MATLAB-Based Visualization of Helical Structures and Their Applications

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Abstract

A helix is a type of smooth space curve, i.e. a curve in three-dimensional space. This study aims to present a MATLAB-based approach to visualize helical structures and to demonstrate their applications in various fields. As a qualitative study, primary and secondary sources of data have been consulted. Primarily, observing the real-life activities of humans and secondly, the analysis of archival documents such as related research articles, analytical books, and related reports. In conclusion, most of the architectural work in the world is possible by different types of geometrical design through the helix. It also found that we can visualize the different types of helices by using MATLAB in mathematics teaching. The implication of helix is everywhere in different fields and areas such as constructing a basket, screw, etc. Finally, we used helical concepts related to daily life activities in the field of instruction in the mathematics classroom.

Keywords: Bertrand curve, cylindrical helix, circular helix, space curve, MATLAB.

Background of the Study

A helix is a three-dimensional shape that is characterized by a smooth, curved surface that winds around a central axis (Du et al., 2018). The shape of a helix is often compared to that of a spiral staircase, where the steps form a curve that rises or descends around a central column. Helical structures can be found in a wide range of natural and man-made systems, including DNA molecules, proteins, seashells, springs, and screws. Standard screws, nuts, and bolts are all right-handed, as are both the helices in a double-stranded molecule of DNA (Gardner 1984, pp. 2-3). Also, large helical structures in animals (such as horns) usually appear in both mirror-image forms, although the teeth of a male narwhal, usually only one which grows into a tusk, are both left-handed. Gardner (1984) contains a fascinating discussion of helices in plants and animals, including an allusion to Shakespeare's *A Midsummer Night's Dream.*

Helix was discovered around 1968 that the DNA in living organisms on Earth has a double helix structure. A similar double Helix structure of some galaxies was discovered recently (Humi, 2019). This raises the possibility that other life forms might use as building block molecules with different geometrical structures. The shortest path between two points on a cylinder is a fractional turn of a helix, as can be seen by cutting the cylinder along one of its sides, flattening it out, and noting that a straight line connecting the points becomes helical upon re-wrapping (Steinhaus 1999, p. 229). It is for this reason that squirrels chasing one another up and around tree trunks follow helical paths.

A helix is a type of smooth space curve, i.e. a curve in three-dimensional space. It has the property that the tangent line at any point makes a constant angle with a fixed line called the axis. Helices are important in biology, as the DNA molecule is formed as two intertwined helices, and many proteins have helical substructures, known as alpha helices (Huang et al., 2015). The alpha helix in biology, as well as the A and B forms of DNA, are also right-handed helices. The Z form of DNA is left-handed. The pitch of a helix is the height of one complete helix turn, measured parallel to the axis of the helix. A double helix consists of two (typically congruent) helices with the same axis, differing by a translation along the axis. A curve is called a slant helix if it's principal normal makes a constant angle with a fixed line in space. It can be constructed by applying a transformation to the moving frame of a general helix. There were many attempts in architectural history, to build helical towers and other constructions. Most of the architectural work in the world is possible by different types of geometrical design. The work constructions which are in helical shapes are also the most important and interesting works in the world such as the Guggenheim Museum, Canton Tower, and Agora Garden Tower (Vahid, 2017). It would seem that the success of the helix as a shape in biological molecules is a case of nature working the best it can with the constraints at hand," Kamien said. "The spiral shape of DNA is dictated by the space available in a cell much like the way the shape of a spiral staircase is dictated by the size of an apartment (University of Pennsylvania, 2005).

The purpose of this study is to present a MATLAB-based approach to visualize helical structures and to demonstrate its applications in various fields. This article is qualitative in nature and the content analysis method is used to analyze the data. This method involves systematically analyzing the content of documents or other materials to identify patterns, themes, and other relevant information. This method is particularly useful when analyzing textual data and can be used to answer research questions related to attitudes, beliefs, and other subjective phenomena.

