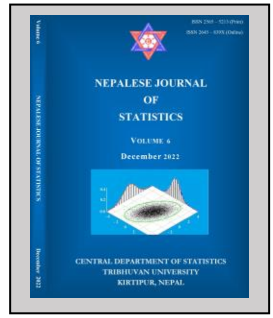
# Inverse Exponentiated Odd Lomax Exponential Distribution: Properties and Applications




**Arun Kumar Chaudhary[1]*, Lal Babu Sah Telee[2] and Vijay Kumar[3]**

***Submitted: 21 October 2022; Accepted: 15 December 2022***
**Published online: 27 December 2022**
**DOI: 10.3126/njs.v6i01.50801**

## ABSTRACT

**Background:** New family of distributions has important functions in generalization of distributions by modifying some existing distributions for getting more flexible irrespective to applied and practical view point. The Inverse Exponentiated Odd Lomax Exponential Distribution (IEOLE) having four parameters is suggested. Proposed model is based on T-X family of distribution which is the extended form of beta-generated distribution. Based on the LSE, MLE, and CVM methods, the parameters of the proposed distribution are estimated. Different model validation criteria and model comparisons are done by considering other existing models.

**Materials and methods:** IEOLE is compound distribution derived by using theoretical concept of Odd Lomax Exponential distribution and T-X family of distribution. The parameters of the proposed distribution are estimated through the least square, Cramer–Von Mises and maximum likelihood methods. The applicability of the proposed model is evaluated using R programming on two real-life time data sets.

**Results:** The statistical properties and different characteristics like the hazard rate function, the cumulative distribution function, quantile function, skewness, and kurtosis of the proposed model are discussed. Box plots, TTT plot, density fits etc. shows that the proposed model fits better to considered two real data sets. Different model validation criteria such as AIC, BIC, and CAIC are obtained and compared with some existing well-known probability distributions.

**Conclusion:** This study presents new probability model called *Inverse Exponentiated Odd Lomax Exponential distribution*. The density curves of the model show that it is unimodal having right skewed. The suggested model has proven to be versatile for modeling real-world data due to its increasing-decreasing, right-skewed form. Also, the hazard rate function (HRF) graph is decreasing, decreasing-increasing or inverted bathtub shaped according to the value of the model constants. Different validation criteria show that the suggested model fits data well and the goodness-of-fit shows that proposed model has lesser test statistic value and higher p- value with respect to some existing models.

**Copyright & License @ Central Department of Statistics, TU, 2022** 

**Keywords**: Estimation, hazard function, maximum likelihood estimation, odd Lomax exponential distribution, survival function, T-X family of distribution.

**Address correspondence to the authors:** Department of Management Science, Nepal Commerce Campus, Tribhuvan University, Nepal. Email: akchaudhary1@yahoo.com[1*] (Corresponding author email); Email: lalbabu3131@gmail.com[2]; Department of Mathematics and Statistics, DDU Gorakhpur University, Gorakhpur, India.
Email: vkgkp@rediffmail.com[3]

## INTRODUCTION

Probability distribution is useful not only in the field of statistics but also in the field of life science, economics and applied science. Due to these broad applications of probability distributions in different fields, advancement of the probability distribution has been growing continuously. Since last couple of decades, many new probability distributions have been introduced by researchers using different techniques. Although there are many existing techniques to solve the problem of real life data, new distributions are required to solve the problem more effectively and precisely. New family of distributions have key role in generalization of several models. There are several methods of getting new models like compounding, exponentiation, modifying and adding some constants to well-known distributions. Such models get more flexibility for applied and practical view point (Usman, Haq, & Talib, 2017). Present literature contains numerous distributions. Lomax distribution is extended by using Marshall and Olkin family of distribution which have been suggested by (Ghitany, Al-Awadhi & Alkhalfan, 2007). Many new distributions have been derived by using the Lomax distribution such as McDonald Lomax distribution developed by (Lemonte & Cordeiro, 2013). The power Lomax distribution with three parameters suggested by (Rady, Hassanein & Elhaddad, 2016) is more flexible compared to Lomax distribution. This distribution has decreasing and inverted bathtub hazard rate functions. Proposed model is increasing, decreasing and j-shaped hazard rate functions. A novel distribution known as the alpha power inverted exponential distribution is created by (Ceren, Cakmakyapan & Gamze, 2018) using alpha as a power from the inverted exponential distribution. The Lomax random variable has been used as a generator to construct the Odd Lomax exponential (type III) distribution (Ogunsanya, Sanni & Yahya, 2019). An odd generalized exponential is compounded with inverted Lomax distribution to derive Odd generalized exponentiated Inverse Lomax distribution which have been introduced by (Maxwell, Chukwudike & Bright, 2019). Ijaz and Asim (2019) have suggested the Lomax exponential distribution derived from Lomax distribution with decreasing and increasing hazard shapes. Inverse Lomax distribution is used as generator to form the inverse Lomax- exponentiated G- family which have been introduced by (Falgore & Doguwa, 2020). When modeling the lifetime dataset, Chaudhary and Kumar (2020) proposed the Logistic Lomax distribution by taking the Lomax distribution as the parent distribution and found that the modal showed the inverted bathtub shaped hazard function. Chaudhary and Kumar (2021) have also presented the ArcTan Lomax Distribution by taking the Lomax distribution as baseline model. In addition, Joshi and Kumar (2021) used the inverse Lomax distribution as the parent distribution to introduce the Logistic Inverse Lomax distribution. In order to create the Exponentiated Odd Lomax Exponential (EOLE) distribution, Dhungana and Kumar (2022) combined an exponentiated odd function with the Lomax distribution as a baseline distribution. They discovered that the resulting distribution was unimodal and positively skewed in contrast to the hazard rate function, which continued to increase monotonically and exhibit inverted bathtubs. By fusing the inverse Weibull distribution and the odd Lomax-G family, Almetwally (2021) created the Odd Lomax-G Inverse Weibull Distribution to predict the mortality rate for the COVID-19 pandemic data and found that the probability density function (PDF) of the model can be decreasing curves, symmetric, or right-skewed and the model's HR function includes numerous intriguing shapes, including upside-down curves, constant, and decreasing, all of which are desirable characteristics for lifetime models. Different models derived so far are described and verified using different real life data sets. In this chapter, we have considered two real life data sets. Using the data sets, we have demonstrated the applicability of the $IEOLE(\alpha,\lambda,\theta,\delta)$.

## Model analysis

Exponential distribution has significant role in probability theory as well as in overall statistics. It is a special case of Rayleigh, Gamma and Weibull distribution where events are independent, continuous with constant average rate. It is also continuous analog of geometric distribution. Inverse exponential distribution is modified form of exponential distribution where inverse of random variable is considered as variable. Inverse exponential distribution is used as baseline distribution in different family of distribution to get new distributions. The cumulative distribution function of inverse exponential distribution is given as:

$$G(x) = e^{-\alpha/x} \tag{1}$$

We can define $\overline{G}(x)$ as $\overline{G}(x) = 1 - e^{-\alpha/x}$ (2)

Also, define function, $W(x) = \left[\frac{G(x)}{\overline{G}(x)}\right]^{\theta} = \left[\frac{e^{-\alpha/x}}{1-e^{-\alpha/x}}\right]^{\theta} = (e^{\alpha/x}-1)^{-\theta}$ (3)

where W($x$) is the inverse exponentiated odd function and $\theta > 0$ is an auxiliary parameter on odd function developed by (Tahir & Nadarajha, 2015) and (De Brito, Rêgo, De Oliveira & Gomes-Silva, 2019). T-X family of distributions, which is an expanded version of beta generated distributions with non-negative continuous random variable T, was proposed by (Alzaatreh, Lee & Famoye, 2013) and defined by:

$$F(X) = \int_0^{W(x)} r(t)dt \tag{4}$$

The generator r(t) uses the probability density function of the Lomax distribution (Pareto type II distribution). Lomax distribution is well known distribution having strong applications in modeling, reliability theory, actuarial sciences and economics etc., (Chakrabortya, 2019). Probability density function of Lomax distribution is given as:

$$r(t) = \frac{\lambda}{\delta}\left\{1+\left(\frac{t}{\delta}\right)\right\}^{-(\lambda+1)} \quad ; \; t>0, \lambda>0, \delta>0 \tag{5}$$

