Tribhuvan University Journal
Vol. 39, No. 1: 16-34, June 2024
Research Directorate, Tribhuvan University (TU),
Kathmandu, Nepal
DOI: https://doi.org/10.3126/tuj.v39i1.66665

EXPLORING THE COGNITIVE GROWTH STAGES IN PERRY THEORY AND THEIR APPLICATION FOR HIGHER LEVEL MATHEMATICS LEARNING

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Received date:13 Mar. 2024- Accepted date: 2 May 2024

ABSTRACT

This article explores the cognitive growth stages in Perry's theory, its applications, and relevancies in higher-level mathematics learning using a review-based method. Perry's theory of intellectual and ethical development provides a framework for understanding and applying effective principles in mathematics education. The article examines the developmental progressions outlined in Perry's stages and their alignment with the cognitive shifts required for advanced mathematical understanding. The study focuses on the application of Perry's theory in the context of higher-level mathematics education to gain insights into how to nurture students' cognitive development and equip them with the skills for advanced mathematical competency. The article concludes by discussing the relevancies of Perry's theory in different dimensions of mathematics learning shaping effective mathematics curriculum and pedagogy. The findings highlight the importance of considering Perry's framework in designing instructional strategies that promote deeper understanding, cognitive flexibility, and critical thinking in higher-level mathematics learning.

Keywords: Higher-level mathematics, Intellectual development, Perry theory, Relevancies, Schemes.

INTRODUCTION

Mathematics education in Nepal faces ongoing challenges in nurturing critical thinking and conceptual understanding (Mishra, 2024).

Incorporating both theoretical instruction and problem-solving activities can improve teachers' and students' conceptual understanding of mathematical concepts (Boye & Agyei, 2023). To address these challenges, cognitive development theories provide frameworks for scaffolding learners' progression toward advanced logical-mathematical proficiencies (Ahmed *et al.,* 2023). Perry's theory of intellectual and ethical development proposes stages through which learners transition from dualistic to relativistic ways of comprehending complex ideas (Moore, 2002).

Piaget, Vygotsky, and Bruner present developmental models that outline stages associated with a person's cognitive development. However, the Perry model provides a framework for understanding the intellectual and ethical growth of college students as they progress through different stages of cognitive and moral development during their higher education experience (Perry, 1981). These models suggest a generally linear progression through distinct stages during childhood and early adulthood. Vygotsky emphasizes the impact of social interactions on cognitive development (Vygotsky, 1978), while Bruner focuses on active learning, prior knowledge, and cognitive strategies in knowledge construction (Bruner, 1960). Perry's theory focuses on conceptual change in facilitating learners' cognitive readiness, especially in higher-level learners (Moore, 2002).

Sequencing content and pedagogy based on developmental stages can help learners confront mathematical concepts from an increasingly sophisticated lens (Hodgen *et al.,* 2018). Exploring cognitive development theories helps educators understand the intellectual growth involved in learning complex concepts like mathematics and informs instructional practices (Christina, 2022; Nadelson *et al.,* 2018; Taber, 2018). Aligning pedagogy with learners' cognitive levels supports transitions between stages of mathematical reasoning and nurtures critical thinking abilities (Gilmore, 2023; Ahmed *et al.,* 2023). Culturally adapting theories to the local context is important to understand the conceptualization of knowledge processes (Zhang *et al.,* 2022).

Context and Rationale

Perry's theory is a significant framework for understanding student intellectual/ethical development and effective higher education practices, including mathematics education by focusing on epistemological development and engagement with mathematical ideas (Perry, 1981). The Perry theory is recognized as part of the mathematics curriculum in the

M.Ed. program at Tribhuvan University, Nepal. It provides students with a theoretical foundation for understanding progression through intellectual and ethical developmental stages when encountering new knowledge (Perry, 1970).

Educators can benefit from Perry's theory by tailoring instruction to meet stage-specific student needs (Thoma, 1993). By recognizing diverse epistemological beliefs, teachers can promote growth and higher-stage advancement (Thoma, 1993). The theory assists curriculum design fostering growth by appropriately challenging complex problems and fostering critical thinking, facilitating dualistic thinking transition (Nilson, 2007). This enhances mathematical reasoning and problem-solving skills.

