Modified exponentiated inverted exponential distribution with applications to life time dataset

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Abstract

In this article, we have introduced a new probability distribution called Modified Exponentiated Inverted Exponential (MEIE) Distribution. The introduced model is right tailed increasing- decreasing hazard rate function. We have discussed some statistical properties like probability density function, cumulative distribution function, survival function and hazard rate function etc. Model parameters are estimated via Maximum Likelihood, Least Squares, and Cramer-Von Mises estimation techniques. To check the applicability a real data set is taken and the model validation is checked using different information criteria as well as using some graphical techniques. For model comparison we have considered some published probability model.

Keywords: Density function, Model Validation, Information Criteria, Cumulative Distribution

Introduction

Medical, environmental, architecture, biological sciences, applied statistics, ecology, reliability and accounting are the different fields where life time dates play crucial role during modeling. In theory there are a lot of models and distribution using different sets of life time data. Validity and reliability of the techniques used in data analysis is mostly based on the distributions used. Different ways of formulating the new distribution can be found in theory during past few years. Although there many distributions available but still there are some real-life data are available that cannot be explained and analyzed by the models available.

Some typical approaches for generating new continuous distributions found in the literature are (i) distribution compounding, (ii) distribution mixing, (iii) using any distribution as a generator, (iv) power transformation techniques, and (v) inverse transformation techniques.

The exponential distribution has played powerful role in the study of data. The exponential distribution, in probability and statistics, forms a continuous family of probability distributions used for generating new ones. It was the pioneering model for lifetime analysis and statistical techniques. The memory-less property of the exponential distribution is used for life testing of the products that do not age with time. Failure rate of various types of devices does not depend upon their age and, therefore, the Exponential distribution is considered for study of the failure rate in those cases.

Several modifications and generalization of the exponential distribution for life time data analysis have been established by different researchers in the literature by taking the exponential distribution as the baseline model. Some of novel models developed by modifying and generalizing exponential model aregeneralized exponential model (Gupta &Kundu, 2001) and beta exponential model (Nadarajah & Kotz, 2006). Other modified models are beta generalized exponential given by (Barreto-Souza et al. ,2010), Kumaraswamy exponential (Cordeiro& de Castro, 2011), and gamma exponentiated exponential model (Ristic & Balakrishnan, 2012). Merovci, (2013) found transmuted exponentiated exponential distribution and Louzada et al. (2014) gave exponentiated exponential geometric model. Other modified models are Kumaraswamy transmuted exponential distributions(Afifya et al., 2016), modified exponential distribution (Rasekhi et al.,2017),Logistic

Modified Exponential Distribution (Chaudhary& Kumar,2020a), Half Logistic Modified Exponential Distribution(Chaudhary & Kumar, 2020b), Arctan Exponential Extension Distribution(Chaudhary & Kumar, 2021), Modified NHE Distribution (Chaudhary & Sapkota, 2021),Modified Inverse NHE Distribution (Chaudhary et al., 2022) and Half Cauchy-Modified Exponential Distribution (Chaudhary & Kumar, 2022).

Lemonte (2013) recommended a novel exponential-type model with a failure rate function that is inverted bathtub, constant, decreasing, bathtub-shaped and increasing. Inverse exponential (IE) model was given by (Keller, et al., 1982) which is very suitablefor real life modeling phenomena where failure rate is of inverted bathtub shaped. TheInverted exponential distribution (IED) has studied by (Dey, 2007) as a life distribution model from a Bayesian viewpoint.

The IE distribution is extended and generalized for formulating, analyzing the Generalized Inverse Exponential (GIE) model by (Abouammoh, et al.,2009), the beta inverted exponential distribution (Singh & Goel, 2015) and Logistic Inverse Exponential Distribution (Chaudhary & Kumar, 2020c).

Exponentiated distributions have been extensively explored in statistics since 1995. Numerous authors have introduced different classes, including Mudholkar et. al., (1995), who proposed the Exponentiated Weibull distribution. After that researcher proposed many standard models using the Exponentiated distributions. Exponentiated Exponential distribution having two constants was firstly given by (Gupta & Kundu ,2001). The Exponentiated Weibull, Exponentiated Gamma, Exponentiated Gumbel as well as Exponentiated Frechet models, defined and studied by Nadarajah and Kotz (2006). Flaih et al. (2012) used extra parameter for extending the IW distribution resulting the exponentiated inverted Weibull (EIW) distribution. Models of Exponentiated type is being applied in many areas of biology and engineering; see Cordeiro et al (2013) for details. A generalization of the Generalized Inverse Exponential distribution called the Exponentiated Generalized Inverse Exponential distribution has been defined and studied by (Oguntunde et al ,2014). Chaudhary and Kumar (2014) have been developed three-parameter exponentiated log-logistic distribution under Bayesian approach. Fatima and Ahmad (2017) introduced the new family of distribution called Exponentiated inverted Exponential distribution (EIED) and found that that the density function of EIED distribution is unimodal and positively skewed and the hazard rate function is increasing- decreasing and it shows an inverted bathtub shape.

