AMC JOURNAL (DHANGADHI) Year 7; August, 2024; Vol. 5; Issue 1, PP25-32
DOI: 10.3126/amcjd.v5i1.69116

Teachers' Practices on Using CRA Approach in Teaching Grade Ten Mathematics

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Article History: Article Received: May 2, 2024 Article Accepted: July 16, 2024

Abstract

The mathematical concept examines the connection between prior mathematical learning experiences and new information. Similarly, increased learner interest fosters a positive attitude toward mathematics. To improve teaching methods, teachers must understand current teacher practices. The CRA strategy includes three stages—concrete, representational, and abstract, to enhance conceptual learning and encourage active participation in teaching. In online journals, e-library (TU and KU), and Google scholars, I did not find any research study regarding practice of CRA approach in grade ten mathematics, especially in context of Nepal. So, this study aims to investigate how secondary mathematics teachers apply the CRA approach. The research question is: How do secondary-level mathematics teachers use concrete materials, representations, and abstract symbols in their instruction?The paper is based on qualitative research, involving interviews with five secondary mathematics teachers from different schools, each with at least five years of experience, and classroom observations to gather data on the use of the CRA pedagogical strategy. Data was collected through classroom observations, in-depth teacher interviews, and the author's personal teaching experiences. The interviews and other data were transcribed, categorized, themed, and analyzed to derive interpretations and findings (Saldana, 2016). It was discovered that most teachers implement the CRA approach in various ways, influenced by factors such as school environment, educational policy, curriculum content, student and parent characteristics, teacher abilities and experience, and available resources. Ultimately, for comprehensive learner development, teachers must be mindful of their pedagogy, content, and instructional materials to enhance learning in their subjects.

Key words: CRA pedagogical strategy, pedagogical practice, teaching activities in mathematics, instructional materials.

Background of the study

Mathematics is viewed as the basic foundation and essential tool for educational, scientific, and technological development in all sectors of human life (Ogunleye, 2019). Mathematics plays vital roles in the observation of facts, collecting information, measuring, arranging logical arguments, hypothesizing, identifying the relationship between variables, and predicting the result for solving any real-life problems. Although mathematics is considered the backbone of education and all other sectors, there are some difficulties in mathematics learning so that learners are unable to use mathematical

knowledge to solve their daily problems properly. The main challenges of mathematics learning are learners' interest, decontextualized nature of mathematics i.e. no connection of the mathematical content knowledge with the context of the learners, teachers' pedagogy, centrally prepared content-based mathematics curriculum, (Luitel, 2005, 2019; Shrestha, et. al, 2020) along with disengaged nature and teacher-centered transmitting pedagogy (Pant, & Luitel, 2016). Furthermore, subject-centric, content-centric, and mark-centric pedagogy approaches to teaching (Luitel& Taylor, 2005) are the main problems in mathematics teaching.

On the other hand, there are several issues of mathematics teaching in Nepal such as issues from theoretical perspectives, social issues, cultural issues, political issues, and technological issues (Panthi and Belbase, 2017). Furthermore, Panthi, et. al., (2021) pointed out several issues in teaching mathematics such as Equity and Justice, Social and Cultural Issues, Technological and Pedagogical Issues, Political and Social Issues, Professional Issues, and Theoretical Issues in Nepal. The Decreasing interest of learners has difficulties in completing mathematical concepts and decreases the achievement of learning mathematics (Meke et. al. 2019). Meke and colleagues (2019) focus on the fact that students having difficulties in mathematical concepts will have difficulties in the next level which makes students less interested and less active participation in mathematics learning which might be one of the causes of low achievement. Among many issues in mathematics teaching, in this study authors are focusing on the pedagogical issues only. Teachers have to be aware of the content and pedagogy which can develop the mathematical concepts and enhance the interest of learners in learning mathematics.

The mathematical concept is not only the result of the mathematical process and result of seeing but is the knowledge of the logical-mathematical necessity of a particular relationship (Simon, 2020). So, the mathematical concept seeks the relation between the mathematical previous learning experience and new facts. For example, the student who wants to understand the formula for the rectangle, he/she should have to understand the meaning of multiplication and reach the conclusion that why it is the product of length and breadth.

On the other hand, students' interest is one of the most influencing factors in learning mathematics which is the preference in someone's soul with happiness (Azmidar. et. al., 2017). From the above argument, it is clear that the higher interest of learners will develop a positive attitude towards the mathematics learners. For the reformation of the pedagogy of teachers, researchers have to understand how teachers are practicing. In this context, are teachers practicing the CRA approach or any other method of pedagogy?

