Risk Behavior of Different Weekdays in NEPSE Index

Kiran Kumar Shrestha[1] and Rabindra Kayastha[2*]

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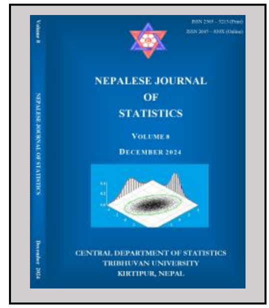

ABSTRACT

Background: In some societies some day of week is favored for financial transactions over other days. In this regard, in this research work, it is intended to study effect of different days of week in Nepal Stock Exchange (NEPSE) index.

Objective: This study aims to assess effect of different days of week on NEPSE index.

Materials and Methods: Data on closing indices of NEPSE, available online on official website of NEPSE are used for analysis purpose. Trading activities in NEPSE market runs on Sunday, Monday, Tuesday, Wednesday and Thursday and remains closed on Friday and Saturday. To observe the difference in NEPSE indices on different trading days at first ARIMAX model is applied with different weekdays as exogenous variable. Since the volatility in values are observed to be clustered, so GARCH model is implemented to describe variances. The means and variances predicted by the models are used to identify value-at-risk and expected-shortfall on different days of week and by observing risk in terms of these measures effect of different weekdays on NEPSE market is assessed.

Results: The optimum model resulted for Sunday as exogenous variable is ARIMAX (1, 1, 1) + GARCH (0, 2). For Monday, it is ARIMAX (3, 1, 3) + GARCH (0, 4). Similarly, for Tuesday, Wednesday and Thursday, ARIMAX (1, 1, 1) + GARCH (0, 5), ARIMAX (3, 1, 3) + GARCH (0, 2) and ARIMAX (1, 1, 1) + GARCH (2, 3), respectively are found as the optimum models. Next, values of value-at-risk on these days, calculated as 95% quantile of residuals of corresponding models are found to be 18.07, 17.86, 18.09, 17.86 and 21.86 on Sunday, Monday, Tuesday, Wednesday and Thursday, respectively. Similarly, expected shortfall on these respective days, calculated as mean of values below value-at-risk, are found to be 38.23, 38,29, 38.23, 38.28 and 38.74, respectively.

Conclusion: There is no noticeable effect of different days of week on NEPSE index when viewed with the aspect of value-at-risk and expected shortfall.

Keywords: ARIMA, ARIMAX, expected-shortfall, GARCH, NEPSE, value-at-risk.

Address correspondence to the author: Department of Mathematics, School of Science, Kathmandu University, Dhulikhel, Nepal[1, 2]. Email: kiran.shrestha@ku.edu.np[1]; rkayastha@ku.edu.np [2*](Corresponding author email) [2*]

INTRODUCTION

Fluctuations in prices of different commodities with reference to time is a hot subject of study for many researchers and investors since past few decades. The mathematical description of fluctuations in values of a time series observations started from workings of George Box and Gwilym Jenkins (Box et al., 1982). The approach they initiated is termed as Autoregressive Integrated Moving Average (ARIMA) model. ARIMA model is able to describe autocorrelation among different observations of a time dependent data. This model assumes that residuals are uncorrelated random variables with zero mean and some finite variance and which does not vary with time. However, this model cannot encompass different external variables that may influence the time related financial values. In this regard, ARIMA with exogenous inputs model (ARIMAX model) was developed to extend the functionality of ARIMA framework by integrating exogenous variables, which are external factors that can influence the time series being studied. This integration allows the model to leverage additional information that can significantly enhance forecasting accuracy. Pankratz (2012) refers to the ARIMAX model as dynamic regression model. According to Dickey and Fuller (1979), if the assumption of the constancy in variance of ARIMA and ARIMAX model is violated then estimates of parameters will be inefficient and significance of coefficients will be invalid. To encompass the concept of heteroscedasticity in time stamped data Autoregressive Conditional Heteroscedastic (ARCH) model of Engle (1982) and Generalised Autoregressive Conditional Heteroscedastic (GARCH) model of Engle and Bollerslev (1986) were introduced. In this regard, it is attempted to observe whether investors in Nepal Stock Exchange (NEPSE) market favor some particular day of week for transactional activities of stock with respect to other days by using ARIMAX model with different days of week as exogenous variables. Moreover, assumption of homoscedasticity of variance is tested and is described by using GARCH model.

Objective

The objective of this research work is to find whether there is noticeable difference in NEPSE index on different days of week, i.e., whether participants of NEPSE market prefer some day of week over others days.

Literature review

A large number of research work is going on with modeling mean behavior and volatility behavior of financial markets. Some studies are focused on data on indices of some volume, some are related to price and some are related to fluctuations, i.e., returns. The study of price fluctuations in financial activities is considered to be started with researches on efficient market by Fama (1970) who mentions that a market in which prices always "fully reflect" available information is called "efficient". Pollet and Wilson (2010) argues that changes in stock market risk holding average correlation constant can be interpreted as changes in the average variance of individual stocks. Such changes have a negative relation with future stock market excess returns. Attempts to identify a variance-in-mean relationship in financial observations are carried by number of researchers including Corrado and Miller Jr (2006), Campbell (2007), French (1987) and so on. Describing volatility pattern of stock market using GARCH model is carried by Emenike (2010) in article entitled "Modelling stock returns volatility in Nigeria using GARCH models" and explained that modelling volatility will improve the usefulness of stock prices as a signal about the intrinsic value of securities, thereby, making it easier for firms to raise fund in the market.