Ilori and Jolayemi (2021) introduced the Weighted Exponentiated Inverted Exponential distribution as a modification of the Exponentiated Inverted Exponential distribution (WEIED). The hazard function of the Weighted Exponentiated Inverted Exponential Distribution exhibits unimodal (inverted bathtub) and decreasing shapes.

Exponential distribution, a special case of Rayleigh, Gamma and Weibull distribution has important role in statistics. Events of this distribution are independent, continuous having constant average rate. Exponential distribution was modified to generate Inverted exponential distribution. In literature, inverted exponential distribution (Dey, 2007) is used as baseline distribution withcumulative distribution function given as:

$$G(x) = e^{-\lambda/x}, x > 0, \lambda > 0 \tag{1.1}$$

Mudholkar et al. (1995) developed the Exponentiated Weibull Family. An extra positive parameter is raised to the cumulative distribution function for generating Exponential distribution. The additional parameter characterizes the shape of the resulting distribution (Lemonte et al., 2013). If X is a r.v. having distribution G then by exponentiation of G, cumulative distribution function can be obtained as,

$$Q(x) = [G(x)]^{\alpha} \tag{1.2}$$

where G(*x*) is the CDF of the parent distribution.

Let X be a random variable of an inverted exponential distribution (IED) with CDF G(x). Then $Q(x) = [G(x)]^{\alpha}$ is a cdf of the exponentiated inverted exponential (EIE) distribution with scale parameter λ and shape parameter α.

This model has been developed by (Fatima & Ahmad ,2017). Thus, the cumulative distribution function of the exponentiated inverted exponential (EIE)distribution takes the following form:

$$Q(x) = [e^{-\lambda/x}]^{\alpha}, x > 0, \lambda > 0, \alpha > 0. \quad (1.3)$$

To get more flexibility of the distribution, we have proposed a new continuous probability called *modified exponentiated inverted exponential distribution* by modifying the exponentiated inverted exponential distribution with introducing one more scale parameter β. We denote it by $MEIE(x;\alpha,\beta,\lambda)$. Thus, the cumulative distribution function of MEIE distribution is given by

$$F(x) = \left[\exp\left\{(-\lambda/x)(\exp(-\beta x))\right\}\right]^{\alpha} \quad ; \; x > 0, \alpha, \beta, \lambda > 0 \quad (1.4)$$

Here, a real data set is considered for testing the applicability of the proposed model $MEIE(x;\alpha,\beta,\lambda)$.

The study is structured into distinct sections. The introduction covers literature and related probability models. The model analysis section presents graphs for cumulative distribution and probability density function. Additionally, it explores properties like quantile function, skewness, and kurtosis. Parameters estimation techniques are maximum likelihood, least squares, and Cramer-Von Mises estimation. The model is used on a one real data set in the Applications to Real Data Sets section. Next section is model comparison section where model is compared with some previously defined models. The conclusion section summarizes the study, while the final section lists its references.

Model Analysis

Modified Exponentiated Inverted Exponential Distribution

Let X is continuous random variable following $X : MEIE(x;\alpha,\beta,\lambda)$ then CDF and PDF of MEIE are given as,

$$F(x) = \left[\exp\left\{(-\lambda/x)(\exp(-\beta x))\right\}\right]^{\alpha} \quad ; \; x > 0, (\alpha, \beta, \lambda) > 0, \quad (2.1)$$

$$f(x) = \alpha\lambda\left[\exp\left\{(-\lambda/x)(\exp(-\beta x))\right\}\right]^{\alpha}\left[\frac{\exp(-\beta x)}{x}\right]\left(\beta + \frac{1}{x}\right) \quad (2.2)$$

Reliability/Survival Function

The survival function of MEIE is given as,

$$S(x) = 1 - F(x) = 1 - \left[\exp\left\{(-\lambda/x)(\exp(-\beta x))\right\}\right]^{\alpha}$$

(2.3)

Hazard Rate Function

The hazard rate function HRF of the proposed model MEIE is given as,

$$h(x) = \left[\alpha\lambda\left[\exp\left\{(-\lambda/x)(\exp(-\beta x))\right\}\right]^{\alpha}\left[\frac{\exp(-\beta x)}{x}\right]\left(\beta + \frac{1}{x}\right)\right]$$
$$\left[1 - \left\{\exp\left\{(-\lambda/x)(\exp(-\beta x))\right\}\right\}^{\alpha}\right]^{-1} \quad (2.4)$$

Reversed Hazard Rate Function

Reversed Hazard rate function of MEIE is given below in equation (2.5)