Here, because the Lomax distribution contains one of each scale parameter δ and shape parameter λ and the inverse exponential distribution has only one scale parameter α, we combine the inverse exponentiated odd function [W(*x*)] and the PDF of the Lomax distribution as a generator. Another shape parameter defined in new model is θ. The compounded new model having two shape parameters and two scale parameters will make model more flexible and robust capturing different types of data such as skewed, truncated, etc. We name it as Inverse Exponentiated Odd Lomax Exponential distribution (IEOLE) with four parameters. The cumulative distribution function (CDF) of the proposed model is:

$$F(x,\alpha,\lambda,\theta,\delta)=\int_0^{(e^{\alpha/x}-1)^{-\theta}} \frac{\lambda}{\delta}\left\{1+\left(\frac{t}{\delta}\right)\right\}^{-(\lambda+1)} dt;$$

$$=1-\left\{1+\frac{1}{\delta}(e^{\alpha/x}-1)^{-\theta}\right\}^{-\lambda} \quad ; x\geq 0,(\alpha,\lambda,\theta,\lambda)>0 \qquad (6)$$

The corresponding probability density function (PDF) of IEOLE is given by

$$f(x)=\begin{cases} \dfrac{\lambda\theta\alpha}{\delta}x^{-2}e^{\alpha/x}\left(e^{\alpha/x}-1\right)^{-(\theta+1)}\left[1+\dfrac{1}{\delta}\left(e^{\alpha/x}-1\right)^{-\theta}\right]^{-(\lambda+1)} & ; x\geq 0,(\alpha,\lambda,\theta,\lambda)>0 \\ 0 & ; \text{Otherwise} \end{cases} \qquad (7)$$

where, $\int_0^{\infty} f(x)dx=1$

Similarly, the survival function S(*x*) of the IEOLE is given as,

$$S(x)=1-F(x)=1-\left[1-\left\{1+\frac{1}{\delta}(e^{\alpha/x}-1)^{-\theta}\right\}^{-\lambda}\right]$$

$$=\left\{1+\frac{1}{\delta}(e^{\alpha/x}-1)^{-\theta}\right\}^{-\lambda} : x\geq 0,(\alpha,\lambda,\theta,\lambda)>0 \qquad (8)$$

The conditional density function (h(x)) also called the hazard rate function (HRF) is given as

$$\text{h(x)} = \lim_{\Delta x\to 0}\frac{\text{Prob}(x\leq X\leq \Delta x)}{\Delta x S(x)}=\frac{f(x)}{1-F(x)},$$

$$=\frac{\lambda\theta\alpha}{\delta}x^{-2}e^{\alpha/x}\left(e^{\alpha/x}-1\right)^{-(\theta+1)}\left[1+\frac{1}{\delta}\left\{e^{\alpha/x}-1\right\}^{-\theta}\right]^{-1} ; x>0,(\alpha,\lambda,\theta,\lambda)>0 \qquad (9)$$

The HRF function of probability models may be decreasing, decreasing-increasing, j - shaped; reversed j shaped etc. based on parameters values. In proposed model, the shape of hazard rate function is decreasing at (α = 0.01, λ = 8.1, θ = 0.5) and (α = 0.05, λ=8.2, θ= 0.9) when δ = 1. As α is greater than 0.1, hazard function is monotonically increasing-decreasing. Varying the values of the parameters, hazard rate curves of different shapes can be obtained. Figure 1, represents the probability density function and hazard rate function of the proposed model.

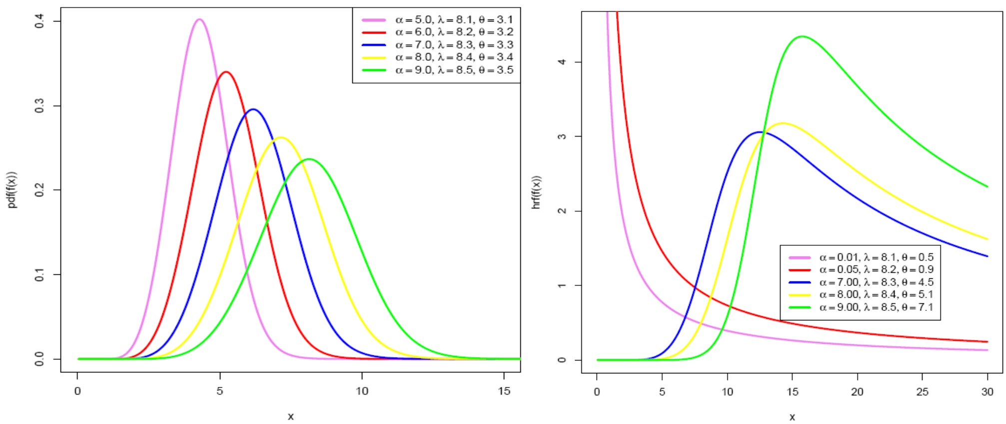



**Fig.1.** PDF (left panel) and HRF (right panel) for different values of α, λ and θ for constant value for δ = 1.

The reversed hazard rate function r(*x*) i.e., the ratio of the density function to the cumulative distribution function is given by:

$$r(x)=\frac{\frac{\lambda\theta\alpha}{\delta}x^{-2}e^{\alpha/x}\left(e^{\alpha/x}-1\right)^{-(\theta+1)}\left[1+\frac{1}{\delta}\left\{e^{\alpha/x}-1\right\}^{-\theta}\right]^{-(\lambda+1)}}{\left[1-\left\{1+\frac{1}{\delta}(e^{\alpha/x}-1)^{-\theta}\right\}^{-\lambda}\right]};x>0,(\alpha,\lambda,\theta,\lambda)>0 \qquad (10)$$

We have also defined the cumulative hazard rate function *H(x)* in expression (11). That is

$$H(x)=-\ln S(x)=\lambda\ln\left\{1+\frac{1}{\delta}(e^{\alpha/x}-1)^{-\theta}\right\};x>0,(\alpha,\lambda,\theta,\lambda)>0 \qquad (11)$$

## Quantile function

The Quantile function of a model can be defined by $Q(u)=F^{-1}(u)$ where, u follows uniform distribution U (0, 1). Quantile functions are used for theoretical aspects of a probability distribution and are alternative to CDF & PDF. Different statistical measures like median, skewness, kurtosis can be obtained using quantile function. The quantile function Q of the IEOLE is defined as:

$$Q(u)=\alpha\left[\ln\left[1+\left[\delta\left\{(1-u)^{-1/\lambda}-1\right\}\right]^{-1/\theta}\right]\right]^{-1} \; ; 0 < u < 1$$

(12)

By putting the value of u = ½ in expression (12), the median value can be derived as:

$$Median=\alpha\left[\log\left[1+\left[\delta\left\{(1/2)^{-1/\lambda}-1\right\}\right]^{-1/\theta}\right]\right]^{-1} \qquad (13)$$

Also, replacing Q (u) by *x*, the random deviate generation of the distribution can be defined as:

$$x=\alpha\left[\log\left[1+\left[\delta\left\{(1-u)^{-1/\lambda}-1\right\}\right]^{-1/\theta}\right]\right]^{-1} \; ;0<u<1 \qquad (14)$$

The random deviate generation helps to generate random numbers (observations).

### *Asymptotic behavior and model of the distribution*

Asymptotic behavior of the density function can be determined by checking $\lim_{x\to 0} f(x) = \lim_{x\to\infty} f(x)$. Taking limiting at end points,

$$\lim_{x\to 0} f(x) = \frac{\lambda\theta\alpha}{\delta} x^{-2} e^{\alpha/x} \left(e^{\alpha/x} - 1\right)^{-(\theta+1)} \left[1 + \frac{1}{\delta}\left\{e^{\alpha/x} - 1\right\}^{-\theta}\right]^{-(\lambda+1)} = 0$$

$$\lim_{x\to \infty} f(x) = \frac{\lambda\theta\alpha}{\delta} x^{-2} e^{\alpha/x} \left(e^{\alpha/x} - 1\right)^{-(\theta+1)} \left[1 + \frac{1}{\delta}\left\{e^{\alpha/x} - 1\right\}^{-\theta}\right]^{-(\lambda+1)} = 0$$

That is, $\lim_{x\to 0} f(x) = \lim_{x\to\infty} f(x) = 0$. To calculate the modal value, applying condition $f'(x) = 0$. For simplicity, log density function is used to find $f'(x)$. Taking logarithm of density function defined in equation (7), we get

$$\ln f(x) = \ln\alpha + \ln\lambda + \ln\theta - \ln\delta - 2\ln x + \alpha x^{-1} - (\theta+1)\ln(e^{\alpha/x_i} - 1) - (\lambda+1)\ln\left\{1 + \frac{1}{\delta}(e^{\alpha/x_i} - 1)^{-\theta}\right\} \tag{15}$$

Differentiating equation (15) with respect to $x$, and $f'(x) = 0$ implies that

$$(2x+\alpha) - \alpha(\theta+1)\frac{e^{\alpha/x}}{e^{\alpha/x} - 1} + \alpha\theta(\lambda+1)\frac{e^{\alpha/x}}{\left\{\delta\left(e^{\alpha/x} - 1\right)^{(\theta+1)} + e^{\alpha/x} - 1\right\}} = 0 \tag{16}$$

Equation (16) is nonlinear so it can be solved using numerical method. The first term ($2x + \alpha$ ) of equation (16) is monotonically increasing, being a linear expression in *x*. The 2nd term $\left\{-\alpha(\theta+1)\frac{e^{\alpha/x}}{e^{\alpha/x} - 1}\right\}$ is monotonically decreasing as it is negative and is a combination of increasing exponential expression.