Bollerslev et al. (2018) states that the most critical feature of the conditional return distribution is arguably its second moment structure, which is empirically the dominant time-varying characteristic of the distribution. According to Ladokhin (2009) the main characteristic of any financial asset is its return which is typically considered to be a random variable. The spread of outcomes of this variable, known as assets volatility, plays an important role in numerous financial applications. In the same way, Carroll and Kearney (2009) described that volatility as a phenomenon as well as a concept remains central to modern financial markets and academic research. The link between volatility and risk has been to some extent elusive, but stock market volatility is not necessarily a bad thing. Ade (2023) argues in article entitled "The Role of Exogenous Variables in Time Series Forecasting of Economic Indicators" that future research should continue to explore the impact of a broader array of exogenous factors and employ innovative modeling techniques to enhance predictive performance. The importance of including exogenous variables is also highlighted in conference paper "A Methodology for Calculating the Contribution of Exogenous Variables to ARIMAX Predictions", by Wang et al. (2021) by proposing the approach that accumulates the contributions from past values with calculated weights and eliminates collinearity between exogenous variables, which makes it efficient and feasible for local interpretation, and further provides global understanding of the behavior of an ARIMAX model.

MATERIALS AND METHODS

Data source and description

Data required for analysis purpose are obtained from www.nepalstock.com, the official website of Nepal Stock Exchange (NEPSE) market. Day-wise closing indices available online on this website are used for analysis. Nepal Stock Exchange market opens for 5 days on a week starting from Sunday to Thursday and remains closed on Friday and on Saturday. To assess effect of different weekdays in stock market, 6209 number of closing indices starting from 1997-07-20 to 2024-07-04 are utilized. A glimpse of indices used for the study are presented in Fig. 1.

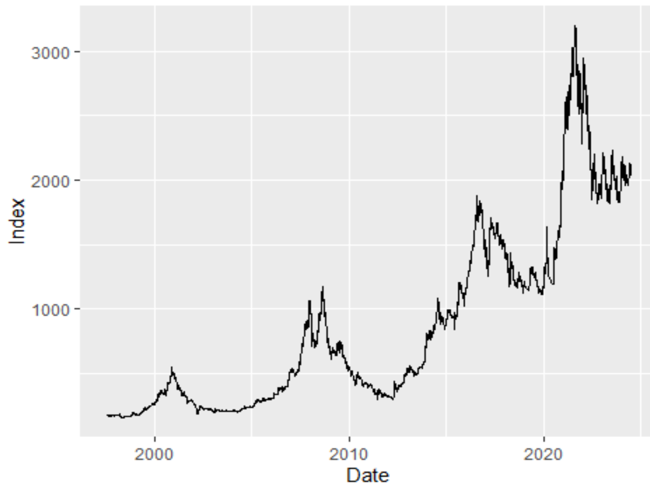

Fig. 1. Plot of indices used for study.

First of all mean of indices as well as respective variations, in term of standard deviation and risk, in term of coefficient of variation on different days of week are observed and analyzed. As these results are obtained by considering different observations of indices to be independent of one another, so no valid conclusion can be ascertained. As such to describe dependencies and correlations in observations ARIMA/ARIMAX models are developed with different weekdays as exogenous variables.

ARIMA / ARIMAX model

In ARIMA models, AR components consider lagged values of different orders as regressors. Similarly, MA components consider errors of one or more lags as regressors. To stationarize the observations successive differences of one or more orders are also considered and they are integrated with AR and MA part and this form is called ARIMA model. If 'p' number of lagged variables $y_{t-1}, y_{t-2}, \ldots, y_{t-p}$, 'q' number of lagged errors $\epsilon_{t-1}, \epsilon_{t-2}, \ldots, \epsilon_{t-q}$ and differencing of order 'd' are considered then in ARIMA (p, d, q) model the value of current observation y_t is given by (1).

$$\nabla^d y_t = \alpha + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} + \cdots + \theta_q \epsilon_{t-q} + \epsilon_t \qquad (1)$$

where $\nabla^d y_t = \sum_{j=0}^{d} \binom{d}{j} (-1)^j y_{t-j}$ and $\alpha, \phi_1, \phi_2, \cdots, \phi_p; \theta_1, \theta_2, \cdots, \theta_q$ are parameters to be estimated. Moreover, in ARIMA (p, d, q) model if 'k' number of exogenous variables $X_1, X_2, \cdots, X_k$ are considered as regressors then ARIMAX (p, d, q) model is expressed as (2).

$$\nabla^d y_t = \alpha + \phi_1 (y_{t-1} - y_{t-2}) + \phi_2 (y_{t-2} - y_{t-3}) + \cdots + \phi_p (y_{t-p-} - y_{t-p+1}) + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} + \cdots + \theta_q \epsilon_{t-q} + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_k X_k + \epsilon_t \qquad (2)$$