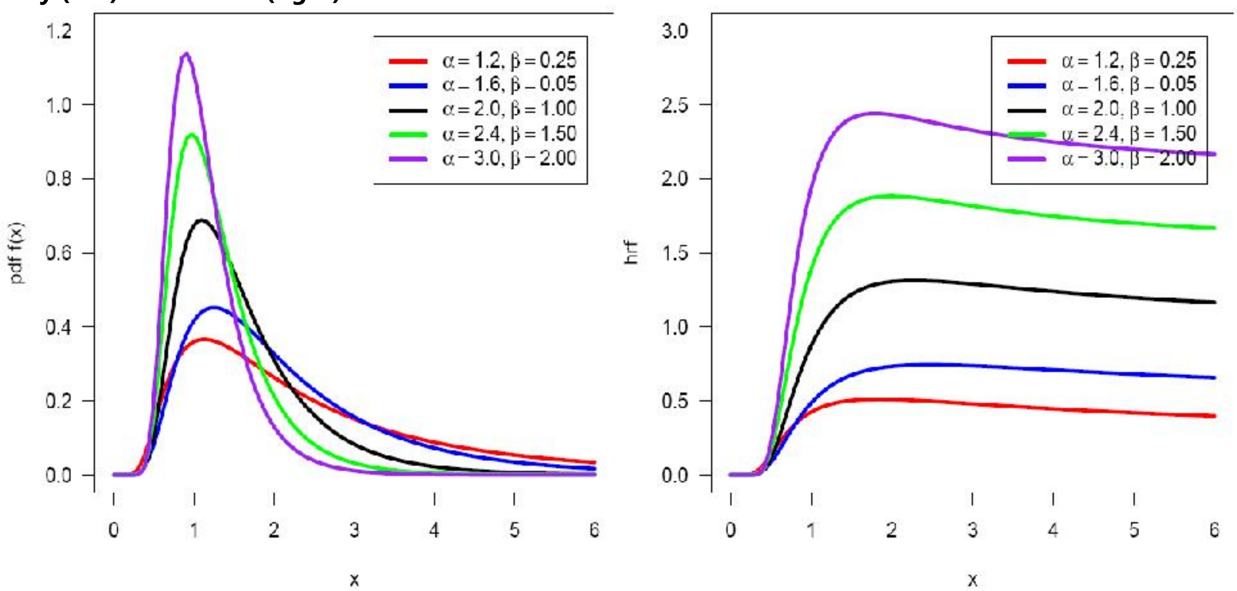
$$H(x) = \left[\alpha\lambda\left[\exp\left\{(-\lambda/x)(\exp(-\beta x))\right\}\right]^{\alpha}\left[\exp(-\beta x)/x\right]\left(\beta + 1/x\right)\right]$$

$$\left\{\exp\left\{(-^{a}/x)(\exp(-\ x))\right\}\right\}^{-\sim} \qquad (2.5)$$

PDF and HRF of the proposed model MEIE are displayed in Figure 1:

Figure 1

Density (left) and hazard (right) curves for λ= 2



Probability density curve is of various shapes and positively skewed for various values of the parameters. Shape of the HRF is initially monotonically increasing and decreasing.

Quantile Function

The quantile function MEIE can be given as,

$$\log\left\{\frac{-\log p}{a\sim}\right\}+\ \ x+\log x=0;\quad 0\le p\le 1 \qquad (2.6)$$

Solving equation (2.7) for *x* we will get the quantile function where p follows uniform distribution [0, 1].

Asymptotic property

Asymptotic property exists if $\lim_{x\to 0} f(x)$ and $\lim_{x\to\infty} f(x)$ exist with the resulting value as 0. That is, if both the limits converge to zero the proposed model satisfies the asymptotic behavior.

$$\lim_{x\to 0} f(x)=\lim_{x\to 0}\sim^{a}\left[\exp\left\{(-^{a}/x)(\exp(-\ x))\right\}\right]^{\sim}\left[\exp(-\ x)/x\right]\left(\ +1/x\right)=0$$
$$\lim_{x\to\infty} f(x)=\lim_{x\to\infty}\sim^{a}\left[\exp\left\{(-^{a}/x)(\exp(-\ x))\right\}\right]^{\sim}\left[\exp(-\ x)/x\right]\left(\ +1/x\right)=0 \qquad (2.7)$$

Skewness and Kurtosis of MEIE distribution

These are the characteristics that describes the nature of any model. Bowley's skewness of the $MEIE(\sim,\ ,^{a})$ distribution based on quartiles has form:

$$Sk(B)=\frac{Q(3/4)-2Q(1/2)+Q(1/4)}{Q(3/4)-Q(1/4)}, \qquad (2.8)$$

Kurtosis formulated by (Moors, 1988) of the $MEIE(\sim,\ ,^{a})$ distribution based on octiles has formed as

$$K(moors)=\frac{Q(7/8)-Q(5/8)+Q(3/8)-Q(1/8)}{Q(6/8)-Q(2/8)}, \qquad (2.9)$$

Parameter estimation techniques

Parameters can be estimated applying different methods. We have applied following methods.