The 3rd term $\alpha\theta(\lambda+1)\frac{e^{\alpha/x}}{\left\{\delta\left(e^{\alpha/x} - 1\right)^{(\theta+1)} + e^{\alpha/x} - 1\right\}}$, which is a combination of exponential expressions, is also monotonically decreasing. Thus, the LHS of the equation (16) is an additive combination of monotonically increasing and decreasing expressions, which is therefore a monotonic expression. One can clearly see that the LHS of (16) is negative when

$$(2x+\alpha)+\alpha\theta(\lambda+1)\frac{e^{\alpha/x}}{\left\{\delta\left(e^{\alpha/x}-1\right)^{(\theta+1)}+e^{\alpha/x}-1\right\}}<\alpha(\theta+1)\frac{e^{\alpha/x}}{e^{\alpha/x}-1}$$

and positive when

$$(2x+\alpha)+\alpha\theta(\lambda+1)\frac{e^{\alpha/x}}{\left\{\delta\left(e^{\alpha/x}-1\right)^{(\theta+1)}+e^{\alpha/x}-1\right\}}>\alpha(\theta+1)\frac{e^{\alpha/x}}{e^{\alpha/x}-1}$$

This shows that the new PDF is unimodal.

## Parameter estimation

We have used three distinct methods of parameter estimation. These methods are,

### Maximum likelihood estimation

Here, we've covered the ML estimators (MLEs) for the IEOLE model in this segment. Let $\underline{x} = (x_1,\dots,x_n)$ be a random sample of size 'n' from $IEOLE(\alpha,\lambda,\theta,\delta)$ then the log likelihood function can be written as,

$$\ell(\alpha,\lambda,\theta,\delta\,|\,\underline{x}) = n\ln\alpha + n\ln\lambda + n\ln\theta - n\ln\delta - 2\sum_{i=1}^{n}\ln(x_i) - (\theta+1)\sum_{i=1}^{n}\ln(e^{\alpha/x_i}-1) + \alpha\sum_{i=1}^{n}(1/x_i) - (\lambda+1)\sum_{i=1}^{n}\ln\left\{1+\frac{1}{\delta}(e^{\alpha/x_i}-1)^{-\theta}\right\} \quad (17)$$

After differentiating (17) with respect to α, λ, θ and δ, we get ,

$$\frac{\partial\ell}{\partial\alpha} == \frac{n}{\alpha} - (\theta+1)\sum_{i=1}^{n}\frac{1}{x_i}(e^{\alpha/x_i}-1)^{-1}e^{\alpha/x_i} + \sum_{i=1}^{n}(1/x_i) + \frac{(\lambda+1)\theta}{\delta}\sum_{i=1}^{n}\frac{e^{\alpha/x_i}}{x_i}\left\{1+\frac{1}{\delta}(e^{\alpha/x_i}-1)^{-\theta}\right\}^{-1}(e^{\alpha/x_i}-1)^{-(\theta+1)}$$

$$\frac{\partial l}{\partial\lambda} = \frac{n}{\lambda} - \sum_{i=1}^{n}\ln\left\{1+\frac{1}{\delta}(e^{\alpha/x_i}-1)^{-\theta}\right\}$$

$$\frac{\partial\ell}{\partial\theta} == \frac{n}{\theta} - \sum_{i=1}^{n}\ln(e^{\alpha/x_i}-1) + \frac{(\lambda+1)}{\delta}\sum_{i=1}^{n}\left\{1+\frac{1}{\delta}(e^{\alpha/x_i}-1)^{-\theta}\right\}^{-1}(e^{\alpha/x_i}-1)^{-\theta}\ln(e^{\alpha/x_i}-1). \quad (18)$$

$$\frac{\partial\ell}{\partial\delta} = -\frac{n}{\delta} + \frac{(\lambda+1)}{\delta^2}\sum_{i=1}^{n}(e^{\alpha/x_i}-1)^{-\theta}\left\{1+\frac{1}{\delta}(e^{\alpha/x_i}-1)^{-\theta}\right\}^{-1}$$

By setting $\frac{\partial\ell}{\partial\alpha} = \frac{\partial\ell}{\partial\lambda} = \frac{\partial\ell}{\partial\theta} = \frac{\partial\ell}{\partial\delta} = 0$ and solving them for α, λ and θ we get ML estimators of $IEOLE(\alpha,\lambda,\theta,\delta)$. It is not possible to solve non-linear equations (18) so with the aid of suitable computer programming, one can solve them easily. Let $\hat{\Theta} = (\hat{\alpha},\hat{\lambda},\hat{\theta},\hat{\delta})$ denote the estimated parameter using MLE of $IEOLE(\alpha,\lambda,\theta,\delta)$ with parameter vector $\Theta = (\alpha,\lambda,\theta,\delta)$, then the asymptotic normality results in, $\left(\hat{\Theta}-\Theta\right)\rightarrow N_3\left[0,\left(I\left(\Theta\right)\right)^{-1}\right]$,where $I\left(\Theta\right)$ stands for the Fisher's information matrix given by:

$$I\left(\Theta\right)=-\begin{pmatrix} E\left(\frac{\partial^2 l}{\partial\alpha^2}\right) & E\left(\frac{\partial^2 l}{\partial\alpha\partial\lambda}\right) & E\left(\frac{\partial^2 l}{\partial\alpha\partial\theta}\right) E\left(\frac{\partial^2 l}{\partial\alpha\partial\delta}\right) \\ E\left(\frac{\partial^2 l}{\partial\lambda\partial\alpha}\right) & E\left(\frac{\partial^2 l}{\partial\lambda^2}\right) & E\left(\frac{\partial^2 l}{\partial\lambda\partial\theta}\right) E\left(\frac{\partial^2 l}{\partial\lambda\partial\delta}\right) \\ E\left(\frac{\partial^2 l}{\partial\alpha\partial\theta}\right) & E\left(\frac{\partial^2 l}{\partial\lambda\partial\theta}\right) & E\left(\frac{\partial^2 l}{\partial\theta^2}\right) E\left(\frac{\partial^2 l}{\partial\theta\partial\delta}\right) \\ E\left(\frac{\partial^2 l}{\partial\delta\partial\alpha}\right) & E\left(\frac{\partial^2 l}{\partial\delta\partial\lambda}\right) & E\left(\frac{\partial^2 l}{\partial\delta\partial\theta}\right) E\left(\frac{\partial^2 l}{\partial\delta^2}\right) \end{pmatrix}$$

In general practice, $\Theta$ cannot be determined so considering that asymptotic variance $\left(I\left(\Theta\right)\right)^{-1}$ of MLE is worthless. Therefore, using the parameters' estimated values, the asymptotic variance can be approximated. Let $O(\hat{\Theta})$ be the observed fisher information matrix. We can obtain $O(\hat{\Theta})$ as an estimate of the information matrix $I\left(\Theta\right)$ in terms of the hessian matrix H define below.

$$O(\hat{\Theta})=-\begin{pmatrix} \frac{\partial^2 l}{\partial\alpha^2} & \frac{\partial^2 l}{\partial\hat{\alpha}\partial\hat{\lambda}} & \frac{\partial^2 l}{\partial\hat{\alpha}\partial\hat{\theta}} \frac{\partial^2 l}{\partial\hat{\alpha}\partial\hat{\delta}} \\ \frac{\partial^2 l}{\partial\hat{\lambda}\partial\hat{\alpha}} & \frac{\partial^2 l}{\partial\hat{\lambda}^2} & \frac{\partial^2 l}{\partial\hat{\lambda}\partial\hat{\theta}} \frac{\partial^2 l}{\partial\hat{\lambda}\partial\hat{\delta}} \\ \frac{\partial^2 l}{\partial\hat{\theta}\partial\hat{\alpha}} & \frac{\partial^2 l}{\partial\hat{\theta}\partial\hat{\lambda}} & \frac{\partial^2 l}{\partial\theta^2} \frac{\partial^2 l}{\partial\hat{\theta}\partial\hat{\delta}} \\ \frac{\partial^2 l}{\partial\hat{\delta}\partial\hat{\alpha}} & \frac{\partial^2 l}{\partial\hat{\delta}\partial\hat{\lambda}} & \frac{\partial^2 l}{\partial\hat{\delta}\partial\hat{\theta}} \quad \frac{\partial^2 l}{\partial\hat{\delta}^2} \end{pmatrix}_{|(\hat{\alpha},\hat{\lambda},\hat{\theta},\hat{\delta})} = -H\left(\Theta\right)_{|(\Theta=\hat{\Theta})}$$