Furthermore, Perry's theory emphasizes creating a supportive, inclusive learning environment (Baxter Magolda, 1992). Instructors can provide guidance, mentorship, and reflection opportunities to navigate cognitive dissonance, and uncertainty between stage transitions (Magolda, 2014). This approach contributes to overall cognitive and personal development.

Objectives of the Study

The objectives of this paper are as follows:

1. To discuss the schemas of Perry's theory of intellectual and ethical development, providing a comprehensive overview of the stages of cognitive growth.
2. To explore the applications and relevancies of Perry's theory in the context of higher-level mathematics learning.

METHODOLOGY

This article conducts a desk review employing a systematic literature review approach to explore the states of the intellectual and ethical development of Perry's theory, schemes, applications, and relevancies for higher-level mathematics learning. Diverse sources are included, such as academic journals, conference proceedings, educational databases, and books. This methodology ensures a rigorous, comprehensive examination of Perry's theory schemes, applications, and relevancies for higher-level mathematics learning.

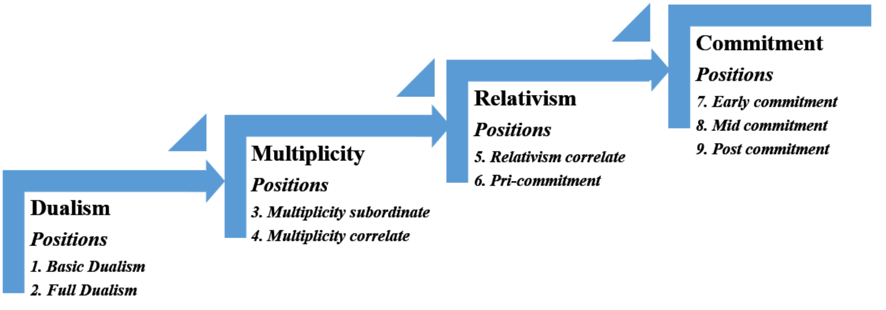
RESULTS AND DISCUSSION

Overview of Perry's Theory of Intellectual and Ethical Development

William Perry's 1950s qualitative longitudinal research on male students led to his developmental theory (Perry, 1970; Thoma, 1993). Perry's theory proposes a 4-category developmental scheme with 9-positions, describing the progression from dualistic to committed thinking as student identities develop (Perry, 1970; Thoma, 1993). The theory primarily focuses on epistemological development, investigating how individuals understand and derive meaning from knowledge (Perry, 1981). Individual conceptions evolve through 4 stages from dualism to commitment (Perry, 1970; Thoma, 1993). The theory emphasizes advancement in each stage through education (Perry, 1981). Perry's model shows individuals change perspectives and thinking as they develop cognition gradually. Progress may vary across topics, individuals can hold different positions on issues (Perry, 1970). Emotional readiness may lag intellectually, necessitating revisiting stages (Perry, 1970). The stages and positions of each cognitive structure are presented in Figure 1 and are discussed below.

Figure 1

Perry's Intellectual Development Model



Dualism

Perry's model identifies the initial stage as dualism, characterized by binary thinking without ambiguity recognition (Perry, 1970). Students perceive knowledge as right or wrong, lacking interpretation (Perry, 1970; Thoma, 1993). Authority figures are solely correct (Thoma, 1993). Dualism fails to acknowledge subjectivity or diverse perspectives and it dismisses alternative opinions (Perry, 1970; Thoma, 1993). Thinking involves memorization rather than engaging complexity and responsibility means obedience over independent judgment (Thoma, 1993). Dualism reflects a simplistic good vs bad mentality lacking perspective diversity acknowledgment (Thoma, 1993).

Position 1: Basic Dualism. In the early phase of the dualistic stage, students view knowledge as consisting solely of absolute right or wrong answers provided by authority figures, without any ambiguity (Perry, 1970). Their thinking operates objectivity in discrete categories of true and false determined by experts.

