Gravity for Geodetic Purpose: Geoid-Ellipsoid Separation and Orthometric Height System

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ABSTRACT

Gravity plays very crucial role in geodetic issues as gravity field determines the shape of the earth. This paper reviews – beginning from defining the gravity, various applications of gravity in geodesy and in other fields, nature of gravity field of our terrain, instruments of gravity measurement, and techniques of gravity survey. This paper first gives information about gravity networks in Nepal, presents an equation to compute orthometric correction (OC) from height obtained from levelling. Gravimetric geoid-determination method is explained here with various gravity reduction methods and topographic-isostatic gravity reduction method is recommended for N- separation computation as this method results gravity anomalies near to zero.

1. INTRODUCTION

Any object on the surface of the earth feels two kinds of force – attraction of gravitation from earth and centrifugal push because the earth is rotating. The combined/resultant attraction of both gravitational acceleration and centrifugal acceleration is termed as gravity. Gravity is then attraction of the earth to unit mass and expressed as acceleration due to gravity. The SI unit of gravity is m/s2 but the unit used in geodesy is Gal unit. 1cm/s2 = 1Gal. The geodetic works such as geoid determination purpose and orthometric correction need to measure gravity value upto milliGal (mGal) to microGal (μGal) accuracy.

The attraction is 978 Gal at equator and 983.2 Gal at pole, 0.53% greater than that of equator and average value is of 980 Gal around the surface of the Earth. The uneven density of Earth's crust causes the variation of gravity around the earth's surface. Longer the distance of an object from center of the earth, weaker the gravity, thus gravity changes according to elevation. And because of the rotation of the earth on its axis, gravity changes according to latitude of a point.

Talking about nature of gravity field of Nepal, terrain of Nepal is one of the most complex geography of the planet. The latitudinal extent of Nepal is 26°N to 30°N. The country has three main physiographic region: plains (Terai), hills and mountains (Pahad) and Himalayan region. The elevation varies from lowest 60m at Terai to 8848m, the highest peak in the world. Terai is fairly level ground so not much of gravity variation occurs, hills are undulating topography and mountains and Himalayas are highly undulating terrain so gentle to extreme variation of gravity can be expected. Geodesist Niraj Manandhar quotes on his article titled "Geoid of Nepal from Airborne Gravity Survey" that more than 400mGal gravity variation is seen while moving from Indian plains to Tibetan plateau. Another fact is that the Himalayas are above the fault line of Indian plate and Eurasian plate and Indian plate is slowly creeping under the Eurasian plate as such gravity variation is expected.

2. APPLICATION of GRAVITY

There are various application of gravity information. This paper focus on use of gravity for geodetic purpose. The gravity is used to determine deflection of vertical, which is used to reduce the baseline length of triangulation points at surface to elliposoidal surface, thus needed in first order triangulation net. The height difference from spirit levelling doesn't equal to the actual orthometric height difference. The level height difference from levelling combined with gravity measurement yields true physical heights. The gravity anomalies are used to compute geoid-ellipsoid separation, and the gravity anomalies are computed from gravity measurements, thus needed for geoid definition of region.

The gravity reveals the subsurface density variation which is useful for exploration for oil, gas, and minerals. Geotechnical engineers use gravity information whether the subsurface voids exists below the planned nuclear building site. Different geological formations in Earth have different density structures and hence different gravity signals. Geophysicist use gravity information to study internal mass distribution of Earth's crust. Gravity field of the earth gives clue to evaluate orbits of satellites.

3. MEASUREMENT of GRAVITY

The measurement of gravity on or above the surface of the earth is called gravimetry. The instrument to measure gravity is gravimeter. Gravimeters are basically of three types based on principle they operate.

- Pendulum gravimeter
- Spring balance gravimeter
- Free-fall gravimeter

When a gravimeter measures/gives directly the value of gravity at a point independent of gravity values of any other point, then this is the absolute measurement of gravity and value is called the absolute gravity. When the gravity is determined relative to base gravity station whose gravity value is already known or determined, then this is called relative gravity measurement and relative gravity value.