Maximization of likelihood produces the observed information matrix using Newton Raphson method. Therefore, variance-covariance matrix can be defined by:

$$\left[-H\left(\underline{\Theta}\right)_{|(\Theta=\hat{\Theta})}\right]^{-1}=\begin{pmatrix} \mathrm{var}(\hat{\alpha}) & \mathrm{cov}(\hat{\alpha},\hat{\lambda}) & \mathrm{cov}(\hat{\alpha},\hat{\theta}) & \mathrm{cov}(\hat{\alpha},\hat{\delta}) \\ \mathrm{cov}(\hat{\lambda},\hat{\alpha}) & \mathrm{var}(\hat{\lambda}) & \mathrm{cov}(\hat{\lambda},\hat{\theta}) & \mathrm{cov}(\hat{\lambda},\hat{\delta}) \\ \mathrm{cov}(\hat{\theta},\hat{\alpha}) & \mathrm{cov}(\hat{\theta},\hat{\alpha}) & \mathrm{var}(\hat{\theta}) & \mathrm{cov}(\hat{\theta},\hat{\delta}) \\ \mathrm{cov}(\hat{\delta},\hat{\alpha}) & \mathrm{cov}(\hat{\delta},\hat{\theta}) & \mathrm{cov}(\hat{\delta},\hat{\theta}) & \mathrm{var}(\hat{\delta}) \end{pmatrix} \tag{19}$$

If $Z_{\gamma/2}$ be upper percentile of standard normal variate, then based on asymptotic normality of MLEs, approximate 100(1-γ) % CI for α, λ , θ and δ can be constructed as:

$$\hat{\alpha}\pm Z_{\gamma/2}\sqrt{\mathrm{var}(\hat{\alpha})},\ \hat{\lambda}\pm Z_{a/2}\sqrt{\mathrm{var}(\hat{\lambda})},\ \hat{\theta}\pm Z_{\gamma/2}\sqrt{\mathrm{var}(\hat{\theta})} \text{ and } \hat{\delta}\pm Z_{\gamma/2}\sqrt{\mathrm{var}(\hat{\delta})}$$

### *Method of least-square estimation*

Let us define a function A given in equation (20) using the CDF of the ordered random variable $X_{(1)} < X_{(2)} < \ldots < X_{(n)}$ where $\{X_1, X_2, \ldots, X_n\}$ is a random sample of size n from a distribution function F (.).

 ISSN: 2565-5213(Print); 2465-839X (Online)

$$A(x;\alpha,\lambda,\theta,\delta)=\sum_{i=1}^{n}\left[F(X_{(i)})-\frac{i}{n+1}\right]^2 \tag{20}$$

where $F(X_{(i)})$ is CDF of ordered statistics.

Here α, λ, θ and δ are unknown constants of $IEOLE(\alpha,\lambda,\theta,\delta)$ that can be obtained by minimizing above function.

$$A(x_{(i)};\alpha,\lambda,\theta,\delta)=\sum_{i=1}^{n}\left[\left[1-\left\{1+\frac{1}{\delta}(e^{\alpha/x_{(i)}}-1)^{-\theta}\right\}^{-\lambda}\right]-\frac{i}{n+1}\right]^2 ;\ x>0,(\alpha,\lambda,\theta,\delta)>0 \tag{21}$$

with respect to α, λ, θ and δ.

Differentiating (21) with respect to α, λ, θ and δ we get,

$$\frac{\partial A}{\partial \alpha}==\frac{-2\lambda\theta}{\delta}\sum_{i=1}^{n}\left[F(X_{(i)})-\frac{i}{n+1}\right]\left\{1+\frac{1}{\delta}(e^{\alpha/x_{(i)}}-1)^{-\theta}\right\}^{-(\lambda+1)}\frac{e^{\alpha/x_{(i)}}}{x_{(i)}}(e^{\alpha/x_{(i)}}-1)^{-(\theta+1)}$$

$$\frac{\partial A}{\partial \lambda}=2\sum_{i=1}^{n}\left[F(x_{(i)})-\frac{i}{n+1}\right]\left\{1+\frac{1}{\delta}(e^{\alpha/x_{(i)}}-1)^{-\theta}\right\}^{-\lambda}\ln\left\{1+\frac{1}{\delta}(e^{\alpha/x_{(i)}}-1)^{-\theta}\right\}$$

$$\frac{\partial A}{\partial \theta}=\frac{-2\lambda}{\delta}\sum_{i=1}^{n}\left[F(x_{(i)})-\frac{i}{n+1}\right]\left\{1+\frac{1}{\delta}(e^{\alpha/x_{(i)}}-1)^{-\theta}\right\}^{-(\lambda+1)}(e^{\alpha/x_{(i)}}-1)^{-\theta}\ln(e^{\alpha/x_{(i)}}-1)$$

$$\frac{\partial A}{\partial \delta}=\frac{-2\lambda}{\delta^2}\sum_{i=1}^{n}\left[F(x_{(i)})-\frac{i}{n+1}\right]\left[\left\{1+\frac{1}{\delta}(e^{\alpha/x_{(i)}}-1)^{-\theta}\right\}^{-(\lambda+1)}\right](e^{\alpha/x_{(i)}}-1)^{-\theta}$$

To estimate the parameters using weighted least square method, we have to minimize the function defined by

$$D(X;\alpha,\lambda,\theta,\lambda)=\sum_{i=1}^{n}w_i\left[F(X_{(i)})-\frac{i}{n+1}\right]$$

where, $w_i=\dfrac{1}{Var(X_{(i)})}=\dfrac{(n+1)^2(n+2)}{i(n-i+1)}$

$F(X_{(i)})$ is the CDF of the order statistics and $w_i$ is weight. The function becomes

$$D(X;\alpha,\lambda,\theta,\delta)=\sum_{i=1}^{n}\frac{(n+1)^2(n+2)}{i(n-i+1)}\left[\left[1-\left\{1+\frac{1}{\delta}(e^{\alpha/x_{(i)}}-1)^{-\theta}\right\}^{-\lambda}\right]-\frac{i}{n+1}\right]^2 \tag{22}$$

### Method of Cramer-Von-Mises estimation (CVME)

The Cramer-Von-Mises estimators of α, λ, θ and δ are found by minimizing the function (23) with respect to α, λ, θ and δ.

$$Z(X;\alpha,\lambda,\theta,\delta)=\frac{1}{12n}+\sum_{i=1}^{n}\left[F(x_{i:n}\,|\,\alpha,\lambda,\theta,\delta)-\frac{2i-1}{2n}\right]^2$$

$$=\frac{1}{12n}+\sum_{i=1}^{n}\left[\left[1-\left\{1+\frac{1}{\delta}(e^{\alpha/x(i)}-1)^{-\theta}\right\}^{-\lambda}\right]-\frac{2i-1}{2n}\right]^{2} \qquad (23)$$

Differentiating (23) with respect to α, λ, θ and δ we get,

$$\frac{\partial Z}{\partial\alpha}=\frac{-2\lambda\theta}{\delta}\sum_{i=1}^{n}\left[F(X_{(i)})-\frac{2i-1}{2n}\right]\left\{1+\frac{1}{\delta}(e^{\alpha/x_{(i)}}-1)^{-\theta}\right\}^{-(\lambda+1)}\frac{e^{\alpha/x_{(i)}}}{x_{(i)}}(e^{\alpha/x_{(i)}}-1)^{-(\theta+1)}$$

$$\frac{\partial Z}{\partial\lambda}=2\sum_{i=1}^{n}\left[F(x_{(i)})-\frac{2i-1}{2n}\right]\left\{1+\frac{1}{\delta}(e^{\alpha/x(i)}-1)^{-\theta}\right\}^{-\lambda}\ln\left\{1+\frac{1}{\delta}(e^{\alpha/x(i)}-1)^{-\theta}\right\}$$

$$\frac{\partial Z}{\partial\theta}=\frac{-2\lambda}{\delta}\sum_{i=1}^{n}\left[F(x_{(i)})-\frac{2i-1}{2n}\right]\left\{1+\frac{1}{\delta}(e^{\alpha/x(i)}-1)^{-\theta}\right\}^{-(\lambda+1)}(e^{\alpha/x(i)}-1)^{-\theta}\ln(e^{\alpha/x(i)}-1)$$

$$\frac{\partial Z}{\partial\delta}=\frac{-2\lambda}{\delta^{2}}\sum_{i=1}^{n}\left[F(x_{(i)})-\frac{2i-1}{2n}\right]\left\{1+\frac{1}{\delta}(e^{\alpha/x(i)}-1)^{-\theta}\right\}^{-(\lambda+1)}(e^{\alpha/x(i)}-1)^{-\theta}$$

Through solving $\frac{\partial Z}{\partial\alpha}=0, \frac{\partial Z}{\partial\lambda}=0$ , $\frac{\partial Z}{\partial\theta}=0$ and $\frac{\partial Z}{\partial\delta}=0$ at the same time, we obtain the CVM estimators.