Position 2: Full Dualism. In late dualism, students start to recognize knowledge is not always clear-cut or definitive (Perry, 1981). They consider answers may depend on perspective rather than just authority (Baxter Magolda, 1992). The thought remains binary but expands beyond absolute facts from experts alone. Students observe issues that can be viewed through alternative lenses rather than a single authority perspective (Perry, 1970). Late dualism represents a transitional phase linking dichotomous thinking with developing multiplicity realization involving uncertainty. It signifies a broadening perspective even if evaluation stays dichotomous. Students acknowledge knowledge as more complex and interpretive than early dualism permitted.

Multiplicity

In the multiplicity stage, individuals begin to recognize multiple perspectives and question authority but still rely on external sources without critical evaluation (Perry, 1970; Thoma, 1993). Students acknowledge knowledge uncertainties and subjectivity (Thoma, 1993) as well as diverse viewpoints rather than definitive facts (Nilson, 2007). While multiplicity signifies fracturing binaries, relativistic reasoning remains underdeveloped, leaving beliefs destabilized (Perry, 1970). As a transitional phase, it marks the initiation of subjective thinking and conflicts between competing views (Perry, 1970). Comprised of early and late phases, multiplicity involves recognizing complexity while lacking the means to fully evaluate competing claims (Perry, 1970).

Position 3: Early Multiplicity/Subordinate. In early multiplicity, students begin to recognize knowledge uncertainties and viewpoints offered rather than facts (Perry, 1970; Elder, 2001). Encountering multiple valid yet diverse perspectives without means for evaluation confuses (Baxter Magolda, 1992). Experts seem less reliable inducing unsettling relativism (Perry, 1981). Students doubt authority figures while lacking evaluation skills (Thoma, 1993). Anxiety results as prior beliefs destabilize before a new framework solidifies (Love & Guthrie, 1999). Some dismiss complexity as "mere opinions" (Perry, 1981). Early multiplicity depicts a turbulent transition as certainty fades but critical reasoning remains emerging (Perry, 1970).

Position 4: Late Multiplicity/Correlate. In late multiplicity, students have largely accepted both the diversity of views within knowledge domains as well as their fundamentally uncertain nature (Perry, 1970). They recognize the importance of actively evaluating claims rather than passively accepting information (Perry, 1981). However, standards for making defensible judgments continue to develop (Elder, 2001). Thought incorporates more context and relativism as ideas are recognized as subjective interpretations rather than objective facts (Baxter Magolda, 1992; Thoma, 1993). Despite these advances, the ambiguity between disparate opinions sometimes leaves students feeling loose (Love & Guthrie, 1999). Thus, late multiplicity is characterized by a growing capacity for relativistic thinking paired with remaining difficulties fully resolving uncertainties, as critical skills are still maturing (Perry, 1981).

Relativism

Relativism marks a pivotal shift as students progress from recognizing knowledge as contextual to comprehending relativism (Perry, 1970). They understand knowledge as diverse interpretations constructed alongside alternatives (Elder, 2001; Baxter Magolda, 1992). Students critically evaluate rather than passively accept new ideas (Perry, 1981). The role of interpretation and how positions are shaped is acknowledged (Baxter Magolda, 1992). Relativism integrates pluralism, interpretation, and contextual judgment into thinking (Thoma, 1993). Knowledge is constructed through evaluating perspectives (Baxter Magolda, 1992). It signifies an important transition to comprehending knowledge as interpreted rather than absolute (Thoma, 1993). This stage comprises two phases of development (Perry, 1970).

Position 5: Contextual Relativism/Correlate. During the transition to relativistic thinking, students begin to understand that knowledge is contextual and subject to interpretation rather than absolutist claims (Perry, 1970). They recognize that even authorities offer opinions informed by evidence rather than perfectly objective facts (Perry, 1981). It involves recognizing that knowledge claims are context-dependent and open to interpretation. Different perspectives offer varying solutions rather than absolute answers. Students learn to consider multiple views and support critically to make reasoned evaluations between alternatives based on evidence for different situations (Perry, 1981). Relativistic thinking replaces dualistic right/wrong with the skilled judgment within pluralism (Thoma, 1993).