During the study, I found some study related to CRA approaches in various online study platform (research journalas). Among various emerging and innovative pedagogical strategies of mathematics, the Concrete-Representational-Abstract (CRA) instructional strategy might be a useful strategy to support the student-centered pedagogy and allows learners more active participation of learners. The concept of CRA strategy is based on the Bruners' concept of an inactive, iconic and symbolic mode of representation (Hoong, et. al. 2015). According to Flores (2018), there are proponent scholars of CRA strategy such as Mercer and Miller (1992), Harris et al. (1995), Morin and Miller (1998), Maccini and Ruhl, (2000) and others are promoters of CRA pedagogical strategy. CRA strategy consists of three-stage teaching-learning activities namely; concrete, representational and abstract for better conceptual learning and active participation in teaching-learning activities.

In the concrete stage, teachers use concrete materials for mathematics teaching, demonstrate mathematical skills, and allow learners to manipulate the materials to construct mathematical knowledge. Concrete knowledge is related to the manipulation of the concrete object for mathematical skills which are essential for solving daily life problems. In this stage, teachers and learners use concrete materials collaboratively and construct knowledge. After the concrete stage, the teacher uses the representational stage in which the teacher uses semi-concrete materials such as drawings, pictures, and two-dimensional materials of concrete materials and tries to establish relations among them. Lastly, in the abstract stage, the teacher tries to model mathematical concepts by using numbers, symbols, and notation. In this way, there is a sequential flow of teaching-learning activities in the mathematics classroom. Through the CRA strategy, there might be a logical connection between different cognitive levels of learners which will be useful to develop the mathematical concept in students.

On the other hand, it is found that initially Mercer and Miller (1993) found that students with high-incidence disabilities had better performance in using the CRA approach than traditional methods of teaching mathematics and they tried to generalize the finding to other students. Letter on, Miller (1998) generalized and extend the finding in middle school students with intellectual disabilities for multiplication in mathematics teaching using the CRA approach, and focused on students demonstration of fluency in basic multiplication and consistency in learning achievement of students.

Similarly, Maccini and Rahul (2000) used the CRA approach in teaching algebraic subtraction for secondary-level students for six weeks with the STAR strategy. The STAR strategy refers to the steps Search the problems, Translate the word into symbols, Answer the problems, and Review the solution. Maccini and Rahul (2000) focused on better performance in solving algebraic subtraction and generalized those skills in solving word problems.

Flores (2010) examined the effect of the CRA approach in subtraction by regrouping tenth and hundredth place simultaneously, and found that there is a functional relationship between the CRA approach and students' performance on subtraction problems with regrouping in the tens and hundreds of places.

Likewise, Ogunleye, (2019) conducted a pretest-posttest control group quasi-experimental research on the effect of the CRA approach on chemistry performance of secondary level students with mathematics difficulties using Concrete-Representational-Abstract Instructional Strategy (CRAIS). The CRAIS was more effective than the control group and recommended to use of CRAIS for teaching mathematical aspects of chemistry (Ogunleye, 2019).

Along the same line, Dwijayanti, et. al. (2017) focused on learning through enactive, Iconic, and symbolic order has a more positive and different influence on the understanding of the fraction concepts with students' self-efficacy than in other orders. Furthermore, the qualitative study showed that positive relationship between students' self-efficacy and students' understanding of the concept of the fraction. Flores, et. al. (2020) found that using the CRA approach demonstrated greater learning than using the MTSS in fraction concept teaching.

Besides these, it is found that from 2012 ministry of education, Singapore implemented and promoted the Concrete-Pictorial-Abstract (CPA) as a national policy in school-level instructional strategy

(Leong, et. al., 2015). CPA instructional model is similar to the CRA approach and has a one-to-one correspondence with Burners' Enactive-Iconic-Symbolic strategy of teaching-learning. In Singapore, Concrete is not restricted to concrete manipulatives but also includes concrete experience and teacher-talk about the activity, and abstract as the end result of the learning process (Leong, et. al., 2015).

Being a Secondary mathematics teacher, these literatures motivated me to think about the practice of CRA approaches in the context of Nepal, especially in teaching secondary mathematics. I found many journals related to the use of the CRA approach in mathematics in the context of other countries then, I search literature related to the CRA approach used in the school of Nepal. Moreover, I cannot find any literature related to the CRA practice in secondary mathematics, in the context of Nepal, especially in the Sudurpaschim province of Nepal. So, the purpose of the journal is to explore the practice of the CRA approach in teaching secondary mathematics to enhance the interest of learners and develop mathematical concepts. I hope this paper will contribute to showing the landscape of pedagogical practice of secondary mathematics teachers in Nepal.