## *Application to real data set*

In this section of the study, applicability of the proposed model $IEOLE(\alpha,\lambda,\theta,\delta)$ is tested. For this purpose, two real data sets are considered.

### *Data set I*

This real dataset is from (Bimbaum & Saunders,1969b), who originally examined data pertaining to the fatigue life of 6061-T6 aluminum coupons that were cut parallel to the direction of rolling and oscillated at a rate of 18 cycles per second with 101 observations and a maximum stress per cycle of 31,000 psi.

196,108, 90, 96, 97, 99, 100, 103, 104, 105, 104, 107, 108, 108, 109, 109,, 112, 112, 113, 114, 114, 114, 116, 119, 120, 120, 120, 121, 121, 123, 124, 124, 124, 124, 128, 128, 129, 129, 130, 130, 130, 131, 131, 131, 131, 131, 132, 132, 132, 132, 133, 134, 134, 134, 134, 134, 136, 136, 137, 138, 138, 138, 139, 124, 139, 141, 141, 142, 142, 142, 142, 142, 144, 144, 145, 146, 148, 148, 149, 151, 151, 152, 155, 156, 157, 157, 157, 157, 158, 159, 163, 163, 164, 162, 166, 166, 168, 170, 174, 212, 70

## *Exploratory data analysis*

Exploratory data analysis written by (Tukey, 1977) refers to the critical procedure of initial calculation of data to determine the pattern with help of summary statistics and graphical representation. It is an approach of statistical analysis that attempts to maximize insight into data. Exploratory data analysis uncovers underlying structure and extracts important variables of the data. Figure 2 represent the box plot and the TTT plot of the given data. The data set is positively skewed and non- normal in shape.

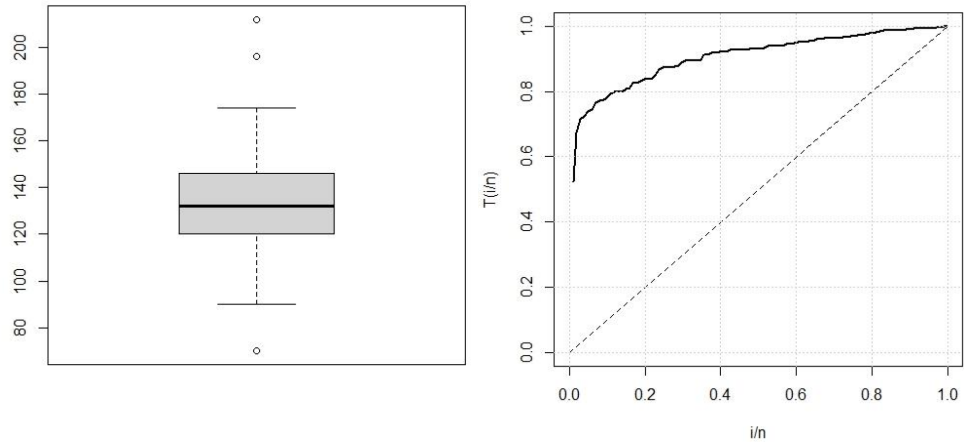



**Fig. 2.** Boxplot (left panel) and TTT plot (right Plot).

**Table1.** Summary statistics.

| Min. | $Q_1$ | Median | Mean | $Q_3$ | SD | Skewness | Kurtosis |
|---|---|---|---|---|---|---|---|
| 70.0 | 120.0 | 132.0 | 133.6 | 146.0 | 22.3409 | 0.3440 | 4.0694 |

### *Parameters*

Applying optim () function in R computation (R Core Team, 2022), MLEs of IEOLE model is calculated by maximizing the likelihood function. Log-Likelihood value of the log-likelihood function is is $l$ = -497.8661. Estimated values of parameters and corresponding standard error (SE) are tabulated in Table 2.

**Table 2.** MLE and SE $\alpha$, $\lambda$, $\theta$ and $\delta$ of IEOLE.

| Parameter | MLE | SE |
|---|---|---|
| Alpha | 76.06047 | 47.2009 |
| Lambda | 1.851201 | 0.8996 |
| Theta | 6.850298 | 1.5970 |
| Delta | 12.92269 | 74.5957 |

In Figure 3, we have the P-P plot and the Q-Q plot exhibited for the graphical model validation. In Table 3, we have also mentioned the parameter estimated values of IEOLE model using above three methods of estimation and corresponding negative log-likelihood, Akaike information criterion (AIC), Bayesian information criterion (BIC) and Kolmogrov -Smirnov (KS) statistics with p-values.

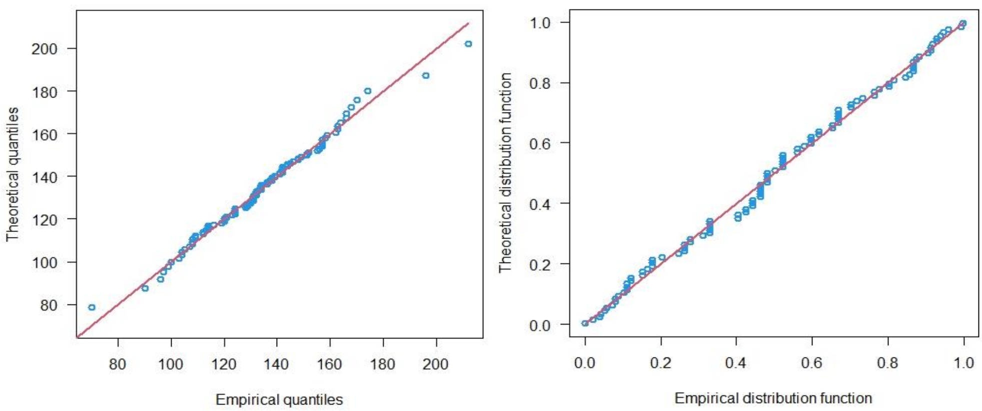



**Fig. 3.** The Q-Q plot (left) and P-P plot in (right) panel of the IEOLE distribution.

**Table 3.** Estimated parameters, log-likelihood, AIC, BIC and KS statistic.

| Methods | $\hat{\alpha}$ | $\hat{\lambda}$ | $\hat{\theta}$ | $\hat{\delta}$ | -LL | AIC | BIC | KS (p-value) |
|---|---|---|---|---|---|---|---|---|
| MLE | 76.06047 | 1.851201 | 6.850298 | 12.92269 | 455.0157 | 918.0314 | 928.4919 | 0.058094 (0.88490) |
| LSE | 78.41200 | 2.505000 | 6.492000 | 12.92500 | 455.2431 | 918.4861 | 928.9466 | 0.060609 (0.85200) |
| CVE | 74.90449 | 2.051043 | 6.794390 | 16.66244 | 455.0477 | 918.0954 | 928.5559 | 0.055006 (0.92000) |

We have plotted the histogram of the data along with the fitted density function of the proposed model taking estimation using MLE, LSE and CVME and shown in figure 4. Figure also displays the empirical cumulative distribution (ECDF) versus fitted CDF.