Position 6: Development of Relativistic Reasoning/Pre-commitment. Students with relativism recognize knowledge varies by context (Perry, 1981). Their approach weighs perspectives, evidence, and factors nuancedly rather than seeking a single answer (Baxter Magolda, 1992). Relativism signifies shifting from absolute to interpretive knowledge comprehension dependent on frames of reference (Elder, 2001). Students can evaluate issues and claims by applying balanced, interpretive reasoning over dualism (Perry, 1981). Relativism develops coordination and critical assessment skills. It cultivates contextual, interpretive thinking (Perry, 1981). This phase signifies pre-commitment thinking.

Commitment

Commitment, the final stage, involves developing personal values/ beliefs commitments while acknowledging multiple perspectives (Perry, 1981). Students recognize subjective knowledge construction yet make informed judgments (Elder, 2001; Nilson, 2007). They actively engage intellectually and remain open to growth (Nilson, 2007; Perry, 1981). Students understand aligning values with disciplinary criteria for decision-making (Perry, 1981). Not all progress linearly through stages, which are not guaranteed (Magolda, 2014). However, Perry's model provides a useful framework for comprehending cognitive development. This stage incorporates three developmental phases (Perry, 1981).

Position 7: Early Commitment. At the first level of the commitment stage, students begin to make real commitments in terms of their personal direction and values. They realize that it is necessary to commit to certain values, beliefs, and actions even in the face of ambiguity (Perry, 1981). This particular stage encapsulates the transformative shift that occurs when one makes a formal commitment (Elder, 2001). It brings about an ontological change, fundamentally altering one's perception and being. After careful consideration and firsthand experience with various alternatives, the student finally made a decision based on reason and personal knowledge.

Position 8: Mid Commitment. During this stage, the student becomes aware of the significant implications of commitment. They delve into matters of responsibility and carefully consider the impact of their decisions. The stage primarily revolves around the process of learning how to make commitments in various domains and expanding one's capacity to do so in different areas of life (Magolda, 2014). Additionally, the student evaluates the consequences and implications of their commitments and endeavors to resolve any conflicts that may arise (Perry, 1981).

Position 9: Post Commitment. Students reach the realization that commitment is ongoing and evolving, not a single event (Perry, 1981). They understand conflicts may persist and accept perpetual struggles of commitment (Perry, 1981). This stage necessitates delicately balancing multiple commitments with wholehearted devotion (Elder, 2001; Magolda, 2014). Students face integrating ambiguities encountered in relativism seamlessly into daily life (Perry, 1981). Rather than categorizing ambiguities, they are now embraced as part of personal growth (Perry, 1981; Magolda, 2014). Ambiguities become integral to student identity (Magolda, 2014). Students no longer resist uncertainty but embrace it naturally with equanimity, grace, and peace of mind (Elder, 2001).

Application of Perry's Theory to Mathematics Learning

Higher education emphasizes intellectual/ethical growth (Moore, 1989). Cognitive theories like Piaget's aim to understand how individuals make sense of experiences and evolve thinking (Rodgers, 1990). Perry's theory proposes hierarchical progression through stages representing distinct thinking approaches (King, 1978). Mathematics aligns with logical reasoning, patterns, and abstract concepts as in Perry's model (Thanheiser, 2023). Engaging with mathematics encompasses more than simply acquiring knowledge and engaging in cognitive processes (Hodgen *et al.,* 2018). Development generally follows sequence, but time in/between stages can vary (Perry, 1981). Thus, Perry's theory offers insights into student progression in thinking and mathematical problem approaches. Some applications of Perry's theory applicable to mathematics learning are discussed.

Knowledge Progression

Perry's theory proposes intellectual development follows hierarchical stages, with each representing distinct thinking (King, 1978). In mathematics, students transition from concrete to more abstract, formal thinking (King, 1978; Piaget, 1960). Mathematical knowledge consists of interconnected factual, procedural, and conceptual knowledge particularly important for navigating tasks by understanding relationships among facts, procedures, and concepts (Donovan & Bransford, 2005; Hodgen *et al.,* 2018). Initially, Perry's theory suggests students exhibit dualistic thinking, perceiving knowledge as black/white and relying heavily on authority. However, dualistic thinking provides a foundation for critical thinking and meaning-making to tackle more complex tasks (Donovan & Bransford, 2005; Hodgen *et al.,* 2018).