Estimation using Maximum Likelihood (MLE)

Defining the log likelihood function for the proposed model in (3.1). Let $\underline{x} = (x_1, \ldots, x_n)$ be a random sample of size 'n' from MEIE then the log likelihood function can be written as,

$$l(\alpha, \beta, \lambda \mid \underline{x}) = n\ln\alpha + n\ln\lambda - \alpha\lambda\sum_{i=1}^{n}\frac{e^{-\beta x_i}}{x_i} - \beta\sum_{i=1}^{n}x_i - \sum_{i=1}^{n}\ln x_i + \sum_{i=1}^{n}\ln\left\{\beta + \frac{1}{x_i}\right\} \qquad (3.1)$$

After differentiating (3.1) with respect to α, β and λ, we get

$$\frac{\partial l}{\partial \alpha} = \frac{n}{\alpha} - \lambda\sum_{i=1}^{n}\frac{e^{-\beta x_i}}{x_i}$$

$$\frac{\partial l}{\partial \beta} = --\alpha\lambda\sum_{i=1}^{n}e^{-\beta x_i} - \sum_{i=1}^{n}x_i + \sum_{i=1}^{n}\left(\beta + \frac{1}{x_i}\right)^{-1} \qquad (3.2)$$

$$\frac{\partial l}{\partial \lambda} = \frac{n}{\lambda} - \alpha\sum_{i=1}^{n}\frac{e^{-\beta x_i}}{x_i}$$

Solving above first order derivatives to zero, parameters of $MEIE(\alpha, \beta, \lambda)$ can be estimated. Solution of above equation is not possible so computer programming can be used. Let $\hat{\Theta} = (\hat{\alpha}, \hat{\beta}, \hat{\lambda})$ and $\Theta = (\hat{\alpha}, \hat{\beta}, \hat{\lambda})$, are estimated constants and parameter vector respectively then resulting asymptotic normality will be, $\left(\hat{\Theta} - \Theta\right) \rightarrow N_3\left[0, \left(I(\Theta)\right)^{-1}\right]$. The Fisher's information matrix $I(\Theta)$ can be given by

$$I(\underline{\Theta}) = -\begin{pmatrix} E\left(\frac{\partial^2 l}{\partial \alpha^2}\right) & E\left(\frac{\partial^2 l}{\partial \alpha \partial \beta}\right) & E\left(\frac{\partial^2 l}{\partial \alpha \partial \lambda}\right) \\ E\left(\frac{\partial^2 l}{\partial \beta . \partial \alpha}\right) & E\left(\frac{\partial^2 l}{\partial \beta^2}\right) & E\left(\frac{\partial^2 l}{\partial \beta . \partial \lambda}\right) \\ E\left(\frac{\partial^2 l}{\partial \lambda . \partial \alpha}\right) & E\left(\frac{\partial^2 l}{\partial \lambda \partial \beta}\right) & E\left(\frac{\partial^2 l}{\partial \beta^2}\right) \end{pmatrix}$$

Asymptotic variance $\left(I(\Theta)\right)^{-1}$ of MLE is worthless because Θ cannot be obtained. Let $O(\hat{\Theta})$ be the observed fisher information matrix. Estimate $O(\hat{\Theta})$ of $I(\Theta)$ hessian matrix H can be obtained as,

$$O(\hat{\Theta}) = -\begin{pmatrix} \left(\frac{\partial^2 l}{\partial \hat{\alpha}^2}\right) & \left(\frac{\partial^2 l}{\partial \hat{\alpha} \partial \hat{\beta}}\right) & \left(\frac{\partial^2 l}{\partial \hat{\alpha} \partial \hat{\lambda}}\right) \\ \left(\frac{\partial^2 l}{\partial \hat{\beta} \partial \hat{\alpha}}\right) & \left(\frac{\partial^2 l}{\partial \hat{\beta}^2}\right) & \left(\frac{\partial^2 l}{\partial \hat{\beta} \partial \hat{\lambda}}\right) \\ \left(\frac{\partial^2 l}{\partial \hat{\lambda} \partial \hat{\alpha}}\right) & \left(\frac{\partial^2 l}{\partial \hat{\lambda} \partial \hat{\beta}}\right) & \frac{\partial^2 l}{\partial \hat{\lambda}^2} \end{pmatrix} = -H(\Theta)\big|_{\left(\Theta = \hat{\Theta}\right)}$$

Variance covariance matrix is,

$$\left[-H(\underline{\Theta})\big|_{\left(\Theta=\hat{\Theta}\right)}\right]^{-1} = \begin{pmatrix} Var(\hat{\alpha}) & Cov.(\hat{\alpha}, \hat{\beta}) & Cov.(\hat{\alpha}, \hat{\lambda}) \\ Cov(\hat{\beta}, \hat{\alpha}) & Var(\hat{\beta}) & Cov(\hat{\beta}, \hat{\lambda}) \\ Cov(\hat{\lambda}., \hat{\alpha}) & Cov.(\hat{\lambda}, \hat{\beta}) & Var(\hat{\lambda}) \end{pmatrix} \qquad (3.3)$$

Here,100(1-γ) % C.I. for α, β and λ are,

$\hat{\alpha} \pm Z_{\gamma/2}\sqrt{Var(\hat{\alpha})}$, $\hat{\beta} \pm Z_{\gamma/2}\sqrt{Var(\hat{\beta})}$ and $\hat{\lambda} \pm Z_{\gamma/2}\sqrt{Var(\hat{\lambda})}$

Estimation using Least-Square (LSE)

Let $X_{(1)} < X_{(2)} < \ldots < X_{(n)}$ is ordered r.v. and a random sample $\{X_1, X_2, \ldots, X_n\}$ of size n is taken from a distribution function F (.). We define a function A using $F(X_{(i)})$ as CDF of ordered statistics by equation (3.3).

$$A(x;\alpha,\beta,\lambda)=\sum_{i=1}^{n}\left[F(X_{(i)})-\frac{i}{n+1}\right]^2 \tag{3.3}$$

Minimizing function (3.3), parameters of $MEIE(\alpha,\beta,\lambda)$ can be obtained.

