a + (1-\mu)c$

Proof. Let T= $\{\mu|\mu a + (1-\mu)c \succsim b\}$. By axiom (2) T is closed subset of unit interval [a, b]. Since a $\succsim$ b. 1 ~T; so T is not empty. Using axiom (2) we can find W = $\{\lambda| \succsim \lambda a + (1-\lambda)c\}$ is closed in [0, 1]; it is not empty since 0 $\in$ W.

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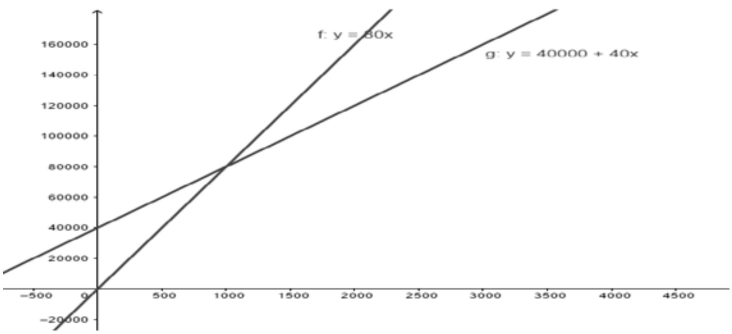
[4] Preference can be defined either as a finite list or as a function that takes the characteristics of two options and returns a decision on them. Any preference relation based on a utility function satisfies completeness and transitivity because the set of real numbers satisfies the property of completeness and transitivity properties.

Now, T ∪ $W = [o, 1]$; the unit internal is connected, so it can not be decomposed into a union, disjoint subsets: Then T ∩ W is not empty. Let$\mu_0 \in T \cap W$; by the definition of and W, b~ $\mu_0 a + (1 - \mu_0)c$.

In real life, preference relation has interesting application in different approach of Economics and management that all costs and selling prices are not entirely under the control of a procedure. Actions by competitors, suppliers, carriers, governments and changes in consumer preferences can affect costs and selling prices, as well as sales volumes at any given time. Changes in any of these variables can be plotted arid the net effect determined at a glance. This glance of assumed case study is presented in this study as a interrelationship between Mathematics and Economical tool- preference relation connecting with management. It gives an application of weak axiom of preference relation. Managers can use break-even analysis to study the relationships among cost, sales volume, and profits but consumers' preference plays a vital role in management.

Case study: Assume in the first case that a company has production associated with each article worth £40 and cost for lease £20000, rental payment £10000 and official vehicle insurance £10000. If a company sold its products at the rate of £80 per unit, in this case we study a case of break-even point. Let the quantity belonging to the production be x, then Total Revenue (TR) = 80x

Total cost (TC) = £(20,000+10,000+10,000) + 40x
Since break-even point is given by, TR = TC
80x = £40,000 + 40x
⇒80x - 40x = £40,000
⇒40x = £40,000) ⇒x= £1,000



Total Cost (TR) =£800,000.

Now, break-even point is (1,000, 800,000).

Figure 1. First case of break- even point

Again, assume that in the second case that the consumer preference reduced to this production and company decreases the selling price to £60.

Total Revenue (TR) = 60x

Total cost (TC)= £(20,000+10,000+10,000) + 40x

Since break-even point is given by, TR = TC

⇒60x = £40,000 + 40x
⇒60x - 40x = £40,000
⇒20x = £40,000) x= £2,000
⇒Total Cost (TR) =£120,000.
Now, break-even point is (2,000, 120,000).

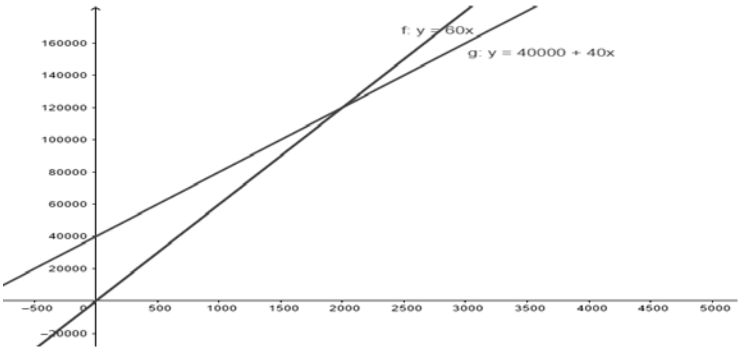


Figure 2. Second case of break-even point

Since in choice theory consumers have variable choices in market thus market managers can use break-even analysis to study the relationships among cost, sales volume, and profits. The break-even quantity does not remain fixed forever. Thus output has to be shifted to the right if more profit is desired. Break-even analysis also provides a rough estimate of profit or loss at various sales volumes. Various scarce resources which have alternative uses that are utilized for the production of various commodities and services in the economy for the satisfaction of unlimited human wants and hence preferences of consumers are different and consumer's preferences determine the management policy. Finally, machinery is considered a fixed expense, but if it is operating at capacity and production is to be increased, it is no longer fixed but consumers preferences depend up on their income and materials availability.

The break-even point refers to the level of output at which total revenue of an institution should be equal to total cost of a particular period. Management has total interest in this level of output. It is much more interested in the broad question of what happens to profits (or losses) at various level of output. It provides guidance to the management to increase level of output to maximize amount of profit through maximum utilization of production capacity.

References

Bisin, A. (2011). *Introduction to Economic Analysis.* Department of Economics, NYU.

Debreu, G. (1954). *Representation of a preference ordering by a numerical function.* New York: Wiley.

Debreu, G. (1958). Stochastic Choice and Cardinal Utility. *Econometrica , 26* (3), 440-444.

Fenchel, W. (1953). *Convex cones, ets, and Functions.* Department of Mathematics, Princeton University.

Herstein, I. N., & Milnor, J. (1953). An Axiomatic Approach to Measurable Utility. *Econometrica , 21* (2), 291 -297.

John, R. (2004). Uses of Generalized Convexity and Generalized Monotonicity in Economics. In *Handbook of Generalized Convexity and Generalized Monotonicity.* Germany: Department of Economics, University of Bonn.

Kannai, Y. (1977). Concavifiability and constructions of concave utility functions. *Journal of Mathematical Economics , 4* (1), 1–56.

Nishimura, H., Ok, E. A., & Quah, J. K.-H. (2017). A Comprehensive Approach To Revealed Preference Theory. *American Economic Review , 107-4*, 1239-1263.

Rubinstein, A. (2012). The Economic Agent. *2nd*. Princeton University Press.