Increasing Complexity and Coherence

In Perry's theory of intellectual development, students progress through stages and develop more complex cognitive structures (Perry, 1981). This progression can be observed in mathematics as students become capable of handling advanced concepts and solving complex problems (Saparbayeva *et al.,* 2024). Prior knowledge has been consistently shown to have a significant impact on the acquisition of new knowledge (Fazio *et al.,* 2016; Byrnes *et al.,* 2018; Kosiol *et al.,* 2019). Alreshidi's research (2023) found that individuals with low initial mathematical knowledge were more likely to encounter difficulties in advancing their understanding of the subject. Perry's theory suggests that students develop foundational knowledge from teachers and gradually improve their learning (Alreshidi, 2023). Perry's theory, with its emphasis on hierarchical intellectual development, aligns with the process of learning mathematics.

Qualitative Growth

Perry's theory highlights intellectual development involves qualitative rather than quantitative growth (Perry, 1981). Mathematical understanding deepens and becomes more sophisticated across stages (Piaget, 1960; Kitchener, 1982; Moore & Hunter, 1993). Piaget emphasized alignment with physical growth, allowing targeted teaching (Piaget, 1960). Progression includes moving from concrete to abstract recognition of uncertainty/probability (Kitchener, 1982). Moore and Hunter (1993) outlined that college students transition from being passive learners of facts and truths to becoming active creators of arguments and knowledge. Early stages focus on procedural memorization while later embrace interpretive, relativistic thinking in concepts (Byrnes *et al.,* 2018). Multiple representations support shifting from concrete to abstract by presenting ideas from diverse angles (Saparbayeva *et al.,* 2024). Perry's framework informs mathematical learning progression.

Dualistic Thinking

Perry's theory highlights that during the early stages of intellectual development, students often exhibit dualistic thinking, where they see knowledge as either right or wrong, and there is a tendency to rely on authorities for answers (Perry, 1981). In mathematics, students in this stage may view mathematical concepts as fixed and rules-based, with little room for interpretation or multiple approaches (Perry, 1981). Educators can help students progress beyond dualistic thinking by encouraging them to explore alternative strategies, consider different perspectives, and engage in problem-solving activities that require critical thinking and reasoning.

Relativistic Thinking

As students progress through the stages of intellectual development, they may enter a stage characterized by relativistic thinking. In this stage, students recognize multiple perspectives and interpretations, including in mathematics. They may question mathematical concepts and seek to understand the underlying principles and assumptions (Perry, 1981). Educators can foster relativistic thinking in mathematics by encouraging students to explore mathematical proofs, analyze different solution methods, and engage in discussions about mathematical concepts and their applications.

Commitment to Understanding

Perry's theory suggests that as students develop intellectually, they move towards a stage where they seek a deeper understanding of knowledge and are willing to invest effort in grappling with complex ideas (Perry, 1981). In mathematics, this can manifest as students' desire to explore mathematical concepts in depth, engage in problem-solving tasks that require perseverance, and actively seek connections between different areas of mathematics (Saparbayeva *et al.,* 2024). Educators can promote this commitment to understanding by providing challenging and thought-provoking mathematical tasks, encouraging student inquiry, and fostering a growth mindset (Alreshidi, 2023; Byrnes *et al.,* 2018).

Developmental Pace and Support

Perry acknowledges that individuals may progress through the stages of intellectual development at different rates, and the time spent within each stage can vary (Perry, 1981). In mathematics learning, students may exhibit different rates of progress and understanding (Piaget, 1960). It is important for educators to provide appropriate support and differentiated instruction to meet the unique needs of each student (Estaiteyeh & DeCoito, 2023). This can include scaffolding, providing additional resources or explanations, and creating opportunities for individualized or small-group instruction (Birnie, 2017). Thus, Perry's theory enhances mathematics learning by promoting cognitive growth, personalized instruction, and a stimulating learning environment.