Pre-knowledge for Helix

Generally, the pre-knowledge of helix is to know about the primary concept of curves, three-dimensional cylinders, and some other angles (Huang et al., 2015). A normal cylindrical surface has an axis perpendicular to one of the principal planes of projection. The axis of the cylindrical surface will appear as a point and the surface will appear as a circle in that view. A cylindrical surface model is proposed to describe the relationship between the surface of the bound document and its image formed on the image plane (Grey, 1998). A three-dimensional space curve is defined in the form of parametric equation $C(s) = \{ x(s), y(s), z(s)\}$ where s is the parameter that represents all values of x, y, and z respectively. Similarly, a space curve can be expressed in the form: $F(x, y, z) = 0$. Also, cylindrical helices can be constructed from the plane curve and Bertrand curves can be constructed from spherical curves. The cylindrical helices and Bertrand curves from the curve on ruled surfaces. The relation between a helix and Bertrand curve as the curve is Bertrand curve if and only if there exist non-zero real numbers A, B such that $A\kappa(s) + B\tau(s) = 1 \; for\; any\; s\epsilon I$ implies circular helix is Bertrand curve (Babaarslan & Yayli, 2013).

Description of the Helix on MATLAB

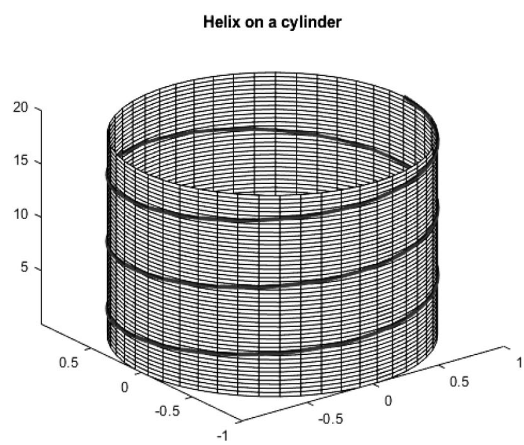
Visualizing helical structures is an essential step toward a better understanding of their properties and applications (Naito et al., 2019). In recent years, there has been a growing interest in the use of computer-based tools for visualizing and analyzing helices (Li et al., 2019). Among these tools, MATLAB has emerged as a popular choice due to its versatility and ease of use. The problems of every helix are visualized by using MATLAB. Here are some helices on MATLAB.

Example: The cylinder is a right circular cylinder of radius 1 centered on the z-axis. It is the set of all points of the form

$(\cos(U),\ \sin(U),\ V), \quad 0 \le U \le 2\pi,\ -\infty < V < \infty$ and here $0 \le V \le 6\pi$

Solution:

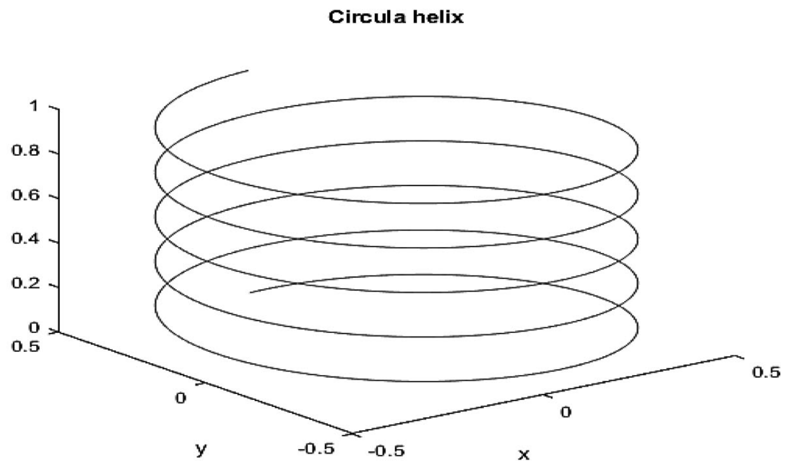
```
>> u=linspace(0,2*pi,50); v=linspace(0,6*pi,50);
[U,V]=meshgrid(u,v);
>> t=linspace(0,6*pi,300);
plot3(cos(t), sin(t),t, 'LineWidth',3), hold on
title('Helix on a cylinder')
surf(cos(U),sin(U),V), colormap white, hold off
```