Here, $\beta_1, \beta_2, \cdots, \beta_k$ are coefficients of exogenous variables $X_1, X_2, \cdots, X_k$. Also, ϵ_t is error associated with y_t and it is assumed that $\epsilon_t{}'s$ are white noise process with 0 mean and some constant variance σ^2, i.e., $\epsilon_t \sim WN(0, \sigma^2)$. To fit ARIMA models observations should be stationary. To observe stationarity of observations plots of NEPSE indices are drawn separately for different days of week. There are a number of formal tests that can be used to test stationarity of observations. These tests include Augmented Diskey-Fuller (ADF) test, Phillips-Perron test, Elliott-Rothenberg-Stock test, Schmidt-Phillips test, Zivot-Andrews test. In this research ADF test (Dickey & Fuller, 1979) is applied since it attempts to find presence of unit root in differenced values, whereas, other tests, for example Kwiatkowski Phillips Schmidt and Shin test (KPSS) tries to find whether values are stationary around a deterministic trend. ADF test is augmented form of Dickey-Fuller test which observes presence of unit root in $AR(1)$ process by using statistics given in Equation (3).

$$t_0 = \frac{\hat{\phi} - 1}{s.e.(\hat{\phi})} \qquad (3)$$

where $\hat{\phi}$ is autoregressive coefficient of $AR(1)$. ADF-test is extended form of Dickey-Fuller test to include more autoregressive terms. Another aspect of fitting of ARIMAX (p, d, q) model is to determine the values of orders of parameters 'p', 'd' and 'q'. Different techniques and approaches are in practice for this purpose. Traditionally, they are identified by observing autocorrelation function (ACF) plot and partial autocorrelation function (PACF) plot. Many researchers observe a number of models with different levels of 'p', 'd' and 'q' and select the optimum one with minimum value of some information criteria such as Akaike's Information Criteria (AIC) or Akaike's Information Criteria Corrected (AICc) or Bayesian Information Criteria (BIC). In this research work, for selecting the optimum ARIMAX model, Hyndman-Khandakar algorithm (Hyndman, 2014), which combines unit root tests, minimization of the AICc and maximum likelihood estimate (MLE), is used. ARIMA / ARIMAX model assumes that residuals are uncorrelated and are homoscedastic. The assumption of uncorrelatedness is validated by applying 'Ljung-Box' test to the residuals of separate models for different days of week. The assumption of homoscedasticity is observed by using ACF-plots as well as PACF-plots of square of residuals of the models. Formally, homoscedasticity is tested by using Autoregressive Conditional Heteroscedasticity (ARCH) test. ARCH test is generally used to observe presence of volatility clustering, a concept used in time series data to observe whether large changes in observations are followed by a large changes in variance and small changes in observations are followed by small changes in the variance. The common test applied to detect volatility clustering in observations is Engle's ARCH test and it is performed to observe the presence of heteroscedasticity.

GARCH / GARCHX model

If heteroscedasticity is observed in time series data it is necessary to model the variance of residuals of ARIMA / ARIMAX models. In GARCH (r, s) modeling technique, 'r' number of squared past observations and 's' number of past conditional variances are used to describe current variance. Here, 'r' and 's' are called orders of GARCH model. GARCH (r, s) model is described mathematically by Equation (4).

$$y_t = \sigma_t . \eta_t$$
$$\sigma_t^2 = \omega + \sum_{i=1}^{r} \alpha_i \, y_{t-i}^2 + \sum_{j=1}^{s} \beta_i \, \sigma_{t-i}^2 \qquad (4)$$

where σ_t^2 is the variance at time t, $\sigma_{t-i}^2; i = 1,2,...,s$ are variances at lagged times and η_t is residual at time t and it is assumed to be white noise process with mean 0 and variance 1, i.e., $\eta_t \sim N(0,1)$. Moreover, $\omega, \alpha's$ and $\beta's$ are coefficients of the model. As ARIMAX models assume variance to be constant, in the same way GARCH models assume mean to be constant. In current research it is attempted to describe both time varying mean as well as time varying variance, so ARIMAX and GARCH models are merged together to get a hybrid form of ARIMA (p, d, q) + GARCH (r, s) model which is represented by Equation (5).

$$\nabla^d y_t = \alpha + \sum_{i=1}^{p} \phi_i (y_{t-i} - y_{t-i+1}) + \sum_{j=1}^{q} \theta_j \epsilon_{t-j} + \sum_{k=1}^{k} \gamma_k X_k + \epsilon_t; \epsilon_t \sim WN(0, \sigma_t^2)$$
$$\epsilon_t = \sigma_t \eta_t; \eta_t \sim N(0,1)$$
$$\sigma_t^2 = \omega + \sum_{i=1}^{s} \alpha_i y_{t-i}^2 + \sum_{j=1}^{r} \beta_j \sigma_{t-j}^2 \qquad (5)$$

The first and foremost step in fitting of GARCH model is to determine the values of orders 'r' and 's'. In the literature of GARCH, the orders are determine by fitting ARIMA model to the squared returns (difference between successive values) of observations and then order of AR-component is considered as the value of 'r' and the order of MA-component is considered as the value of 's'. In the first attempt of fitting of ARIMAX-GARCH models residuals were assumed to be normally distributed with mean 0 and variance 1. But the validation process does not confirm to the assumption. So process of fitting ARIMAX-GARCH model was repeated separately for different weekday by assuming residuals to have t-innovation and this assumption is validated by displaying QQ-plots of residuals of model developed. The degrees of freedom associated with t-innovations of residuals are obtained by using the fact that the kurtosis of t-distribution with 'n' degrees of freedom is $3(n-2)/(n-4)$.