Pendulum gravimeters were extensively used for 300 years till middle of the 20thcentury. The physical pendulum is used as gravimeter implementing formula of mathematical pendulum which use the length of the pendulum and swing period to result the value of g. Gravimeters based on principle of pendulum can be used as both absolute and relative gravimeter. The accuracy achieved was better than 1mGal upto 0.1mGal.

Since 1930 gravimeters based in principle of spring balance developed and most used for relative gravimetry measurement at present. LaCoste & Romberg (LCR) gravimeter implements the principle of astatic spring to determine the gravity value relative to base station. Most used relative gravimeters are LCR and Worden gravimeters. The accuracies achieved is 0.01mGal (10µGal). Even better accuracy of 1µGal can be obtained when used as stationary gravimeters.

The gravimeters based on principle of free-fall are used as absolute gravimeter. Time taken for an object to fall a certain vertical distance is used to get the value of g. Interferometric distance measurement and electronic time recording is implemented to develop free-fall gravimeters. JILA absolute gravimeters, FG5 absolute gravimeters are available for absolute gravity measurements and accuracies is in range of 1-10µGals.

4. NATIONAL CONTEXT OF GRAVITY SURVEY

The department of survey has conducted several gravity survey in different time period in cooperation with different foreign organization with the purpose of establishing national geodetic datum, national gravity network, absolute gravity measurement for crustal movement studies, topographic mapping, nationwide geoid definition. At present Survey Department is working towards defining geoid model of Sagarmatha region.

4.1 Gravity Survey during 1981- 1984

Ministry of Defense UK (MoDUK) established base gravity station network of Nepal comprising of one fundamental gravity base station named KATHMANDU J, 45 gravity base stations, and 375 gravity detail stations located at airports, government buildings, and road accessible places. Fundamental gravity base station KATHMANDU J established at Tribhuvan International Airport (TIA) as a part of International Gravity Standardized Net 1971 (IGSN1971)- the gravity value of this station was transferred from BANGKOK in 1981. The gravity value of same station was also transferred from Srilankan IGSN71 station 029691 COLOMBO was made as a check.

4.2 Absolute Gravity Measurement in 1991

Survey Department and Universtiy of Colorado, Boulder and National Geospatial Agnecy (NGA) USA measured absolute gravity value of Fundamental Absolute Gravity Station (FAGS-1) established at Nagarkot Geodetic Observatory using Joint Institute for Laboratory Astrophysics (JILA) gravimeter with microgal accuracy. Other gravity stations: Nagarkot GPS, Kathmandu airport, Simara airport, and Simara GPS were established transferring gravity value from FAGS-1.

4.3 Gravity Survey in 1993 by ENTMP (Eastern Nepal Topographic Mapping Project)

47 relative gravity stations mostly in hilly area where levelling information lacks were established using FAGS-1 of Nagarkot and gravity base station KATHMANDU J and Simara J as reference gravity stations. These 47 gravity stations and existing 30 gravity stations combined with GPS observations over same gravity stations and with spirit levelling information – a geoid model was developed to orthometric height information for the preparation of topographic map of eastern region.

4.4 Gravity Survey in 1997 by WNTMP (Western Nepal Topographic Mapping Project)

This project utilized data from GPS, spirit levelling, surface gravity information from past gravity survey, EGM96 with new gravity observations at 52 GPS stations and "NEPAL97" geoid was developed by Finnish Geodetic Institute.

4.5 Airborne Gravity Survey in 2010

Survey Department conducted airborne gravity survey within the national boundary in cooperation with DTU-Space, Denmark and NGA USA collecting the gravity information of whole country for the first time. Lacoste Romberg- S type gravimeter and Checkan- AM gravimeter were installed in aircraft with GPS receivers on both onboard aircraft and ground for positioning purpose. The gravity station at Kathmandu airport and at Nagarkot was used as reference station. The accuracy achieved was 3.3mgal for collected gravity data. Geoid thus developed was of accuracy of 10-20 cm.