Example : The circular helix with parametric equation x = a sint, y = a cost, z = t/2πc for a = 0.5 and c = 5.0

```
>> a = 0.5;
c = 5.0;
```

```
t = 0:0.01:10*pi;
x = a*sin(t);
y = a*cos(t);
z = t/(2*pi*c);
figure(1)
plot3(x, y, z);
xlabel('x'); ylabel('y'); title('Circula helix');
```



Mathematical Description of Helix

Mathematically, we shall show that a curve $\boldsymbol{X}(s)$ is a helix if we can find a constant vector **V** so that for all points on the curve $\boldsymbol{V}.\boldsymbol{T}(s) = constant$, a specific example of a helix $\boldsymbol{X}(t) = (a\cos(t), bsin(t), ct)$. Where, a, b, c are nonzero constants. When a = b we have a circular helix and when $a \neq b$ we have an elliptic helix. Here are some theorems on the helix

Theorem 1: Assume $\boldsymbol{X}(s)$ is a helix and show that $\frac{\tau(s)}{k(s)} = constant$

Proof: Since $\boldsymbol{X}(s)$ is a helix there exists a constant vector bfV (which without loss of generality can be taken to be of unit length) so that $\boldsymbol{V}.\boldsymbol{T}(s) = constant$

$$\text{Hence } \boldsymbol{V}.\frac{d\,\boldsymbol{T}(s)}{ds} = 0.$$

But $\frac{d\boldsymbol{T}(s)}{d(s)} = k\boldsymbol{P}$**.** Therefore, since $k \neq 0$, it follows that **V** is orthogonal to **P.** Hence **V** is in the plane of **T** and **b.** Therefore, we can write that $\boldsymbol{V} = \alpha\boldsymbol{T} + \beta\boldsymbol{b}$

However, $\boldsymbol{V}.\boldsymbol{T} = \alpha\boldsymbol{T}.\boldsymbol{T} + \beta\boldsymbol{T}.\boldsymbol{b} = \alpha$

Since both **V** and **T** are of unit length we can find θ so that $\alpha = cos\theta \ and \ \beta = sin\theta$.

Now since **V** is orthogonal to **P** we have $\boldsymbol{V}.\boldsymbol{P} = 0$

Which implies that $\boldsymbol{V}.\frac{d\,\boldsymbol{P}(s)}{ds} = 0$

Therefore, $0 = \boldsymbol{V}.\frac{d\,\boldsymbol{P}(s)}{ds} = (cos\theta\boldsymbol{T} + sin\theta\boldsymbol{b}).(-k\boldsymbol{T} + \tau\boldsymbol{b}) = (-kcos\theta + \tau sin\theta)$.

Finally, we find that $\frac{\tau}{k} = cot\theta$.

Theorem 2: Assume that along the curve $\boldsymbol{X}(s)$, $\frac{\tau(s)}{k(s)}$ is constant and proves that $\boldsymbol{X}(s)$ is a helix.

Proof: To prove this statement we have to find a vector **V** so that $\boldsymbol{V}.\boldsymbol{T}(s)$ is constant for all points on the curve. To this end we consider **V** to be of the form $\boldsymbol{V} = \alpha b\boldsymbol{T} + \boldsymbol{b}$

Where α is a constant whose value will be specified later. Differentiation the expansion for **V** we have using Frenet formulas $\frac{d\boldsymbol{V}}{ds} = \frac{d(\alpha\boldsymbol{T}+\boldsymbol{b})}{ds} = \alpha k\boldsymbol{P} - \tau\boldsymbol{P} = (\alpha k - \tau)\boldsymbol{P}$.

It follows that if we let $\gamma = \frac{\tau}{k} = constant$

Then $\frac{d\boldsymbol{V}}{ds} = 0$ i.e **V** is a constant vector along the curve.