Differentiating (3.3) with respect to α, β, and λ. we get

$$\frac{\partial A}{\partial \alpha}=-2\lambda\sum_{i=1}^{n}F\left(x_{(i)}\right)\left(\frac{e^{-\beta x_{(i)}}}{x_{(i)}}\right)\left[F\left(x_{(i)}\right)-\frac{i}{n+1}\right]$$

$$\frac{\partial A}{\partial \beta}=2\alpha\lambda\sum_{i=1}^{n}F\left(x_{(i)}\right)e^{-\beta x_{(i)}}\left[F\left(x_{(i)}\right)-\frac{i}{n+1}\right]$$

$$\frac{\partial A}{\partial \lambda}=-2\alpha\sum_{i=1}^{n}F\left(x_{(i)}\right)\left(\frac{e^{-\beta x_{(i)}}}{x_{(i)}}\right)\left[F\left(x_{(i)}\right)-\frac{i}{n+1}\right]$$

Parameters can be also obtained by weighted LSE minimizing the function D in (3.4)

$$D(X;\alpha,\beta,\lambda)=\sum_{i=1}^{n}w_i\left[F(X_{(i)})-\frac{i}{n+1}\right] \text{Where, } w_i=\frac{1}{Var(X_{(i)})}=\frac{(n+1)^2(n+2)}{i(n-i+1)}$$

Using the CDF of the order statistics and weight(wi) in above expression with respect to α, β, and λ

$$D(X;\alpha,\beta,\lambda)=\sum_{i=1}^{n}\frac{(n+1)^2(n+2)}{i(n-i+1)}\left[F(X_{(i)})-\frac{i}{n+1}\right]^2 \tag{3.4}$$

Estimation using Cramer-Von-Mises (CVM) method

Using this method, parameters α, β, and λ can be estimated by minimizing the function (3.5) $Z(X;\alpha,\beta,\lambda)=\frac{1}{12n}+\sum_{i=1}^{n}\left[F\left(x_{i:n}\mid\alpha,\beta,\lambda\right)-\frac{2i-1}{2n}\right]^2$ (3.5)

Differentiating (3.5) with respect to α, β, and λ, we get,

$$\frac{\partial Z}{\partial \alpha}=-2\lambda\sum_{i=1}^{n}F\left(x_{(i)}\right)\left(\frac{e^{-\beta x_{(i)}}}{x_{(i)}}\right)\left[F\left(x_{(i)}\right)-\frac{2i-1}{2n}\right]$$

$$\frac{\partial Z}{\partial \beta}=2\alpha\lambda\sum_{i=1}^{n}F\left(x_{(i)}\right)e^{-\beta x_{(i)}}\left[F\left(x_{(i)}\right)-\frac{2i-1}{2n}\right]$$

$$\frac{\partial Z}{\partial \lambda}=-2\alpha\sum_{i=1}^{n}F\left(x_{(i)}\right)\left(\frac{e^{-\beta x_{(i)}}}{x_{(i)}}\right)\left[F\left(x_{(i)}\right)-\frac{2i-1}{2n}\right]$$

Solving $\frac{\partial Z}{\partial \alpha}=0, \frac{\partial Z}{\partial \beta}=0$ and $\frac{\partial Z}{\partial \lambda}=0$, CVM estimates can be obtained.

Application to real data set

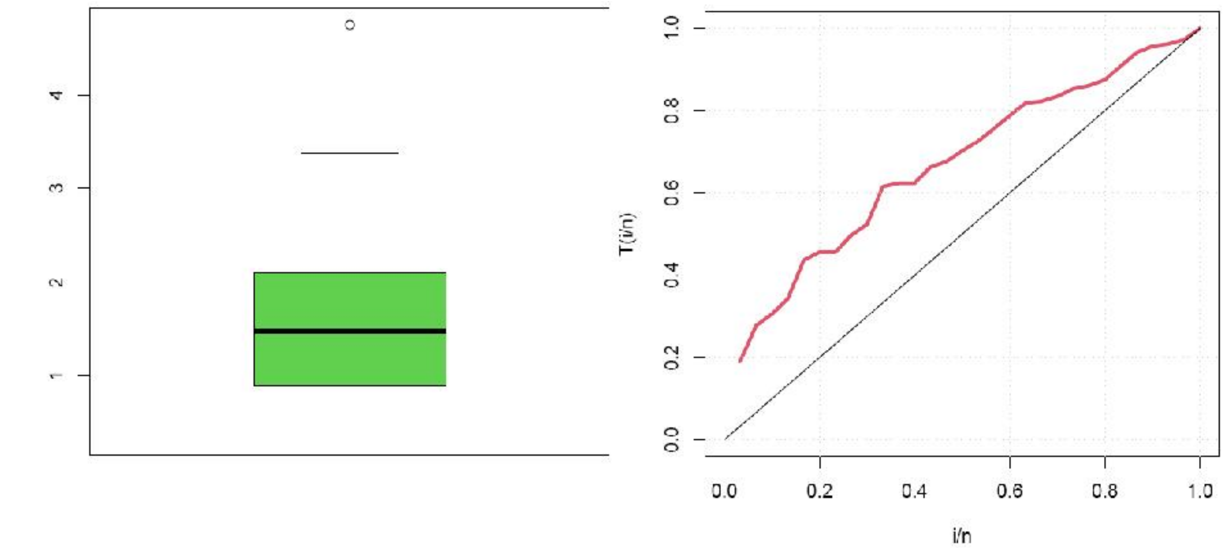
Here, the proposed model MEIE is analyzed using a real data set. The data set is purposed by Hinkley (1977) with the values as given below,