From the above literature, I found that the CRA approach is essential for a better understanding of various mathematical concepts but literatures are silent on the ways of practicing in secondary-level mathematics teaching. I did not find any literature regarding the status of practicing CRA approaches in secondary mathematics. Furthermore, the literature did not show whether are teachers practicing the CRA approach in sequence or any other order for teaching mathematical concepts at the secondary level? In Concrete, representational and abstract, which techniques are mostly practiced in their classroom? So, the present practice of the CRA approach is a gap whose solution might contribute to reform and transforming the pedagogy of mathematics teachers. To fill the literature gap, we formulated the research question as: how do Secondary level mathematics teachers are using concrete materials, representations, and abstract symbols in teaching mathematics?

Using manipulative materials in mathematics classrooms will help students understand abstract concepts, establish the connection between experience, content, and context (Meke et. al., 2019). Furthermore, the use of concrete materials in the mathematics classroom make learning more interesting, challenging, empowering, and motivating students for learning. Hence, CRA pedagogical strategy might address some aspects of mathematics teaching related to the mathematical concepts and interests of the learners along with the exploration of the practice of mathematics teachers regarding to the CRA pedagogical strategy. To develop the mathematical concept in the learners and to enhance their learning interest, teachers have to use innovative and student-centered pedagogy and pedagogical strategy. In my experience, teachers' pedagogy is an appropriate tool to enhance the interest of learners and to develop mathematical concepts for learners. Before reforming the practice of the teachers we have to find the present practice in teaching mathematics. So, the purpose of the study is to explore the practice of teachers regarding the CRA approach in secondary mathematics teaching.

Methodology of the study

The paper is written based on the qualitative inquiry in which facts were collected through interviews and class observation of participant teachers. Participants were selected purposively from five secondary-level schools with a minimum of five years of teaching experience in mathematics. To explore the practice of the mathematics teachers regarding the CRA approach, I observed the classroom

and took in-depth interviews with participant teachers, and being a mathematics teacher, I have included my experience too. Interviews were transcribed, categorized, thematized, and analyzed for interpretation, meaning-making, and findings of the study (Saldana, 2016).

Findings and discussion

From the field work and fact collection, along with the analysis of facts, I reached at the following finding of the study.

Variation in teaching experience of teacher

Generally, every school has a different learning environment for teachers and students. In Nepal, there are three types of schools namely; community schools, institutional schools, and religious schools. Novice teacher started their careers in institutional schools then after some time, they entered community schools for their teaching carrier. It is found that teachers in institutional schools have more pressure to improve the learning of students than in community schools and religious schools from school management and parents of students. Damber states that *'there was pressure for learning from teacher, parents and head teacher in a boarding school'*. This statement shows the pressure of institutional schools on teachers. Likewise, Bhim sir mentioned the same condition regarding institutional schools. He said that *'teachers have to compel students for reading by saying that, you have to learn anyhow, and you have to remember formula and contents'*. Furthermore, Bhim added that *'parents of the institutional school are more aware in the education of their children whereas due to the poor economic condition, and lack of education of parents of government school they are less aware of education'*. The statement shows that the awareness of parents also affects the learning environment of the school.

On the other hand, community school focused on extra co-curricular activities in school. In this context, Damber said that' *in government school there more-free environment, students allow to participate in various extra-curricular activities*. This indicates that institutional schools focused more on content knowledge whereas community schools focus on the holistic development of learners. This shows that the environment of the school depends upon the administration of the school, teachers, students of the school, and parents of the students. This indicated that the learning environment of the school also affects the teaching activities in the classroom.

CRA in Classroom

Most of the mathematics teachers in all types of schools are using traditional, and teacher-centered pedagogy for transmitting content knowledge to the students' minds. Both Damber and Bhim state that, '*generally we are using the chalk-and-talk method for teaching mathematics*. Furthermore, Bhim added, *'I start with the introduction of content, and formula then starts solving the mathematical textbook problem in whiteboard'*. This indicates that most of the teachers are using the teacher-centered traditional method. Further participants indicated that there are various challenges by which they were not using teaching materials regularly in the mathematics classroom. One of the challenges was sitting arrangement in the classroom Damber said that' *there is parallel sitting arrangement in class, and, there are difficulties in teaching mathematics in an open place and difficult to manage sitting plan every day*.

Further, he added that' *there are no any teaching materials available in class, generally, I used to bring from office or home, and there is no place to keep materials in office and classroom'*. Likewise, Bhim added that *'sometimes, I used to bring local materials like Cucumber (KAKRO), Raddish (MULA) for cylinder, Raddish, Carrot (GAJAR) for cone as local, and bracelet for circle but becomes difficult to manage in a classroom, and keep in office for next year'*. I found that there were not only managerial challenges there were other challenges in using teaching materials in the classroom. Bhim said that the *laziness of mathematics teachers is also may be one main cause, on the other hand, the teacher has to complete a huge syllabus in a short time. So, teachers are not using instructional materials in the classroom.'* These facts collected from the participants indicated that due to the various challenges in using teaching materials in the classroom their pedagogy changes accordingly.