Furthermore, $\boldsymbol{V}.\boldsymbol{T} = (\alpha\boldsymbol{T} + \boldsymbol{b}).\boldsymbol{T} = \gamma = \frac{\tau}{k} = constant$

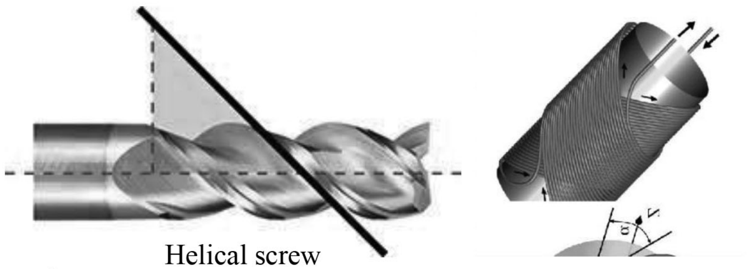
Application of the Helix

Right helicoids are minimal surfaces. Through the centuries helicoids have been applied in architecture mostly for staircase construction as to their minimal space requirement (Nataly et al., 2020). Thus, the developable surfaces as such are cost-efficient and widely used in manufacturing and engineering work. Similarly, in the application of cylindrical helix theory to ultrasonic testing in industrial production, nondestructive testing of components is extensively used and required to ensure product quality (Li et all., 2016). They focused on one of the common nondestructive testing means via ultrasonic waves. Because of the shortcomings of existing test methods for cylindrical parts, we established a mathematical model and developed a set of automatic ultrasonic testing systems to test cylindrical parts. Also, the mathematical model is based on cylindrical helix theory, in which the three-dimensional space is converted into a two-dimensional planar space (Du et al., 2018). Moreover, the helix fusion methods that link two proteins by connecting their terminal alpha-helices into a single and extended alpha helix can be particularly useful because designing fusion helices is conceptually and technically simple. These methods are crucial in obtaining crystals that diffract X-rays to high resolution or attaching large and symmetrical backbone proteins to small target proteins for cryo-EM analysis (Kwon et al., 2020). They believed that the structural rigidity of the fusion helix is crucial for these applications, and the reduction of structural ambiguity and flexibility at the fusion sites will further enhance the usefulness of this method.

Many factors impact the performance of the machining application. Often the helix angle is overlooked. The helix angle controls the angle of the cutting flute entering the workpiece as the tool rotates (Li et al., 2019). The larger the helix angle, the more gradual the entry of the cutting flute. This creates a smoother transition as the cutting flute reaches the desired axial depth of the cut. By using helical gears in applications requiring heavy load efficiencies and noiseless functioning in automotive applications. The gears are a common feature in transmissions. Some of the industries where the helical gears are commonly used are:

- Printing, earth-moving, and fertilizer industries
- Port and power industries, steel and rolling mills
- Textile industries, food industries, plastic industries, elevators, conveyors, compressors, blowers, cutters, and oil industries

Besides the above applications, the gears are used in several additional applications. Each type of gear has its own set of features and attributes making them suitable for different machines and varied operating conditions.



Helical screw

Result and Discussion

Let C be a $C^{(r)}$ curve, with $r \geq 3 \ \ \kappa \neq 0 \ and \ \tau \neq 0$, C is called a general helix if its tangent line makes a constant angle with a fixed line in space (Sy, 2001). A helix in Euclidean 3-space E 3 is a curve where the tangent lines make a constant angle with a fixed direction. A helix curve is characterized by the fact that the ratio κ/τ is constant along the curve, where κ and τ are the curvatures and the torsion of α, respectively. Helices are well-known curves in classical differential geometry of space curves.