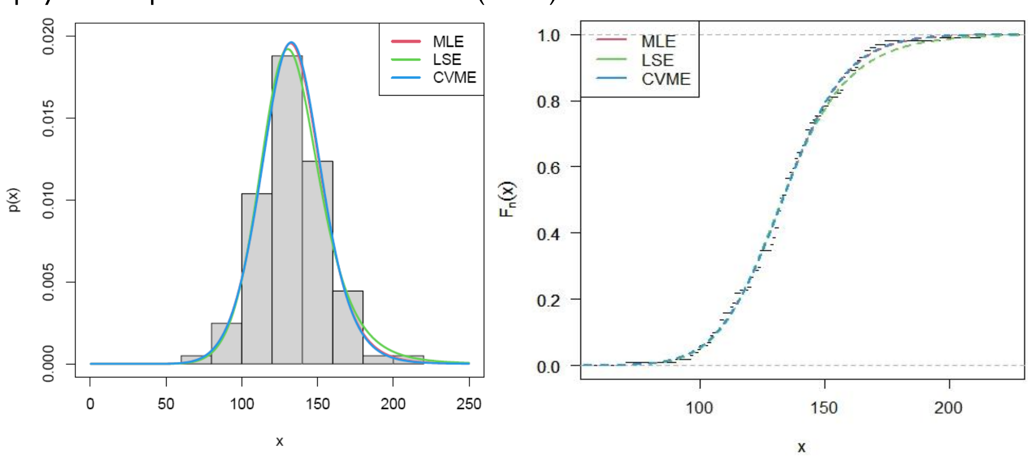


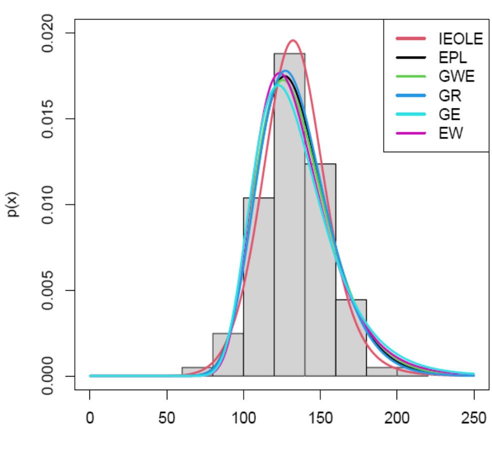

**Fig. 4.** The Histogram Vs. fitted CDF (left panel) and ECDF vs. fitted CDF (right panel) using MLE, LSE and CVM.

Here, we have illustrated the applicability of IEOLE distribution using the real data set I and compared with different published models used in previous research. The existing models are: *Generalized Rayleigh (GR) distribution* (Kundu & Raqab, 2005), *Exponentiated Power Lindley (EPL) distribution* (Ashour & Eltehiwy, 2015), *Generalized Weibull Extension (GWE) distribution* (Sarhan & Apaloo, 2013), *Exponentiated Weibull Distribution (EW) distributon* (Gupta & Kundu, 2001), *Generalized Exponential (GE) distribution* (Gupta & Kundu ,1999). Assessment of the model is checked by comparing different criteria values such as Akaike information criterion (AIC), Bayesian information criterion (BIC), Corrected Akaike information criterion (CAIC), and Hannan-Quinn information criterion (HQIC) of the proposed model IEOLE along with the different published model on the data considered above and are presented in Table 4.

**Table 4.** Log-likelihood (LL), AIC, BIC, CAIC and HQIC.

| Model | -LL | AIC | BIC | CAIC | HQIC |
|---|---|---|---|---|---|
| IEOLE | 455.0157 | 918.0314 | 928.4919 | 918.5121 | 916.2128 |
| GR | 457.3766 | 918.7532 | 923.9835 | 918.8757 | 920.8706 |
| EPL | 457.6517 | 921.3034 | 929.1488 | 921.5508 | 919.8914 |
| GWE | 458.8726 | 923.7452 | 931.5906 | 923.9962 | 922.3332 |
| EW | 461.3052 | 928.6105 | 936.4558 | 928.8524 | 927.1930 |
| GE | 463.7324 | 931.4648 | 936.6951 | 931.5873 | 933.5822 |

The Histogram versus the PDF of fitted models in left panel & Empirical distribution function versus estimated distribution function in right panel of IEOLE including other considered models are presented in Figure 5. From graph, it is clear that the proposed model IEOLE fits better than the other considered existing models.



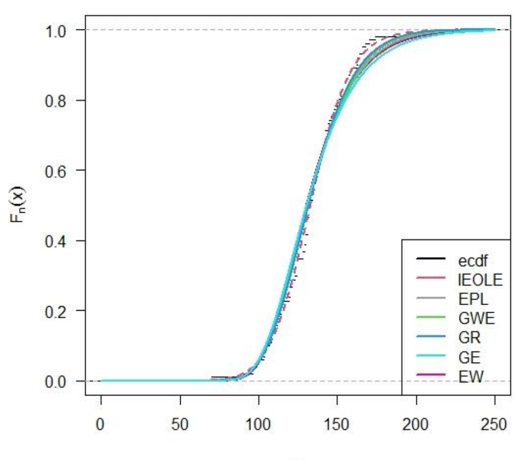




**Fig. 5.** The Histogram and the PDF of fitted model (left) & ECDF with estimated distribution function (right).

Table 5 shows the Kolmogrove-Smirnov (KS), Anderson-Darling ($A^2$), and Cramer's-Von Mises ($W^2$) statistics with corresponding p-values considered distributions including the proposed model. The proposed model has least test statistics and maximum p- values for all the cases than the values of the comparative models. This indicates that the proposed model fits data well than the other considered models.

**Table 5.** KS, $A^2$ and $W^2$ statistics with corresponding p-values of different models.

| Model | KS (p-values) | $A^2$ (p-values) | $W^2$ (p-values) |
|---|---|---|---|
| IEOLE | 0.0581(0.8849) | 0.2598 (0.9649) | 0.0418 (0.9247) |
| GR | 0.0886 (0.4062) | 0.5859 (0.6610) | 0.1001 (0.5851) |
| EPL | 0.0906 (0.3789) | 0.6894 (0.5673) | 0.1148 (0.5177) |
| GWE | 0.0944 (0.3287) | 0.8623 (0.4379) | 0.1395 (0.4239) |
| EW | 0.1102 (0.1719) | 1.1141 (0.3024) | 0.1901 (0.2878) |
| GE | 0.1111 (0.1650) | 1.4023 (0.2016) | 0.2287 (0.2182) |

### *Data set 2*

We have also demonstrated the applicability of the $IEOLE(\alpha, \lambda, \theta, \delta)$ distribution using other real dataset. The data represents the number of deaths per day during the end of December 2020 due to COVID-19 in first wave from 23 January to 24 December (Government of Nepal Ministry of Health and Population, 2020).

2, 2, 2, 2, 2, 2, 3, 2, 3, 3, 4, 2, 5, 5, 3, 2, 4, 4, 8, 4, 4, 3, 2, 3, 7, 6, 6, 11, 9, 3, 8, 7, 11, 8, 12, 12, 14, 7, 11, 12, 6, 14, 9, 9, 11,6, 6, 5, 5, 14, 9, 15, 11, 8, 4, 7 , 11, 10, 16, 2, 7, 17, 6, 8, 10, 4, 10, 7, 11, 11, 8, 7, 19, 9, 15, 12, 10, 14, 22, 9, 18, 12, 19, 21, 12, 12, 18, 8, 26, 21, 17, 13, 5, 15, 14, 11, 17, 16, 17, 23, 24, 20, 30, 18, 18, 17, 21, 18, 22, 26, 15, 13, 13, 6, 9, 17, 12, 17, 22, 7, 16, 16,2 4, 28, 23, 23, 19, 25, 29, 21, 9, 13, 16, 10, 17, 20, 23, 14, 12, 11, 15, 9, 18, 14, 13, 6, 16, 12, 11, 7, 3, 5, 5.

### *Exploratory data analysis*

Exploratory data analysis uncovers underlying structure and extracts important variables of the data. Figure 6 display the boxplot and the TTT plot of the given data.

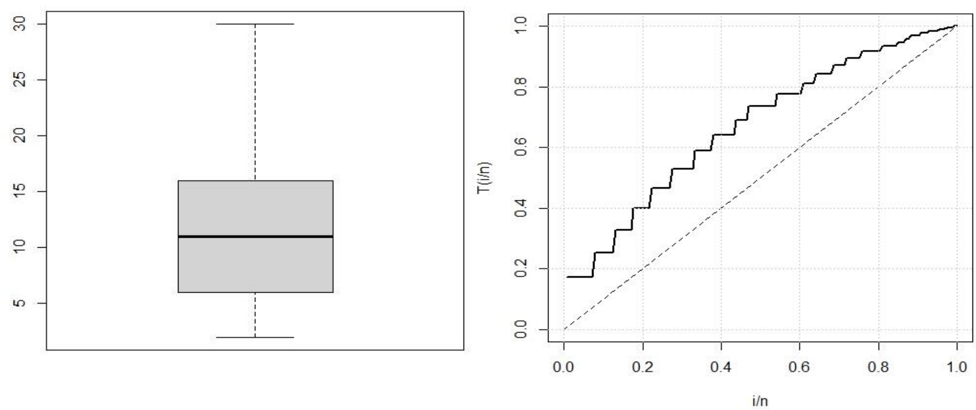



**Fig. 6**. Boxplot (left panel) and TTT plot (right Plot).

**Table 6.** Summary statistics.

| Minimum | $Q_1$ | Median | Mean | $Q_3$ | SD | Skewness | Kurtosis |
|---|---|---|---|---|---|---|---|
| 2.00 | 6.00 | 11.00 | 11.61 | 16 | 6.7591 | 0.508327 | 2.547717 |

The data set is positively skewed and non- normal in shape.