Value at risk (VaR) and expected shortfall on different weekdays

Value-at-risk is a statistical measure of the riskiness of financial entities or portfolios of assets. It is defined as the maximum amount expected to be lost over a given time horizon, at a pre-defined confidence level (Lu et al., 2022). Since the residuals of ARIMAX-GARCH models are observations from which all forms of dependencies are removed they are used to measure one-day value at risk of investing on different days of week. Expected shortfall is defined as the average of all of the returns that are worse than the Value at Risk at a given level of confidence. Nadarajah et al. (2014) states that since value at risk suffers from a number of drawbacks as measure of financial risk, alternative measure referred to as expected shortfall was introduced in late 1990s to circumvent these drawbacks. For the calculation of value-at-risk and expected-shortfall, ARIMAX-GARCH model for different days of week are used to estimate variance on the next day at 5% level with necessary adjustment for t-distribution with specified degrees of freedom. Similarly, for calculation of expected shortfall average of all residuals that are smaller than value-at-risk is considered.

Tools used for analysis

For data analysis purpose **RStudio** (R Core Team, 2022), the statistical analysis programming language, is used. For visual presentation of data **ggplot2** package (Wickham, 2016) of RStudio is used. For data manipulation purpose **dplyr** (Wickham et al., 2023a), **readr** (Wickham et al., 2023b), **tidyr** (Wickham et al., 2023c) packages are used. Similarly, for tabular presentation of results **knitr** (Xie, 2023), **kableExtra** packages are used. For time-series analysis, modeling as well as for forecasting purpose **tseries** (Trapletti & Hornik, 2020), **forecast** (Hyndman et al., 2023) and **rugarch** (Galanos, 2023) are used.

RESULTS

The result of calculation of mean, standard deviation and coefficient of variation of indices grouped by different days of week are presented in descending order of mean in Table 1 and these information are also illustrated in Fig. 2 where the height of different bars corresponds to mean and length of vertical lines at the top of each bar corresponds to standard deviation.

Table 1. Summarized statistics grouped by days.

Day	Mean	Standard deviation	Coefficient of variation
Sunday	1022.2048	713.5928	69.80918
Wednesday	874.5441	714.9899	81.75572
Tuesday	873.0018	720.3080	82.50934
Thursday	863.9226	710.0149	82.18501
Monday	861.9567	707.5231	82.08338

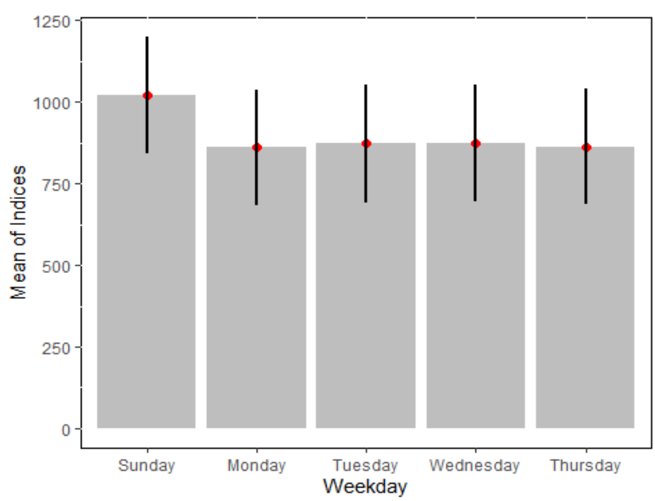

Fig. 2. Plot of mean and variation of index on different days of week.

In fact, these results are obtained by considering that observations of indices are independent of one another. Since financial observations are usually not independent and are correlated so no valid conclusion can be drawn using these results. To describe dependencies in observations ARIMA models are implemented. The plots of indices for different days of week exhibited in Fig. 3 clearly indicates non-stationarity of observations. The result of carrying ADF-test to examine stationarity of indices of different weekdays as shown in Table 2 also point the fact that indices on different days of week are not stationary.