A helix is defined by the parametric equation (r cos(t), r sin(t), pt). Also, The helix is a space curve with parametric equations

$$x = rcost$$

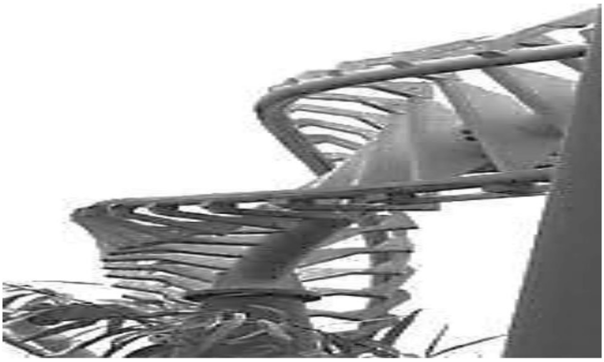
$$y = rsint$$

$$z = ct$$

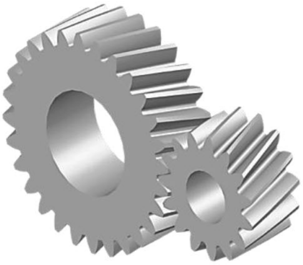
for $t \in [0,\ 2\pi)$, where r is the radius of the helix and $2\pi c$ is a constant giving the vertical separation of the helix's loops.

The curvature of the helix is given by $k = \frac{r}{r^2+c^2}$

The torsion of a helix is given by $\tau = \frac{c}{r^2+c^2}$



There are mainly two types of helix such as cylindrical helix and circular helix. A helix that lies on the surface of a circular cylinder is called a circular helix or a right circular helix (Gupta et al,2010). Space that is traced on the surface of a cylinder and cuts the generators at a constant angle is called a Helix. A cylindrical helix is a curve traced out on the surface of the cylinder and cuts the generators at a constant angle (Gupta et al,2010). Thus, figures 2 & 3 given below be the helices that lie on the cylinder. So, a helix that lies on the cylinder is called a cylindrical helix.



Property I: The necessary and sufficient condition for a curve to be helix is that ratio of the curvature and torsion is constant.

Property II: The spherical curve is a circle if and only if the corresponding Bertrand curves are circular helices.

According to Izumiya and Takeuchi (2002), cylindrical helices can be constructed from plane curves and Bertrand curves can be constructed from spherical curves. Also, they have studied cylindrical helices and Bertrand curves from the viewpoint of curves on ruled surfaces.

Moreover, Schief, W. K. (2003) has given a study of the integrability of Bertrand curves. Similarly, he has studied the spherical images of the tangent indicatrix and binormal indicatrix of a slant helix and they have shown that the spherical images are spherical helices.

Similarly, a helix that lies on the surface of a circular cylinder is called a circular helix or a right circular helix. The curvature and torsion of such helix are both constant. The equation of a circular helix

may be put in the form
$r = (acos\theta, asin\theta, a\theta cot\alpha)$.

Property: If the curvature and torsion are both constant, then the curve is a circular helix.

The necessary and sufficient condition for a curve to be a helix is that the principal normal should all be parallel to a fixed plane.

Conclusion

Helix is a smooth space curve, in its parametric Cartesian equations, this curve presents basic trigonometrical functions and they are in mathematics accepted as functions of natural harmony. Such a geometrical object like a helix has features to be concerned and elaborated for generating more complex models. The architectural point of view obtained the most suitable helical shape has been analyzed in nearly five DNA helical structures to find out the most suitable to further advancement of that and design appliance into building construction. That was triple-helical DNA, just taking out its one helix as tuning to alpha DNA presumably that structure-compatible helical body. Helices have a wide range of applications in various fields and our daily life activities. Helical structures are used in Biology, Material science, Chemistry, Engineering, and daily life activities

Furthermore, the implication of helix is everywhere in different fields and areas. In general, constructing a basket, screws and other materials which are made from wood and bamboo by villagers in Nepal also used the helical concept. Also, it can be used for engineering and architectural design, manufacturing, and efficient material usage. Finally, the helix can be found everywhere surrounding us. It is prevalent from DNA to modern engineering designs. Moreover, the use of MATLAB in the visualization of the helix can be one of the important approaches in teaching-learning.

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