Interpretation of Educational Experiences

Perry's theory emphasizes the active interpretation of educational experiences. In mathematics learning, students engage in sense-making by connecting concepts to real-world situations, evaluating arguments, and applying thinking to solve problems (Gordon, 1998; Li & Schoenfeld, 2019). Mathematics can represent experiences in sense-making involving problem-solving, reasoning, communication, and modeling (Li & Schoenfeld, 2019). Previously, mathematics was often seen as self-contained knowledge with ideal forms not necessarily aligned with perception. As cited by Li and Schoenfeld (2019), Aristotle viewed mathematicians as developing concepts by idealizing experiences with objects. Appropriate experiences can thus contribute to structuring/systematizing formal mathematics (Li & Schoenfeld, 2019).

Relevancies of Perry's Theory

Perry's scheme over 50 years ago remains applicable as it qualitatively describes student thinking evolution and understanding of knowledge, highlighting social-constructivist dimensions of learning (Magolda & King, 2012). Identifying developmental levels helps shape instruction, content, and support tailored to student capacities (Gordon, 1998). The theory remains relevant for education requiring critical reasoning, abstract thinking, knowledge construction, contextualization, and perspective synthesis. It informs curriculum design like capstones. Overall, Perry's framework continues enhancing student intellectual and holistic growth in higher education by understanding development (Baxter

Magolda & King, 2012; Gordon, 1998). The relevancies of Perry's theory are discussed briefly.

Insights for Addressing Challenges

Perry's theory recognizes dualistic views and sees mathematics as absolute rules rather than interpretation, hindering creative and contextual understanding (Perry, 1970; Ernest, 1998). It informs overcoming challenges like viewing mathematics dualistically (Gordon, 1998). Introducing concepts without considering intellectual development can cause disjointed understandings (Perry, 1988). Perry highlights aligning instruction and content with evolving epistemologies (Gordon, 1998). He acknowledged disequilibrium, ambiguity, and multiple solutions as important to growth accommodation (Belenky *et al.,* 1986). In mathematics, generating productive struggle in a supportive environment can help students surmount hurdles to deeper conceptual grasp.

Scaffolding Strategies

Perry's theory informs effective scaffolding strategies like gradual release of responsibility shifting from direct to student-centered, relativistic thinking (Vygotsky, 1978). It utilizes structured supports like modeling, cues, and questions to navigate dualistic-relativistic transitions through productive struggle (van de Pol *et al.,* 2010). Collaborative problem-solving cultivates coordination skills through diverse approaches (Nilimaa, 2023). Reflective discussion and articulating reasoning allow for internalizing relativism (Pantziara & Philippou, 2012). Inquiry/project-based curricula construct knowledge by engaging frameworks (Gordon, 1998). Perry emphasizes formative assessment that provides feedback to personalize scaffolding aligned with different levels (McManus, 2008).

Facilitating Cognitive Development

Perry's theory facilitates intellectual development hierarchically from concrete to abstract, known to unknown, specific to general aligning with mathematics learning. It identifies disequilibrium as necessary to spark new framework accommodation, informing appropriate challenges (Gordon, 1998). The scaffolding support guides the students in conceptual and productive struggle (van de Pol *et al.,* 2010). It links contextual reality in concepts to familiar experiences providing intellectual support to build abstract understanding (Pantziara & Philippou, 2012). It develops cognition and metacognition encouraging reflection/articulation and allowing internalizing relativistic meanings (Gordon, 1998; Pantziara & Philippou, 2012). Inquiry approaches foster more student-driven relativistic thinking through active knowledge construction (Gordon, 1998). It emphasizes instruction according to learner perspectives and facilitates new concept assimilation through productive struggle and meaning-making (Wankat & Oreovicz, 2015).