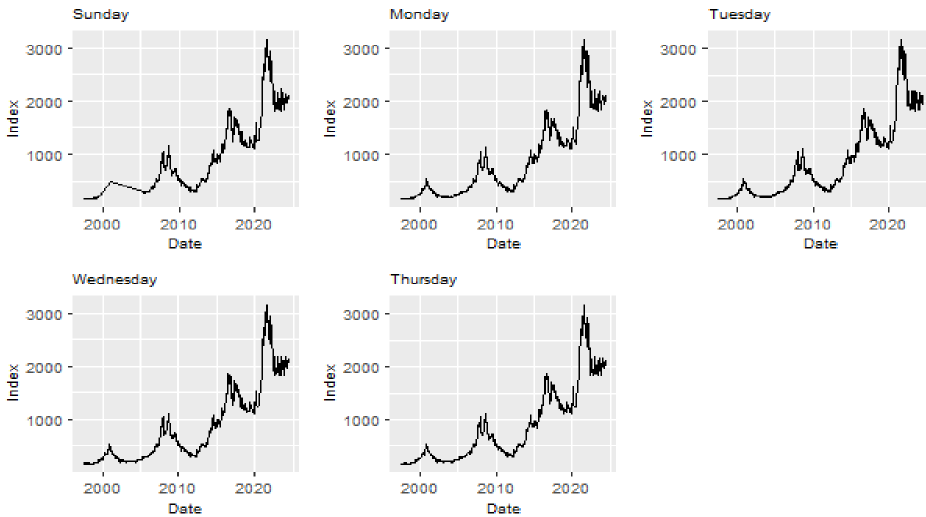

Fig. 3. Plots of indices for different days of week.

Table 2. Result of applying ADF-test on indices of different weekdays.

Weekday	p-Value
Sunday	0.3476124
Monday	0.3784918
Tuesday	0.3032363
Wednesday	0.3851223
Thursday	0.3138612

The result of fitting ARIMAX models to study data with different days of week as exogenous variable separately with AICc values and amount of residual variances of the optimum model are shown in Table 3. The models thus identified are expressed in mathematical form using Equations (6) through (10) for different weekdays.

Table 3. Optimum models with different days of week as exogenous variable.

Exogenous variable	Optimum model	AICc	Residual variance
Sunday	ARIMAX(1,1,1)	51715.46	242.6459
Monday	ARIMAX(3,1,3)	51686.33	241.3916
Tuesday	ARIMAX(1,1,1)	51715.87	242.6618
Wednesday	ARIMAX(3,1,3)	51688.70	241.4836
Thursday	ARIMAX(1,1,1)	51713.45	242.5674

ARIMAX Model with **Sunday** as exogenous variable:

$$y_t - y_{t-1} = 0.314 - 0.4257(y_{t-1} - y_{t-2}) + 0.5746\epsilon_{t-1} - 0.2484Sunday + \epsilon_t \qquad (6)$$

ARIMAX Model with **Monday** as exogenous variable:

$$\begin{aligned} y_t - y_{t-1} &= -0.2899(y_{t-1} - y_{t-2}) + 0.5209(y_{t-2} - y_{t-3}) + 0.4899(y_{t-3} - y_{t-4}) \\ &\quad + 0.4342\epsilon_{t-1} - 0.5412\epsilon_{t-2} - 0.5159\epsilon_{t-3} - 0.5671Monday + \epsilon_t \qquad (7) \end{aligned}$$

ARIMAX model with **Tuesday** as exogenous variable:

$$y_t - y_{t-1} = 0.3139 - 0.4254(y_{t-1} - y_{t-2}) + 0.5747\epsilon_{t-1} - 0.0539Tuesday + \epsilon_t \qquad (8)$$

ARIMAX model wtih **Wednesday** as exogenous variable:

$$\begin{aligned} y_t - y_{t-1} &= -0.3061(y_{t-1} - y_{t-2}) + 0.5202(y_{t-2} - y_{t-3}) + 0.4985(y_{t-3} - y_{t-4}) \\ &\quad + 0.4502\epsilon_{t-1} - 0.5374\epsilon_{t-2} - 0.5254\epsilon_{t-3} + 0.334Wednesday + \epsilon_t \qquad (9) \end{aligned}$$

ARIMAX model with **Thursday** as exogenous variable:

$$y_t - y_{t-1} = 0.3139 - 0.4268(y_{t-1} - y_{t-2}) + 0.576\epsilon_{t-1} + 0.4555Thursday + \epsilon_t \qquad (10)$$

It is observed that all the coefficients of respective models are significant. The result of observing assumption of uncorrelatedness of model residuals using 'Ljung-Box' test are presented in Table 4.

Table 4. Validation of fitted ARIMAX models using Ljung-Box test.

Weekday	p-value
Sunday	0.7423475
Monday	0.9558282
Tuesday	0.7234092
Wednesday	0.9663497
Thursday	0.7341755

The resulting p-values of 'Ljung-Box' test implies that residuals of the fitted models are indeed uncorrelated. The constancy in variance, i.e., homoscedasticity, observed graphically using plots of residuals, ACF plots of squared residuals as well as PACF plots of squared residuals are presented in Figures 4, 5 and 6, respectively.