### *Parameter estimation*

By employing the optim () function in R software, we have calculated the MLEs by maximizing the likelihood function. We have obtained the value of Log-Likelihood is $l$ = -497.8661. In Table 1 we have demonstrated the MLE's with their standard errors (SE) for α, λ, θ and δ.

**Table 7.** MLE and SE α, λ, θ and δ of IEOLE.

| Parameter | MLE | SE |
|---|---|---|
| Alpha | 3.0909770 | 1.235549 |
| Lambda | 18.422761 | 15.97112 |
| Theta | 1.5064570 | 0.154630 |
| Delta | 123.08632 | 103.0589 |

Figure 7 shows the Q-Q plot and P-P plot of IEOLE distribution.

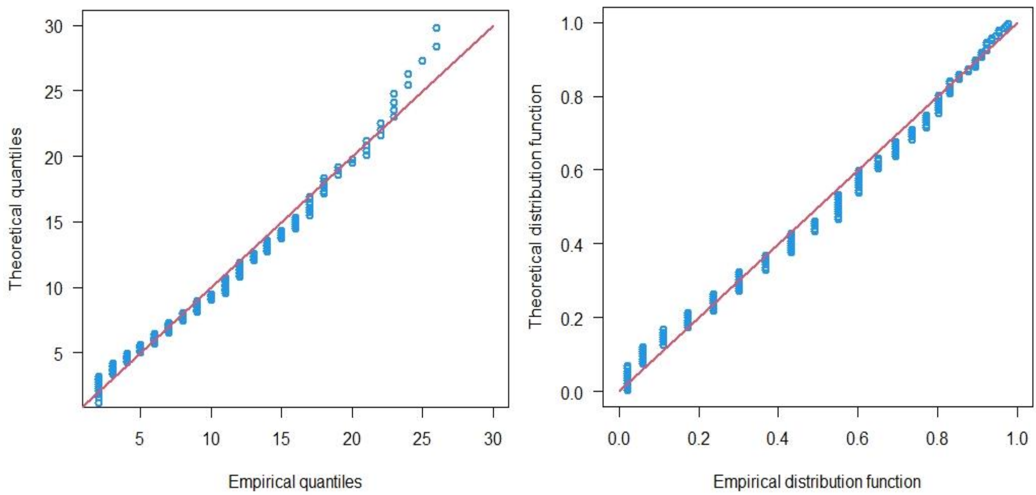



**Fig. 7.** The Q-Q plot (left panel) and P-P plot (right panel) of the IEOLE distribution.

In Table 8 we have presented the estimated value of the parameters of IEOLE based on MLE, LSE and CVE method and their corresponding negative log-likelihood, AIC, BIC and KS statistics with p-value.

**Table 8.** Estimated parameters, log-likelihood, AIC, BIC and KS statistic.

| Methods | $\hat{\alpha}$ | $\hat{\lambda}$ | $\hat{\theta}$ | $\hat{\delta}$ | -LL | AIC | BIC | *KS (p-value)* |
|---|---|---|---|---|---|---|---|---|
| MLE | 3.0910 | 18.4230 | 1.5065 | 123.0863 | 497.8661 | 1003.732 | 1015.854 | 0.0858 (0.2097) |
| LSE | 1.5010 | 11.6810 | 1.5300 | 265.5784 | 498.8482 | 1005.696 | 1017.818 | 0.0889 (0.1779) |
| CVE | 1.7327 | 12.7628 | 1.5650 | 265.5784 | 498.4380 | 1004.876 | 1016.998 | 0.0603 (0.6343) |

In figure 8 we have plotted the histogram of the data along with the fitted density function of the proposed model taking estimation using MLE, LSE and CVME.

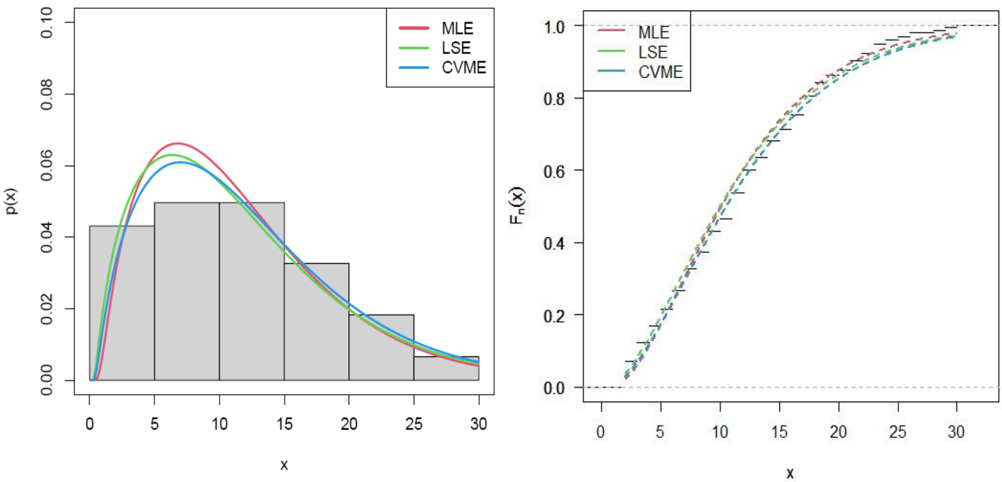



**Fig. 8.** The Histogram vs. fitted CDF (left panel) and ECDF vs. fitted CDF (right panel).

We have also illustrated the applicability of IEOLE distribution using real data set 2 and compared with different published models by researchers. Potentiality of the proposed model is compared with existing models: *Half Logistic Nadarajah Haghighi (HLNHE) distribution* (Joshi & Kumar, 2020a), *Lindley Inverse Weibull (LIW) distribution* (Joshi & Kumar, 2020b), *Exponentiated Generalized inverted Exponential (EGIE) distribution* (Oguntunde, Adejumo & Balogun, 2014), *Lomax Exponentiated Weibull (LEW) distribution* (Ansari & Nofal, 2020), *Generalized Inverted Generalized Exponential (GIGE) distribution* (Oguntunde & Adejumo, 2015). For the assessment of the fit of the proposed model, we have calculated negative log likelihood values, AIC, BIC, CAIC, and HQIC of the models and the values obtained are presented in Table 9.

**Table 9.** Log-likelihood (LL), AIC, BIC, CAIC and HQIC of IEOLE.

| Model | -LL | AIC | BIC | CAIC | HQIC |
|---|---|---|---|---|---|
| IEOLE | 497.8661 | 1003.732 | 1015.854 | 1005.967 | 1010.621 |
| HLNHE | 506.3778 | 1018.756 | 1027.847 | 1018.917 | 1022.440 |
| EGIE | 507.7668 | 1021.534 | 1030.625 | 1021.695 | 1025.225 |
| LEW | 512.8346 | 1031.669 | 1040.761 | 1031.830 | 1035.362 |
| LIW | 514.9643 | 1035.929 | 1045.020 | 1036.090 | 1039.622 |
| GIGE | 520.2300 | 1046.460 | 1055.551 | 1046.621 | 1050.153 |

The histogram versus the density function of fitted distributions and Empirical distribution function versus estimated distribution function of IEOLE distribution and some selected distributions are presented in Figure 9.

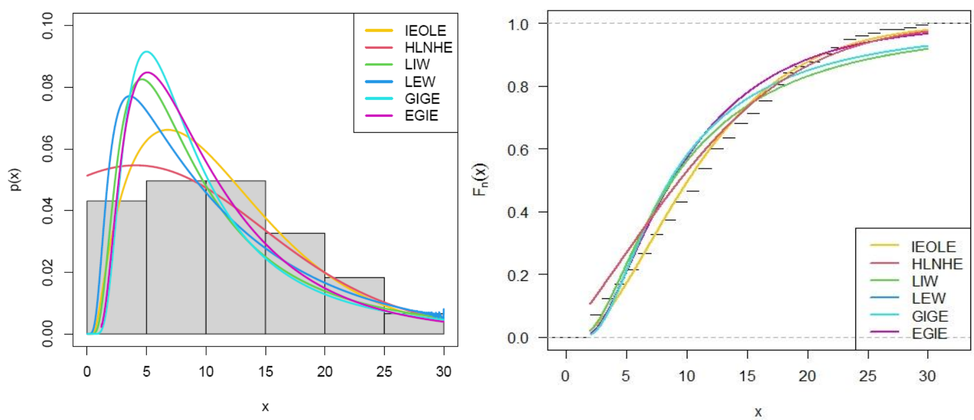



**Fig. 9.** The Histogram and the density function of fitted distributions (left panel) and empirical distribution function with estimated distribution function (right panel).