0.77, 1.74, 0.81, 1.20, 1.95, 1.20, 0.47, 1.43, 3.37, 2.20, 3, 3.09, 1.51, 2.10, 0.52, 1.62, 1.31, 0.32, 0.59, 0.81, 2.81, 1.87, 1.18, 1.35, 4.75, 2.48, 0.96, 1.89, 0.90, 2.05

Exploratory Analysis of Data

As Tukey (1977), exploratory analysis of data contains study of pattern, summary, structure as well as the graphical representation of the data. Figure 2 represent the box plot indicating that data is positively skewed and the TTT plot indicating that the hazard rate is increasing – decreasing.

Figure 2
Boxplot (left panel) and TTT plot (right Plot)



Summary analysis is presented in table 1 showing that data is right tailed with non normal curve.
Table 1
Summary Statistics

Min.	Q_1	Median	Mean	Q_3	Sd	Skewness	Kurtosis	Max.
0.32	0.915	1.47	1.675	2.087	1.001	1.088	4.207	4.75

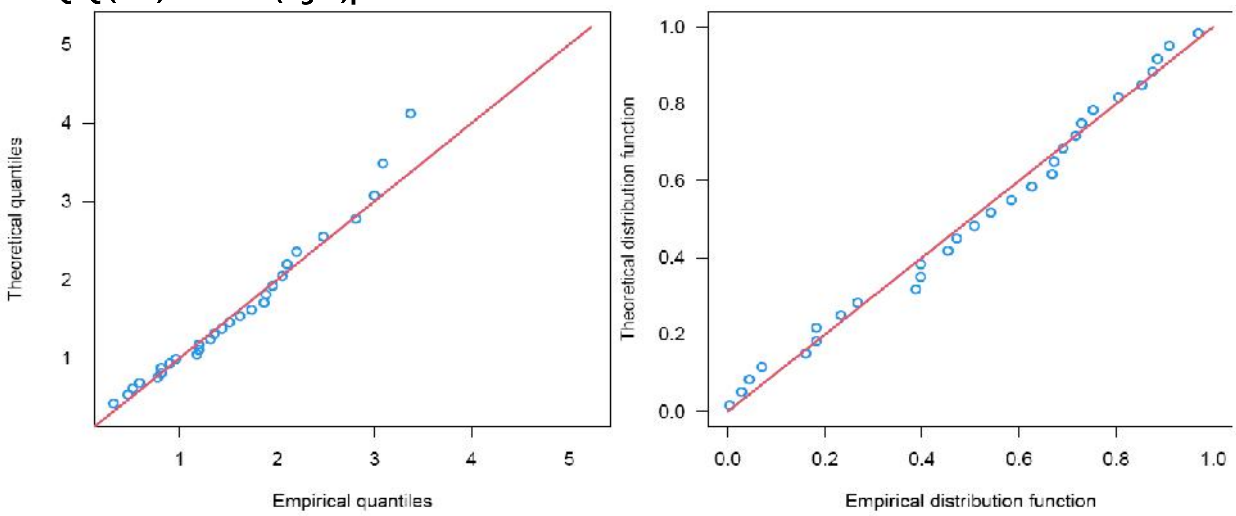
Estimation and testing of validity

Parameters of model is calculated using optim () function provided by R Core Team (2022). Estimated parameters using MLE, LSE and CVME of MEIE model arecalculatedand tabulated in table 2.

Table 2
MLE and SE □ , β and λ of MEIE

Parameter	MLE	LSE	CVME
alpha	0.8645	0.8778	0.9490
beta	0.5695	0.5014	0.5613
lambda	2.3336	2.3380	2.3651

For model validation, P- P plot as well as theQ-Q plotsare used and displayed in the figure 3.
Figure 3
The Q-Q (left) and P-P (right)plots



Log-likelihood, information criteria values and different test statistics as well as p- values are mentioned in Table 3.