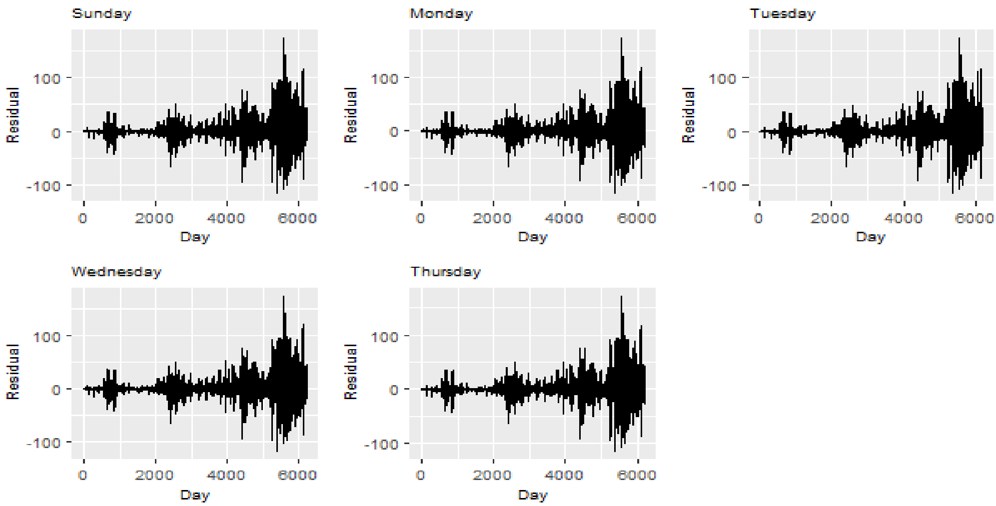

Fig. 4. Residual plots of models with different days of week as exogenous variable.

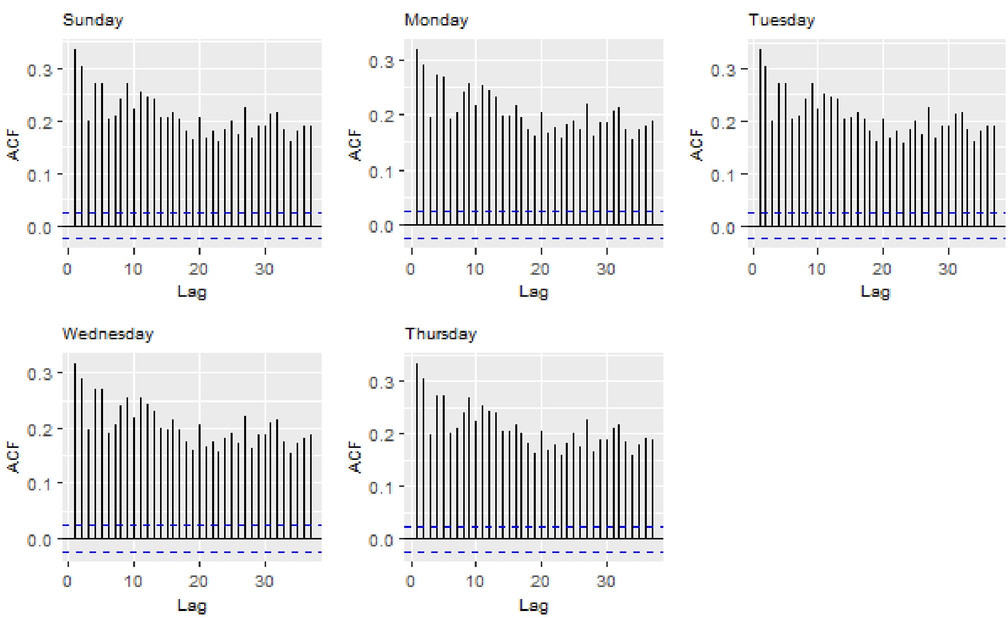

Fig. 5. ACF plots squared residual of models with different days of week as exogenous variable.

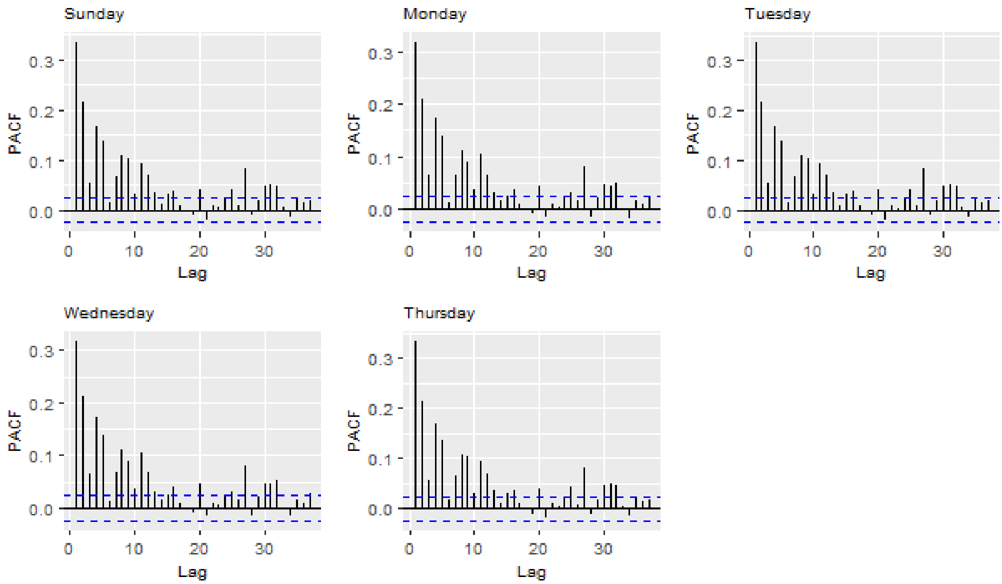

Fig. 6. PACF plots squared residual of models with different days of week as exogenous variable.