The practice of the CRA approach in teachers' pedagogy

In secondary level mathematics, a few of the contents are suitable for the CRA approach in which teachers can use concrete, representational, and abstract techniques of teaching. But it is found that some teachers are using CRA sequence i.e. concrete, representational, and abstract in which some of the teachers are using in RCA order i.e. representational, concrete, and abstract without considering any theory of teaching. Damber said *'firstly I allow students to observe the shape, size, and sides of objects in the classroom with some brainstorming questions. Secondly, I allowed students to draw a picture in their notebooks and compare pictures and solid objects. Lastly, I used calculation to solve mathematical problems.* This shows that some teachers were using concrete, representational, and abstract ways of teaching. On the other hand, Bhim said that *'I used to picture, concrete and then symbol in teaching such types of content in mathematics'*. This statement indicates that some teachers are not using the CRA approach in sequence. Furthermore, Bhim said *'I don't know about any such theory in teaching, which we are using these techniques. But, I built this sequence from my own practice and experience. I felt better teaching by using a picture, concrete, and symbol in sequence'*. Whatever be the sequence of concrete, representational, and abstract the main goal is to provide a better learning environment for the students. CRA is a sequence in which students learn a given mathematics concept along with a systematic presentation of concrete, representations, and abstracts for better learning (Flores, & Hinton, 2021). This indicates that the use of the CRA approach applies to understanding the mathematical concepts of all students.

On the other hand, in some content of mathematics, teachers were using representational figures and abstract only or representational and abstract techniques only. In some chapters or content of mathematics, teachers use pictures and symbols. Supporting this fact, Bhim said that, *'in domestic mathematics* (He used "*domestic mathematics"* for contextual mathematics), *I use a bill of electricity or water. Initially, they observed bills, discussed themselves, and tried to make meaning of concepts and try to calculate'*. Likewise, Damber said that, *' I have used pictures in the set chapter as Venn diagram, electricity and water bills for domestic mathematics. And in geometry, I used various figures in the classroom and finally allow students to solve problems symbolically'*. These facts indicate that, depending upon the nature of the content of mathematics, a teacher used to picture and finally calculate by using symbols. Likewise, Smith (2009) indicates that good instructional materials are the bridge between formal and informal mathematics. These facts indicate that, whatever be the pedagogy of the teachers, instructional materials play a vital role in learning.

Likewise, in some cases, teachers are using concrete and symbols only in teaching mathematics. In this context, Damber said that, *'in the chapter Mensuration, to give the concepts of sides of cone, cylinder, prism, pyramid, and so on, I used the solid models in the classroom'*. Further, he adds his view by saying, *'Also, I used solid materials in geometry to show and demonstrate the properties of a circle, sphere, parallelograms, and other geometric figures. We can use concrete material in height and distance, probability, and algebra'*. In some content of mathematics of grade ten, teachers are using concrete materials for a better concept. Another participant Bhim said that, *' using radish, one concept of a cylinder, cone, length, and breadth can teach by using radish. To show Lateral surface, slant height, height, and cubic objects can teach by using radish. Length and breadth concepts can give by using these materials.* These views of participants show that some contents of mathematics are suitable for using concrete materials. Furthermore, Jones and Tiller (2017) focus on the concrete materials can lead to higher retention rates, a more positive attitude towards learning, and connection of mathematical concepts in their context. This shows that concrete material tries to connect mathematical concepts from hand to head.

All participants' voice indicates that the ultimate goal of using concrete or representational technique is to provide a better understanding of mathematical concepts and to solve a mathematical problem by using symbols. Bhim said that, *'most of the chapters are suitable for direct use of symbol'*. Furthermore, teachers are using symbols only for other reasons *Such as Damber said that 'Schools are conducted properly, takes more time to collect instructional materials, nature of mathematical content, and we have to finish the huge syllabus in a short time. So we are using symbolic techniques mostly'*. According to the nature of the content, context, and time of mathematics teachers have to choose suitable techniques.

Conclusion

In conclusion, it was found that most teachers are using the CRA approach in their own ways in teaching secondary mathematics, and they are trying to provide a better learning environment for mathematical concepts. From the study, I found that learning activities in the classroom are influenced by the administrative environment of the school, educational policy, content of the disciplines, nature of students and their parents, abilities and experience of the teacher, and resources in school. It will be better to use students-centered activities and instructional materials in the classroom which are the foundation of the knowledge construction process. Although teaching-learning activities are the root of learning, teachers are practicing their own pedagogy based on skills, content, curriculum, time, the experience of the teacher, and the learning environment of the school. For the holistic development of the learner, teachers have to be aware of his/her pedagogy, content, and instructional materials used in the classroom for better learning of the respective disciplines.

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