Table 3

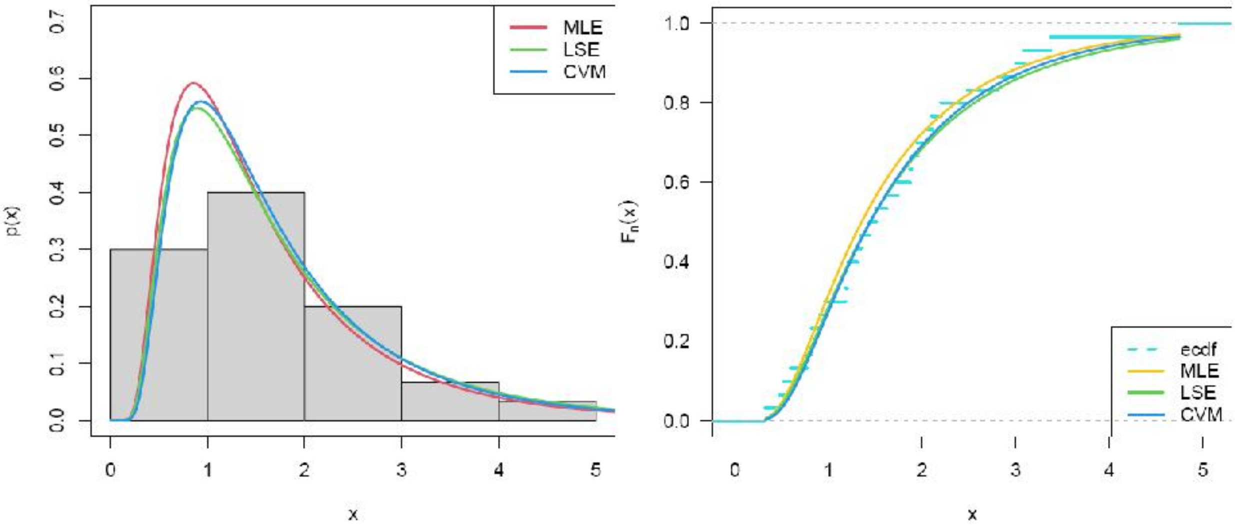
Estimated parameters, log-likelihood, AIC, CAIC, BIC, HQIC, values with KS, An and W statistics

Method	-LL	AIC	CAIC	BIC	HQIC	KS(p-value)	An(p-value)	W(p-value)
MLE	38.8286	83.6571	84.5802	87.8607	85.0019	0.1176(0.811)	0.3182(0.923)	0.0559(0.843)
LSE	38.9956	83.9912	84.9143	88.1949	85.3360	0.0819(0.988)	0.2728(0.957)	0.0296(0.979)
CVE	39.0316	84.0632	84.9863	88.2668	85.4079	0.0750(0.996)	0.2861(0.948)	0.0274(0.988)

In figure 4, histogram versus density curve as well as empirical versus fitted cdf corresponding to applied estimation techniques is displayed.

Figure 4

The fitted density against the histogram (left) as well as ecdf against fitted cdf (right).



Model comparison

In this sub section of study, proposed model is compared with other probability models. Following probability models are chosen to compare the potentiality of the MEIE.

i. Half Logistic Nadarajah Haghighi (HLNHE) Distribution

This is three parameter extension of the exponential distribution with continuous density function (Joshi &Kumar, 2020).

$$f_{HLNHE}(x)=\frac{2\sim\ {}^{a}(1+\sim x)^{(\ \ -1)}\,exp\left({}^{a}\left(1-(1+\sim x)\ \ \right)\right)}{\left[1+exp\left({}^{a}\left(1-(1+\sim x)\ \ \right)\right)\right]^{2}}\qquad ;\sim,\ \ ,{}^{a}>0,x>0$$

ii. A Weighted Inverted Exponential Distribution (WIED)

This is two parameters distribution given by (Hussian, 2013)

$$f(t;{}^{a},\sim)=(1+\sim)\frac{a}{t^{2}}e^{\frac{-a}{t}}(1-e^{\frac{-a}{\sim t}}),t>0,\sim>0,{}^{a}>0$$

iii. Generalized Inverted Generalized Exponential (GIGE)

This is Generalized Inverted Generalized Exponential (GIGE) probability model(Oguntunde et al., 2014)

$$f_{GIGE}(x)=\sim{}^{a}¥\,x^{-2}e^{-¥({}^{a}/x)};\left(1-e^{-¥({}^{a}/x)}\right)^{\sim-1}\ ;\sim,{}^{a},¥>0\,,x>0,$$

iv. Logistic inverse Exponential (LIE) distribution

Logistic inverse exponential distribution is a two parameter univariate continuous distribution(Chaudhary et al., 2020d)

$$f_{LIE}(x)=\frac{\sim a}{x^2}\frac{\exp\{a/x\}\left[\exp\{a/x\}-1\right]^{\sim-1}}{\left[1+\{\exp(a/x)-1\}^{\sim}\right]^2};(\sim,a)>0,x>0$$

Table 4 contains the estimated parameters as well as standard error of estimates of the proposed model and the model considered for the comparisons.