The p-values of Engle's ARCH test for indices on different weekdays starting from Sunday to Thursday are observed to be 8.649243e-194, 8.547239e-255, 1.500364e-252, 1.345995e-255 and 1.631318e-253, respectively. Negligibly small p-values resulted in Engle's ARCH test is the indication of the presence of heteroscedasticity in observations. Thus the assumption of homoscedastic nature of residuals is not resulted to be true and so Generalized Autoregressive Conditional Heteroscedasticity (GARCH) models are developed to describe changes in variance over time. The orders 'r' and 's' of GARCH model, determined after fitting ARIMA model to squared daily fluctuations of indices separately for different weekdays are presented in Table 5.

Table 5. GARCH orders for different weekdays.

Weekday	r-Order	s-Order
Sunday	0	2
Monday	0	4
Tuesday	0	5
Wednesday	0	2
Thursday	2	3

The ARIMAX-GARCH model developed with different days of week as exogenous variable separately are described in Equations (11), (12), (13), (14) and (15). It is to be mentioned here that residuals are assumed to have t-innovation with degree of freedom mentioned in the equations. Moreover, non-significant coefficients are not mentioned in the equations.

ARIMAX-GARCH model with **Sunday** as regressor:

$$y_t - y_{t-1} = 0.1071 - 0.1287(y_{t-1} - y_{t-2}) + 0.303\epsilon_{t-1} - 0.1189 Sunday + \epsilon_t$$
$$\epsilon_t \sim WN(0, \sigma_t^2)$$
$$\epsilon_t = \sigma_t \eta_t ; \eta_t \sim t_{2.1}$$
$$\sigma_t^2 = 0.1371\sigma_{t-1}^2 + 0.8636\sigma_{t-2}^2 \qquad (11)$$

ARIMAX-GARCH model with **Monday** as regressor:

$$y_t - y_{t-1} = 0.019 + 0.0627(y_{t-1} - y_{t-2}) + 0.6231(y_{t-2} - y_{t-3}) + 0.28(y_{t-3} - y_{t-4}) + 0.102\epsilon_{t-1} - 0.6578\epsilon_{t-2} - 0.355\epsilon_{t-3} + 0.1365 Monday + \epsilon_t$$
$$\epsilon_t \sim WN(0, \sigma_t^2)$$
$$\epsilon_t = \sigma_t \eta_t ; \eta_t \sim t_{2.1}$$
$$\sigma_t^2 = 1\sigma_{t-2}^2 \qquad (12)$$

ARIMAX-GARCH model with **Tuesday** as regressor:

$$y_t - y_{t-1} = 0.068 - 0.1285(y_{t-1} - y_{t-2}) + 0.3027\epsilon_{t-1} + 0.1285 Tuesday + \epsilon_t$$
$$\epsilon_t \sim WN(0, \sigma_t^2)$$
$$\epsilon_t = \sigma_t \eta_t ; \eta_t \sim t_{2.1}$$
$$\sigma_t^2 = 1\sigma_{t-2}^2 \qquad (13)$$

ARIMAX-GARCH model with **Wednesday** as regressor:

$$y_t - y_{t-1} = 0.0334 + 0.0196(y_{t-1} - y_{t-2}) + 0.6586(y_{t-2} - y_{t-3}) + 0.2869(y_{t-3} - y_{t-4}) + 0.1452\epsilon_{t-1} - 0.6855\epsilon_{t-2} - 0.3685\epsilon_{t-3} + 0.1365 Wednesday + \epsilon_t$$
$$\epsilon_t \sim WN(0, \sigma_t^2)$$
$$\epsilon_t = \sigma_t \eta_t ; \eta_t \sim t_{2.1}$$
$$\sigma_t^2 = 0.0012\sigma_{t-1}^2 + 0.9996\sigma_{t-2}^2 \qquad (14)$$

ARIMAX-GARCH model with **Thursday** as regressor:

$$y_t - y_{t-1} = 0.0555 + 0.2814(y_{t-1} - y_{t-2}) - 0.0231\epsilon_{t-1} + 0.0195 Thursday + \epsilon_t$$
$$\epsilon_t \sim WN(0, \sigma_t^2)$$
$$\epsilon_t = \sigma_t \eta_t ; \eta_t \sim t_{2.8}$$
$$\sigma_t^2 = 0.0867 + 0.7392y_{t-1}^2 + 0.5011\sigma_{t-1}^2 + 0.1376\sigma_{t-3}^2 \qquad (15)$$

Degrees of freedom of associated t-distribution for different days of week are resulted to be 2.1 for Sunday, 2.1 for Monday, 2.1 for Tuesday, 2.1 for Wednesday and 2.8 for Thursday. The result of performing validation of fitted models by using QQ-plots for different weekdays are presented in Fig. 7. The result of calculation value-at-risk and expected shortfall for the next at 95% confidence level using forecast of variance obtained from ARIMAX-GARCH model and using t-innovation for residuals are exhibited in Table 6.

Table 6. Comparison of risk on different weekdays in terms of VaR and ES.