4.6 Gravity Survey at Present

With purpose of measuring height of Sagarmatha after 2015 April major earthquake on its own effort, Survey Department is determining orthometric height of a Sagarmatha peak using ellipsoidal height from GNSS survey combined with geoidal undulation value of a peak. This geoid-ellipsoid separation at a peak will be extracted from geoid around Sagarmatha region. For that gravity stations are being planned at spacing of 10km and the work is carrying out. Another approach that Survey Department adopting to get height of Sagarmatha peak is trigonometrical/triangulation levelling. The data from trigonometrical/triangulation levelling need to be refined using gravity information of surrounding.

While carrying out gravity survey of any region for any purpose, first absolute gravity survey is carried out at least on two points to establish base gravity station which then have gravity value and then relative gravity survey of all other points relative to those base gravity station is carried out. In our context, base gravity stations are established during ENTMP and WNTMP including Nagartkot FAGS-1, KATHMANDU J, SIMARA J etc. Those absolute gravity stations can be used as base station to carry out relative gravity survey. But it has been more than 30 years that gravity value of those stations measured and gravity has characteristics of temporal variation. There have been shift of masses due to April 25, 2015 major earthquake causing change in gravity. So values could be no longer true. This puts the requirement of new absolute gravity measurement and then only relative gravity survey.

At present, in context of our country, we need to conduct gravity survey for two geodetic purpose: geoid determination of country and orthometric correction (OC) of heights obtained from leveling. For geoid determination purpose, gravity stations are distributed gridwise basis. Such points are distributed sparse in fairly level ground such as in Terai, dense in undulating regions such as in Pahad and much denser networks of gravity points in highly undulating mountainous regions. In case of gravity measurements to determine OC of leveling heights, gravity is measured at every PBMs. Relative gravity survey is conducted for gridwise surface gravity points for geoid computation and for PBMs of leveling relative to base station.

5. ORTHOMETRIC CORRECTION

What's wrong with heights obtained from precise levelling? Spirit levelling is used to determine height difference between points situated at a distance. Spirit levelling assume level surfaces are parallel- but when levelling is carried out in long run- the level difference as measured by differential levelling does not exactly equal to actual orthometric height differences. When a levelling is carried out in close circuit following different level route, the sum of level differences must be zero but it doesn't. This is because of non-parrallel nature of equipotential surfaces. Equipotential surfaces become close to each other where gravity is strong and distant where gravity is weak. Hence to define the unique height of a point which is orthometric height, the level difference (measured height difference) obtained from levelling must be corrected with orthometric correction (OC). So the levelling combined with gravity measurements with mathematical modelling yields true orthometric height of point. So gravity need to be measured at every PBMs of level alignment.

Peter Vanieck mentioned in his book that while levelling in Alpine road from Biasca to St Bernardo of distance 50km climbing from elevation of 300m to 2000m, the OC computed is 23cm which is 30 times larger than the tolerance limit for precise levelling. Mader (1954) experienced while levelling from 754m altitude to 2505m altitude, the orthometric correction is about 15cm per 1km of measured height difference.

C. Hwang developed the equation to compute OC to be applied to measured level difference to get orthometric height difference between any two PBMs. According to him, if point A is sufficiently close to point B in horizontal distance (below 2km), following equation provides the OC:

$$OC_{AB} = \frac{1}{g_B}\left[\frac{(g_A + g_B)}{2} - \overline{g_B}\right]\Delta n_{AB} + H_A(\frac{\overline{g_A}}{\overline{g_B}} - 1)$$

Where

OC_{AB} is the orthometric correction to be applied.

g_A is the gravity value measured at point A

g_b is the gravity value measured at point B

$\overline{g_A}$ is the mean gravity value along the plumbline through point A

$\overline{g_b}$ is the mean gravity value along the plumbline through point B

Δn_{AB} is the measured level difference between A and B

The mean gravity value $\overline{g}$ along plumbline through point is computed by following equation:

$$\overline{g} = g + 0.042H^M \qquad (2)$$

Where g is the gravity is value at a point (PBM) and H^M is height from levelling without applying OC. Since our PBMs of national leveling alignment is situated at distance of 2km and gravity can be assumed to be linear function of height as well as horizontal distance, above equation of determining OC applicable in our case. Helmut Mortiz mentioned that it is sufficient to measure gravity values at distance of some kilometers with the purpose of computing OC.