Table 10 shows the Kolmogrov - Smirnov (KS), Anderson-Darling ($A^2$), and Cramer's-Von Mises ($W^2$) statistics with corresponding p-values considered distributions including the proposed model.

**Table 10.** KS, $A^2$ and $W^2$ statistics with corresponding p-values of different models.

| Model | KS (p-values) | $A^2$ (p-values) | $W^2$ (p-values) |
|---|---|---|---|
| IEOLE | 0.0859 (0.2095) | 1.2322 (0.2556) | 0.1608 (0.3590) |
| HLNHE | 0.1099 (0.0497) | 3.4407 (0.0165) | 0.0906 (0.3489) |
| LIW | 0.1421 (0.0042) | 4.2426 (0.0067) | 0.7318 (0.0106) |
| LEW | 0.1362 (0.0069) | 3.8155 (0.0108) | 0.7286 (0.0108) |
| GIGE | 0.1622 (0.0006) | 5.6256 (0.0015) | 0.9111 (0.0040) |
| EGIE | 0.1603 (0.0008) | 4.7960 (0.0036) | 0.8598 (0.0052) |

The proposed model has least test statistics and maximum p- values. This indicates that the proposed model fits data well than the other considered existing models.

## CONCLUSION

In this study, we have presented a new distribution called *Inverse exponentiated odd Lomax exponential distribution*. Study of some statistical properties of the proposed distribution including the expressions for its survival function, hazard rate function, and the random deviate generation are done. The PDF curves of the proposed distribution have shown that its shape is increasing-decreasing and right- skewed. Applicability of the model has been tested by considering two sets of real data. Three important estimation methods, maximum likelihood estimation (MLE), least-square estimation (LSE), and Cramer-Von-Mises estimation (CVME) methods are used for the parameter estimation. The Q-Q plot, the P-P plot and other information criteria are applied for testing the validity of the model. From the analysis of two real life time data sets, we have found that the suggested model fits well as compared to some considered existing models. Also, the graph of the hazard function is decreasing, monotonically increasing and inverted bathtub shaped according to the value of the model parameters. The applicability and suitability of the proposed distribution shows that the proposed distribution is more flexible as compared to some considered existing distributions and therefore, the proposed model can be used as an alternative model in the future to analyze the survival and life time data.

## CONFLICT OF INTEREST

We, the authors, hereby declare that there is absence of conflict of interest.

## REFERENCES

Almetwally, E. M. (2021). Application of COVID-19 pandemic by using odd Lomax-G inverse Weibull distribution. *Mathematical Sciences Letters*, *10*(2), 47-57.

Alzaatreh, A., Lee, C., & Famoye, F. (2013). A new method for generating families of continuous distributions. *Metron*, 7(1), 63-79.

Ansari, S. I., & Nofal, Z. M. (2020). The Lomax exponentiated weibull model. *Japanese Journal of Statistics and Data Science*, *4*(1), 21-39.

Ashour, S. K., & Eltehiwy, M. A. (2015). Exponentiated power Lindley distribution. *Journal of advanced research*, *6*(6), 895-905.

Birnbaum, Z. W., Saunders, S. C. (1969b). Estimation for a family of life distributions with applications to fatigue. *Journal of Applied Probability*, *6*, 328347.

Ceren, Ü. N. A. L., Cakmakyapan, S., & Gamze, Ö. Z. E. L. (2018). Alpha power inverted exponential distribution: Properties and application. *Gazi University Journal of Science*, *31*(3), 954-965.

Chakrabortya, T. (2019). A note on the Lomax distribution. *Math. Stat,* ArXiv.1911.12612v1.

Chaudhary, A. K., & Kumar, V. (2020). The Logistic Lomax Distribution with Properties and Applications. *International Journal of New Technology and Research*, *6*(12), 74-80.

Chaudhary, A. K., & Kumar, V. (2021). The ArcTan Lomax distribution with properties and applications. *International Journal of Scientific Research in Science, Engineering and Technology*, *8*(1), 117-125.

De Brito, C. R., Rêgo, L. C., de Oliveira, W. R., & Gomes-Silva, F. (2019). Method for generating distributions and classes of probability distributions: The univariate case. *Hacettepe Journal of Mathematics and Statistics*, *48*(3), 897-930.

Dhungana, G. P., & Kumar, V. (2022). Exponentiated Odd Lomax Exponential distribution with application to COVID-19 death cases of Nepal. *PlOS one*, *17*(6), e0269450.

Falgore, J. Y., & Doguwa, S. I. (2020). Inverse Lomax-exponentiated g (il-eg) family of distributions: properties and applications. *Asian Journal of Probability and Statistics*, *9*(4), 48-64.

Ghitany, M. E., Al-Awadhi, F. A., & Alkhalfan, L. (2007). Marshall–Olkin extended Lomax distribution and its application to censored data. *Communications in Statistics-Theory and Methods*, *36*(10), 1855-1866.

Government of Nepal Ministry of Health and Population (2020). *Health sector response to COVID-19.* SitRep#319.

Gupta, R. D., & Kundu, D. (1999). Theory & methods: Generalized exponential distributions. *Australian & New Zealand Journal of Statistics*, *41*(2), 173-188.

Gupta, R. D., & Kundu, D. (2001). Exponentiated exponential family: an alternative to gamma and Weibull, *Biometrical Journal*, *43*, 117- 130.

Ijaz, M., & Asim, S. M. (2019). Lomax exponential distribution with an application to real-life data. *PloS one*, *14*(12), e0225827.

Joshi, R. K., & Kumar, V. (2020a). Half Logistic NHE: Properties and Application. *International Journal for Research in Applied Science & Engineering Technology, 8*(IX), 741-753.

Joshi, R. K., & Kumar, V. (2020b). Lindley inverse Weibull distribution: Theory and Applications. *Bull. Math. & Stat. Res*, *8*(3), 32-46.

Joshi, R.K. & Kumar, V. (2021). The Logistic Inverse Lomax Distribution with Properties and Applications. *International Journal of Mathematics & Computer Research*, *9*(1) 21692177.

Kundu, D., & Raqab, M. Z. (2005). Generalized Rayleigh distribution: different methods of estimations. *Computational Statistics & Data Analysis*, *49*(1), 187-200.

Lemonte, A. J. (2013). A new exponential-type distribution with constant, decreasing, increasing, upside-down bathtub and bathtub-shaped failure rate function. *Computational Statistics & Data Analysis*, *62*, 149-170.

Maxwell, O., Chukwudike, N. C., & Bright, O. C. (2019). Modeling lifetime data with the odd generalized exponentiated inverse Lomax distribution. *Biom Biostat Int J*, *8*(2), 39-42.

Mudholkar, G. S., & Srivastava, D. K. (1993). Exponentiated Weibull family for analyzing bathtub failure-rate data. *IEEE Transactions on Reliability*, *42*(2), 299-302.

Ogunsanya, A. S., Sanni, O. O., & Yahya, W. B. (2019). Exploring some properties of odd Lomax-exponential distribution. *Annals of Statistical Theory and Applications*, *1*, 21-30.

Oguntunde, P. E., Adejumo, A., & Balogun, O. S. (2014). Statistical properties of the exponentiated generalized inverted exponential distribution. *Applied Mathematics*, *4*(2), 47-55.

Oguntunde, P. E., & Adejumo, A. O. (2015). The generalized inverted generalized exponential distribution with an application to a censored data. *Journal of Statistics Applications & Probability*, *4*(2), 223-230.

Rady, E. H. A., Hassanein, W. A., & Elhaddad, T. A. (2016). The power Lomax distribution with an application to bladder cancer data. *SpringerPlus*, *5*(1), 1-22.

Sarhan, A. M., & Apaloo, J. (2013). Exponentiated modified Weibull extension distribution. *Reliability Engineering & System Safety*, *112*, 137-144.

R Core Team. (2022). *R: A language and environment for statistical computing*. R Foundation for Statistical Computing, Vienna, Austria. https://www.R-project.org/

Tahir, M. H., & Nadarajah, S. (2015). Parameter induction in continuous univariate distributions: Well-established G families. *Anais da Academia Brasileira de Ciências*, *87*, 539-568.

Tukey, J. W. (1977). *Exploratory Data Analysis*. Pearson Modern Classic.

Usman, R. M, Haq, M., & Talib, J. (2017). Kumaraswamy half-logistic distribution: properties and applications. *J Stat Appl Probab.*, *6*,597–609.

**Reference** to this paper should be made as follows:
Chaudhary, A. K., Telee, L. B. S., & Kumar, V. (2022). Inverse exponentiated odd Lomax exponential distribution: Properties and applications. *Nep. J. Stat, 6,* 29-50.