Weekday	Value at Risk (VaR)	Expected Shortfall (ES)
Sunday	18.07877	38.23594
Monday	17.86420	38.29189
Tuesday	18.09298	38.23818
Wednesday	17.86046	38.28489
Thursday	21.86019	38.74582

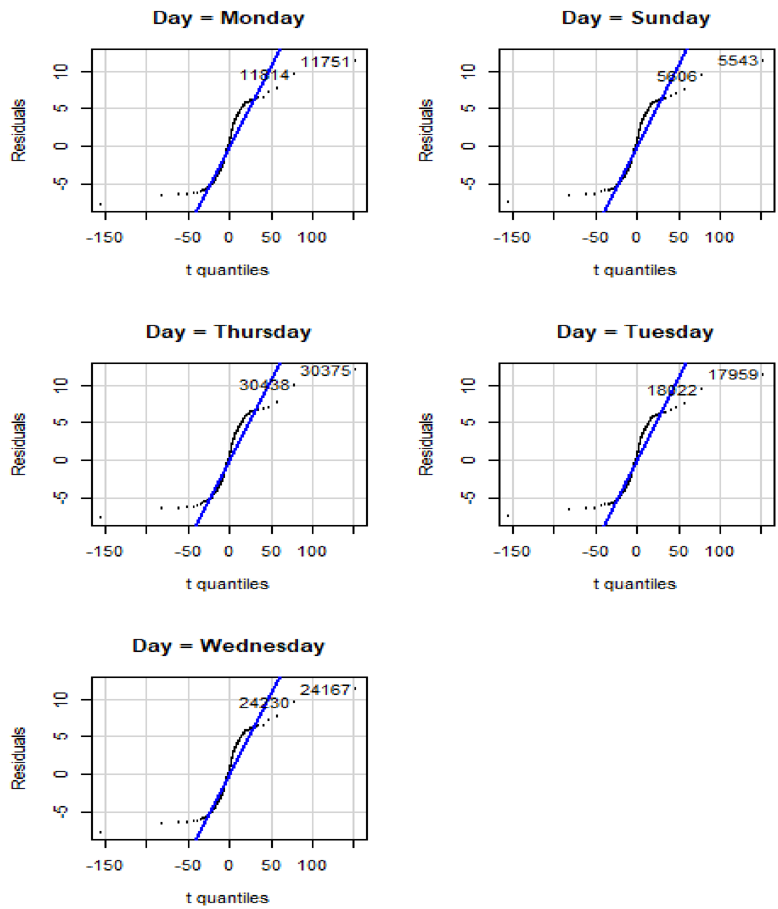

Fig. 7. QQ plot of residuals of GARCH models for different days of week.

DISCUSSION

Observing summarized mean, standard deviation and variance of indices on different days of week, shown in Table 5.1, it can be revealed that mean of NEPSE index is relatively higher on Sunday in comparison to other days of week. Similarly, the value of coefficient of variation of the index on Sunday is relatively smaller than that on rest of the days. Thus assuming different values of indices to be uncorrelated Sunday is observed to possess less risk in comparison to other days. Next, observing ARIMAX-GARCH models for different days of week, as they can describe both mean as well as volatile behavior, it is revealed that there is noticeable rise in value of indices on Monday, Tuesday and on Wednesday. On Thursday there is minor rise in value and on Sunday there is minor fall in the value of NEPSE index. However, observing coefficients of weekdays used as exogenous variables of ARIMAX model, it can be said that there is no much effect of weekdays on NEPSE index. Moreover, since the coefficients of GARCH components with weekdays as exogenous variable in ARIMAX-GARCH models are observed to be non-significant on all weekdays, it can be said that the fluctuations in variances of index on different days of week are not noticeably different. Finally, observing QQ-plots of residuals of ARIMAX-GARCH model it is revealed that using t-innovation for residuals of the model for NEPSE indices is more reliable than that using normal innovation. The figures of value-at-risk, calculated as 95% quantiles of residuals of ARIMAX-GARCH models for different days of week, indicate that risk of carrying transactions on Thursday is slightly greater in comparison to other days of week. Similarly, values of expected shortfalls, calculated as average of values that are less than value-at-risk, also indicate that there is slightly greater risk of investing on Thursday.

CONCLUSION

Regarding different values of NEPSE indices as independent observations indicate that the risk of transactions on Sunday is slightly less, however, when dependency of observations is taken into account by using ARIMAX-GARCH model the risk is nearly same on all days of week except on Thursday on which value-at-risk and expected shortfall are slightly more in comparison to other days of week. Thus it can be concluded that risk of carrying transactions on different days of week in NEPSE market are not noticeably different.

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CONFLICT OF INTEREST

The authors declare that they have no conflict of interest.

AUTHOR CONTRIBUTION

Both the authors have made substantial contributions on the concept and designing of the article, obtaining, analyzing, and interpreting data, drafted the article, reviewed the results and approved the final version of the manuscript critically for important intellectual content, agreed to be accountable for all aspects of the work in ensuring the questions related to the accuracy and maintained integrity of the work which were appropriately investigated and resolved.

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DATA AVAILABILITY

Data is available in the official website of NEPSE.

ETHICAL STATEMENT

Ethical approval was not sought for the present study because of open data source.

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