6. GRAVITY MEASUREMENT and GEOID DETERMINATION

Though there are various approach of geoid determination, one approach is gravimetric geoid determination using gravity. The gravity can be measured through terrestrial gravity measurement, airborne gravity measurement and spaceborne gravity measurement. This paper discusses computation of N separation using surface gravimetry data.

GNSS survey combined with geoidal undulations gives the orthometric height. The latitude and longitude gives the horizontal position of a point and the carefully measured ellipsoidal height in addition with geoid-ellipsoid separation information yields physically meaningful orthometric height with following equation:

$$H = h - N \qquad (3)$$

Where H is the height above geoid called orthometric height, h is height above ellipsoid surface called geometric height and N is the geoid – ellipsoid separation.

GNSS survey technology has become popular, timely, precise, and cost effective. As Geodeist Niraj Manandhar tells in his article titled "Concept towards cm-geoid for Nepal and GPS to replace conventional levelling using airborne gravity" that vertical component from GNSS survey paired up with geoid information can replace costly, tedious, and cumbersome spirit leveling as well as providing the elevation information where leveling seems inaccessible and cannot be carried out. This puts out having knowledge of *N*- separation is must.

Gravity anomalies which is the difference between observed then reduced gravity value to sea level and theoretical/normal gravity value, are input to Stoke's formula, which generates *N*- value, vertical distance between geoid and ellipsoid. Before computing the anomalies, the surface gravity value is reduced to sea level using free-air reduction approach, Bouger reduction approach, and topographic-isosatic reduction approach.

Free – air reduction is carried out such that gravity station is assumed to hang in free air above certain height from sea level. The gravity value is reduced certain height down to sea level then the difference between free-air reduced gravity value and normal gravity value yields free-air anomalies. The Bouger reduction takes care of attraction of masses between geoid and surface gravity station, the reduction of surface gravity value by subtracting the attraction of Bouger plate and combined with free-air reduction yields the Bouger reduced gravity value. The deviation of Bouger gravity value at sea level form normal gravity value is called Bouger anomalies. Bouger reduction can be more refined by taking terrain deviation from Bouger plate and applying terrain correction to get the refined Bouger reduction and thus anomalies. Theory of isostasy reveals that there exist some kind of mass deficiency under the mountains below sea level and mass surplus at ocean. The reduction of gravity value by taking consideration of attraction of compensating mass below sea level is called isostatic reduction. When gravity value at surface is reduced such that the attraction of topography above sea level and attraction of compensating masses along with free-air reduction, then such reduction is called topographic-isostatic reduction. The equation of topographic-isostatic reduction:

$$g_{TI} = g - A_T + A_C + F \qquad (4)$$

Where

g_{TI} is topographic – isostatically reduced gravity value

g gravity value at surface

A_T attraction of topography

A_C attraction of compensation

F free-air reduction

Then topographic-isostatic anomlies is given by

$$\Delta g_{TI} = g_{TI} - Y \qquad (4)$$

$$\Delta g_{TI} = g_{TI} - \gamma \qquad (5)$$

According to Helmut moritz, topographic – isostatic anomalies are best suited to compute geoidal separation/ N-computation.

7. CONCLUSION and RECOMMENDATIONS

Gravity varies in magnitude and direction both around the surface of the earth and should be measured and refined for geodetic problems solving. Nepal has rugged terrain beside Terai region. Gravity survey should be carried out covering and distributed all over the country to get the nationwide gravity model. The absolute gravity stations established in past must be reobserved because of temporal varying nature of gravity. All the levelling networks that we have must be orthometrically corrected using above mentioned equation as gravity field can produce larger magnitude of OC compared to tolerance of precise levelling. For gravimetric geoid-determination approach using data from surface gravimetry topographic-isostatic anomalies should be used.

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