Table 4

Estimated parameters and the standard error of estimates

Models	A	β	λ	γ
MEIE	0.8647 (3.7368)	0.5695 (0.1811)	2.3336 (10.065)	
WIED	2.6782(17.6573)		2.3725(0.4330)	
HLNHE	26.818(18.2725)	1.5259 (0.2273)	0.0036(0.0013)	
GIGE	3.3196(1.0657)		9.8260(96.5594)	0.2261(2.2223)
LIE	1.8792(0.2906)		0.9453(0.1102)	

Table 5 display the model assessment is verified here comparing Akaike, Bayesian, Corrected Akaike and Hannan-Quinn information criteria of MEIE against considered models.

Table 5

Negative oflog-likelihood, AIC, BIC, CAIC and HQIC

Model	-LL	AIC	BIC	CAIC	HQIC
MEIE	38.8286	83.6571	87.8607	84.5802	85.0019
HLNHE	39.1288	84.2577	88.4613	85.1808	85.6025
GIGE	39.6596	85.3192	89.5228	86.2423	86.6640
LIE	40.0598	84.1196	86.9220	84.5641	85.0161
WIED	41.8902	87.7805	90.5829	88.2249	88.6770

In figure 5, histogram versus density curve as well as empirical versus fitted cdf corresponding to competing models is displayed.

Figure 5

The fitted density against the histogram (left) as well as ecdf against fitted cdf (right) for competing models

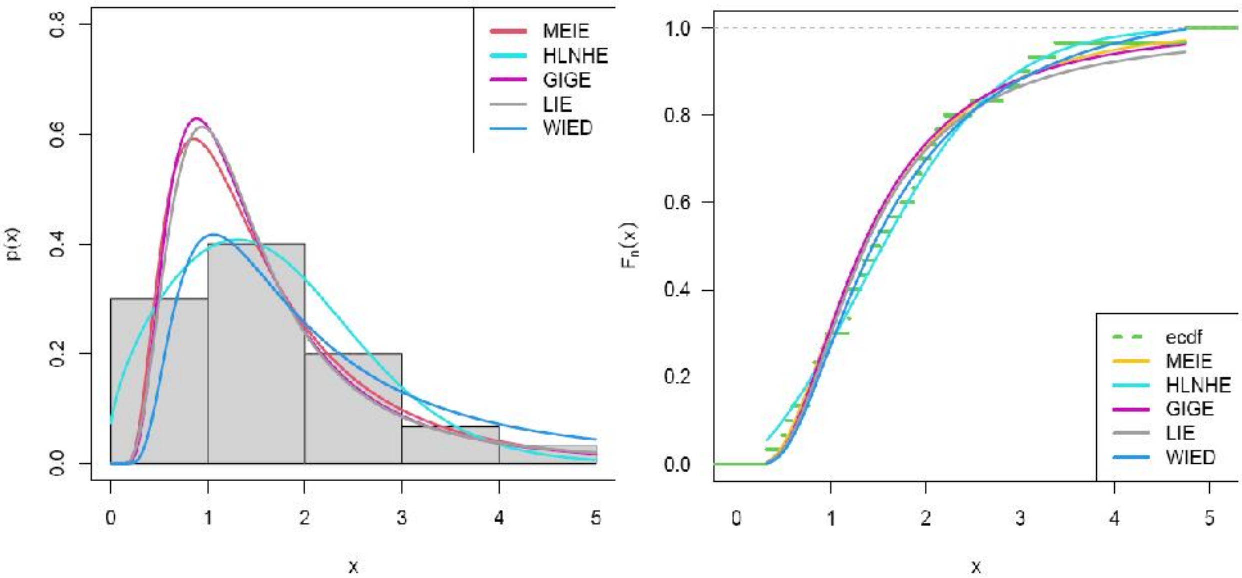


Figure 5: The fitted density against the histogram (left) as well as ecdf against fitted cdf (right) for competing models.

Now the KS, An, and W withcorresponding p-values for different models taken in consideration with MEIE are mentioned in table 6. Result shows that MEI Fits well compared to most of the competing models.

Table 6
KS, An& W statistics havingrespectivevalues of p.

Models	KS (p - value)	An (p - value)	W (p - value)
MEIE	0.0749(0.9959)	0.2861(0.9479)	0.0274(0.9883)
HLNHE	0.0689(0.9988)	0.2131(0.9865)	0.0244(0.9918)
GIGE	0.1218(0.7654)	0.4361(0.8111)	0.0716(0.7454)
LIE	0.1047(0.8976)	0.3987(0.8490)	0.0540(0.8550)
WIED	0.0739(0.9967)	0.3216(0.9206)	0.0267(0.9871)

Conclusion

This study is based on formulation of a model Modified Exponentiated Inverted Exponential Distribution. This presents survival, hazard rate, and quantile functions for certain properties. The density curve model MEIE shows that its shape is of different shape. Testing of applicability of MEIEis done by taking a real data set. Estimation of parameters are done by MLE, LSE and CVM methods. The Q-Q and P-P plots indicate a strong fit of MEIE to real data. Information criteria and validity tests confirm the model's good fit. The hazard graph varies with parameter values, displaying increasing-decreasing patterns. For model comparison, proposed model is compared with four other models. The proposed MEIE model demonstrates superior fit to the data compared to other considered models, as indicated by information criteria and goodness of fit. Moreover, the proposed distribution's adaptability to real data underscores its applicability and flexibility.

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