Developing Reflective Learners

Perry's theory of intellectual and ethical development emphasizes the cultivation of reflective learners, tracing a progression from dualistic to relativistic thinking (Perry, 1970; Gordon, 1998; Perry, 1981). This developmental progression aligns with the learning process in advanced mathematics, where students transition from concrete rules to abstract principles (Lyons, 2010). Advanced stages of development involve commitment, paralleling the process of independently solving open-ended mathematics problems and developing mathematical maturity (Conley *et al.,* 2011). Teachers play a crucial role in facilitating student development by presenting alternative problem-solving strategies, respecting multiple approaches, and discussing the complexities of knowledge (Holma & Hyytinen, 2015; Moore, 2002). Perry's theory provides a framework for understanding how higher-level mathematics students develop sophisticated epistemological views and cultivate reflective thinking, which is essential for advanced mathematics.

Development of Critical Thinking

Perry's theory emphasizes critical thinking for mathematics learning (Perry, 1981; Gordon, 1998). It describes a progression from dualistic to commitment aligning with critical thinking skills like questioning assumptions and considering perspectives (Moore, 2002). Advanced stages involve critically evaluating despite uncertainty (Gordon, 1998). Encouraging reflection examines reasoning and cultivates critical thinking (Gordon, 1998). Reflective exercises foster abilities like recognizing flaws and evaluating arguments (Gordon, 1998). Presenting paradoxes/counterexamples promotes reconciling uncertainty/conflict strengthening evaluation (Lyons, 2010). Thus, Perry provides insights into facilitating the development of critical thinking skills necessary for advanced mathematics competence.

Rote vs Conceptual Learning

In Perry's theory, early dualism aligns with rote memorization while later relativism involves understanding concepts (Perry, 1981; Gordon, 1998; Moore, 2002). Rote learners perceive discrete facts whereas conceptual learners recognize interrelationships/patterns (Perry, 1970). Conceptual teaching like reflection promotes relativism by organizing knowledge meaningfully (Moore, 1989). Rote learning stems from anxiety while conceptual learning embraces ambiguity/viewpoints (Perry, 1970). Contextualizing concepts through problems fosters deeper understanding and reflecting commitment (Moore, 2002; Perez *et al.,* 2014). Perry's model supports conceptual mathematics pedagogy by emphasizing meaning-making, ambiguity, reflection, and recognizing active knowledge construction roles (Perry, 1970; Moore, 2002). This facilitates conceptual learning and higher-order knowledge development.

Designing Curriculum and Pedagogy

Perry's theory provides insights for effective mathematics curriculum and pedagogy. It suggests facilitating the transition from dualistic to relativistic thinking by exploring exceptions, ambiguity, and knowledge construction (Moore, 2002; Gordon, 1998). The curriculum aims to sequentially develop a deeper understanding of commitment. Mathematics involves pattern recognition, generalization, and logical reasoning skills and nurtures abstract thinking as emphasized by John Locke. Therefore, the curriculum should sequence from concrete to abstract rules to integrate principles, maximizing cognitive development (Perez *et al.,* 2014). Pedagogies like problem-based learning, reflection, and discussing viewpoints encourage taking ownership of knowledge and commitments within uncertainty, reflecting advanced stages (Tursynkulova *et al.,* 2023). Formative assessment also emphasized, providing feedback to support conceptual changes in mathematics assessment (Gordon, 1998).

CONCLUSION

Perry's theory offers a valuable framework for fostering effective higher-level mathematics learning. The developmental stages align with the cognitive shifts needed to progress in mathematics. Perry's model informs pedagogical practices and assessments that support conceptual growth, such as scaffolding, reflective exercises, and exposure to alternative perspectives. Formative assessments and an inclusive classroom climate are also well-aligned with Perry's theory. Examining mathematics education through this the lens provides insights into nurturing cognitive development and producing strong, independent thinkers ready for higher-level mathematics. Similar to Piaget, Perry recognized cognitive dissonance as a driver of growth. However, Perry focused more on shifts from simplistic to interpretive, relativistic understanding and personal perspective formation. His examinations emphasized comprehending concepts within one's abilities and learning as a progressive internalization of complex ways of knowing, making meaningful contributions regarding changing views of knowledge over the course of education.

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